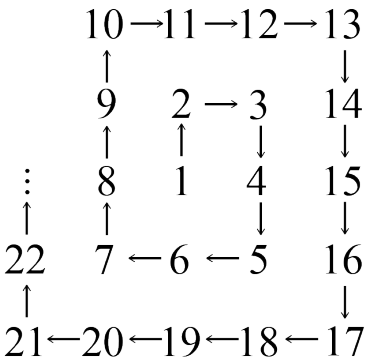
Problem of the Week
Problem E and Solution
The Spiral Sequence Challenge

Problem

A spiral of the positive integers, placed into rows and columns, is created in the following way:
The integer 1 is placed.
Moving up one row, the integer 2 is placed.
Moving right one column, the integer 3 is placed.
Moving down one row, the integer 4 is placed, then moving down one more row, the integer 5 is placed.
Moving left one column, the integer 6 is placed, then moving left one more column, the integer 7 is placed.
Moving up one row, the integer 8 is placed, then moving up one more row, the integer 9 is placed, then moving up one more row, the integer 10 is placed.
Moving right one column, the integer 11 is placed, then moving right one more column the integer 12 is placed, then moving right one more column, the integer 13 is placed.
The pattern continues, alternating moving down across rows, then left across columns, then up across rows, then right across columns to create a spiral shape.



Where do the integers 2024, 2025, and 2026 appear in the spiral?

Solution

Consider the integers 1, 2, 3, and 4 in the spiral. These integers form a 2 integer by 2 integer "square". This square has 1 in the bottom-left corner, 2 in the top-left corner, 3 in the top-right corner, and 4 in the bottom-right corner.

When we consider the integers 1 through 9, we note that they form a second "square". This square is 3 integers by 3 integers, and has the integer 9 in the top-left corner.

$$\begin{array}{ccc} 9 & 2 \rightarrow & 3 \\ \uparrow & \uparrow & \downarrow \\ 8 & 1 & 4 \\ \uparrow & & \downarrow \\ 7 & \leftarrow 6 \leftarrow & 5 \end{array}$$

When we consider the integers 1 through 16, we note that they form a square that is 4 integers by 4 integers, and has the integer 16 in the bottom-right corner.

$$\begin{array}{cccc} 10 \rightarrow & 11 \rightarrow & 12 \rightarrow & 13 \\ \uparrow & & & \downarrow \\ 9 & 2 \rightarrow & 3 & 14 \\ \uparrow & \uparrow & \downarrow & \downarrow \\ 8 & 1 & 4 & 15 \\ \uparrow & & \downarrow & \downarrow \\ 7 & \leftarrow 6 \leftarrow & 5 & 16 \end{array}$$

Continuing in this manner, the integers 1 through 25 form a square that is 5 integers by 5 integers, and has the integer 25 in the top-left corner.

$$\begin{array}{ccccc} 25 & 10 \rightarrow & 11 \rightarrow & 12 \rightarrow & 13 \\ \uparrow & \uparrow & & & \downarrow \\ 24 & 9 & 2 \rightarrow & 3 & 14 \\ \uparrow & \uparrow & \uparrow & \downarrow & \downarrow \\ 23 & 8 & 1 & 4 & 15 \\ \uparrow & \uparrow & & \downarrow & \downarrow \\ 22 & 7 & \leftarrow 6 \leftarrow & 5 & 16 \\ \uparrow & & & & \downarrow \\ 21 & \leftarrow 20 & \leftarrow 19 & \leftarrow 18 & \leftarrow 17 \end{array}$$

This pattern continues, and the next square will have $6 \times 6 = 36$ in the bottom-right corner. The following square will have $7 \times 7 = 49$ in the top-left corner.

In general, the integers from 1 to n^2 will form a square with n rows of integers and n columns of integers. Since each new square is completed by alternating creating a row from right to left and then a column from bottom to top, with creating a row from left to right and then a column from top to bottom, all odd perfect squares are in the top-left corners and all even perfect squares are in the bottom-right corners.

Now, $45^2 = 2025$, so 2025 is an odd perfect square.

Thus, 2025 will be the top-left corner of the 45 integer by 45 integer "square". The integer 2024 will appear below 2025 and the integer 2026 will appear above 2025.