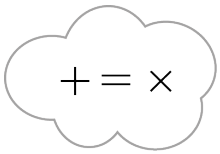

Problem of the Week
Problem E and Solution
A Productive Sum

Problem

When the four positive integers 1, 1, 2, and 4 are multiplied together, the result is 8 since $1 \times 1 \times 2 \times 4 = 8$. When they are added together, the result is also 8 since $1 + 1 + 2 + 4 = 8$.

We can also find a collection of n positive integers, with $n > 1$, that multiply to 2020 and add to 2020. Determine all possible values for n.

Solution

To form such a collection of integers, our strategy is to determine all collections of integers larger than 1 whose product is 2020, and then for each collection add enough 1s to make the sum of the numbers in the collection equal to 2020.

Since we want to consider integers whose product is 2020, we should find the divisors of 2020.

The prime factorization of 2020 is $2 \times 2 \times 5 \times 101$. We can use this factorization to determine all collections of integers greater than 1 that multiply to 2020. The only collection with 4 integers is $2, 2, 5, 101$. The collections with 3 integers are $4, 5, 101$, and $2, 10, 101$, and $2, 5, 202$, and $2, 2, 505$. A systematic count determines that the only collections with 2 numbers are $4, 505$, and $5, 404$, and $10, 202$, and $2, 1010$, and $20, 101$. It is left for the reader to verify this. We will not consider the collection with just 2020, as it is given that the number of integers in the collection is to be greater than 1.

We list these collections in the first column of the table below. We also determine the sum of the integers in this collection, the number of 1s needed (found by subtracting the sum from 2020), and the value of n, which is equal to the number of integers in first column plus the entry in third column.

Collection of integers greater than 1	Sum of integers greater than 1 in collection	Number of 1s needed to sum to 2020	Value of n
$2, 2, 5, 101$	110	1910	$1910 + 4 = 1914$
$4, 5, 101$	110	1910	$1910 + 3 = 1913$
$2, 10, 101$	113	1907	$1907 + 3 = 1910$
$2, 5, 202$	209	1811	$1811 + 3 = 1814$
$2, 2, 505$	509	1511	$1511 + 3 = 1514$
$4, 505$	509	1511	$1511 + 2 = 1513$
$5, 404$	409	1611	$1611 + 2 = 1613$
$10, 202$	212	1808	$1808 + 2 = 1810$
$2, 1010$	1012	1008	$1008 + 2 = 1010$
$20, 101$	121	1899	$1899 + 2 = 1901$

Therefore, the possible values for n are 1010, 1513, 1514, 1613, 1810, 1814, 1901, 1910, 1913, and 1914.