Problem of the Week

Problem E

Mapping the Beats 3

Music streaming services often describe both songs and listeners using numerical characteristics called features. Artificial intelligence systems then use these features to predict what songs a listener is most likely to enjoy. One type of AI system is a neural network. In addition to music recommendation systems, neural networks are also used in image recognition and large language models (LLMs). Although real neural networks contain many hidden layers and millions of parameters, they are built on the same mathematical idea explored in this problem: information is transformed through a sequence of functions to produce a prediction.

Real recommendation systems often use many features at once. Songs and listeners can therefore be represented as points in a multi-dimensional coordinate system called a feature space. Recommendations for a listener can then be made by looking for songs represented by points that are close in distance to the point representing the listener's preferences.

A particular music app represents songs and listeners using three scores: Energy, Danceability, and Happiness. These are expressed as points (E, D, H) where E, D, and H are integers from 1 to 10. Suppose we have songs $Q = (5, 8, 5)$, $R = (8, 5, 2)$, $S = (6, 6, 7)$, $T = (3, 9, 3)$ and listener $P = (6, 7, 4)$.

(a) Use the three-dimensional distance formula $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$ to calculate the distance from P to each song. Which of the four songs would the streaming service recommend for this listener?

(b) Determine the number of points (E, D, H) whose distance from $P = (6, 7, 4)$ is at most 3.

