

Problem of the Week
Problem E and Solution
An Average Mistake

Problem

Students in three different classes wrote the same Calculus exam, which was marked out of 100. One class of 22 students averaged 87%, the second class of 27 students averaged 83%, and the third class of 31 students averaged 81%.

Three students, Abby, Ben, and Cara, discussed their results. Abby obtained a mark one less than Ben and Cara obtained a mark one more than Ben. Upon reviewing their exams, Abby and Ben both discovered addition errors, and their marks were each increased to 92. Cara found an unmarked question, which also raised her mark to 92.

These corrections changed the combined exam average for all students across the three classes to exactly 84%. What marks did Abby, Ben, and Cara originally receive before the errors were corrected?

Solution
Solution 1

Let Ben's original mark be b. Then Abby's original mark is $b-1$ and Cara's original mark is $b+1$.

The total number of marks for a class can be calculated by multiplying the number of students by the class average.

The total number of marks in the first class before the error was discovered was $22 \times 87 = 1914$. The total number of marks in the second class before the error was discovered was $27 \times 83 = 2241$. The total number of marks in the third class before the error was discovered was $31 \times 81 = 2511$.

The total of the marks from the three classes before correcting the errors was $1914 + 2241 + 2511 = 6666$. To correct the errors, we subtract the three incorrect marks from the total and add the three correct marks. The new total is therefore

$$6666 - (b-1) - b - (b+1) + 3 \times 92 = 6666 - 3b + 276 = 6942 - 3b$$

The total number of students in the three classes combined was $22 + 27 + 31 = 80$. Since the average after correcting the three errors was exactly 84%, the total number of marks for the three classes was $80 \times 84 = 6720$.

Thus, $6942 - 3b$ represents the total 6720. Therefore, $6942 - 3b = 6720$. It follows that $3b = 222$ and $b = 74$.

Since $b = 74$, $b - 1 = 73$ and $b + 1 = 75$.

Therefore, Abby's original mark was 73, Ben's original mark was 74 and Cara's original mark was 75.

Solution 2

Let Ben's original mark be b. Then Abby's original mark is $b-1$ and Cara's original mark is $b+1$.

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The total of the marks from the three classes before correcting the errors was $1914 + 2241 + 2511 = 6666$.

The total number of students in the three classes combined was $22 + 27 + 31 = 80$. Since the average after correcting the three errors was exactly 84%, the total number of marks for the three classes was $80 \times 84 = 6720$.

The total mark change is then $6720 - 6666 = 54$.

But the total mark change can also be calculated by subtracting each of Ben's, Abby's, and Cara's old marks from 92, and then adding the three differences. That is,

$$\begin{aligned}
[92-(b-1)]+[92-b]+[92-(b+1)] &= 54 \\
92-b+1+92-b+92-b-1 &= 54 \\
276-3b &= 54 \\
222 &= 3b \\
74 &= b
\end{aligned}$$

Since $b = 74$, $b-1 = 73$ and $b+1 = 75$.

Therefore, Abby's original mark was 73, Ben's original mark was 74 and Cara's original mark was 75.