Problem of the Week
Problem E and Solution
Corner Connection

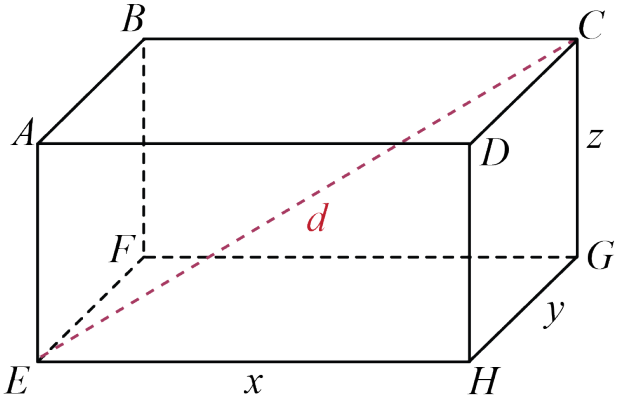
Problem

In rectangular prism $ABCDEFGH$, the sum of the lengths of all of the edges is 28 cm, and the total surface area is 13 cm^2.

What is the length of EC, the diagonal of the prism?

Solution

Let $EH = x$, $HG = y$, and $CG = z$. We construct EC and label it d.



By the Pythagorean Theorem in $\triangle EHG$, $EG^2 = EH^2 + HG^2$.
By the Pythagorean Theorem in $\triangle EGC$, $EC^2 = EG^2 + CG^2$.
Therefore, $EC^2 = EG^2 + CG^2 = EH^2 + HG^2 + CG^2$.
That is, $d^2 = x^2 + y^2 + z^2$.

Since the sum of the lengths of all the edges is 28, then $4x + 4y + 4z = 28$ or $x + y + z = 7$.
Since the surface area of the prism is 13, we know $2xy + 2yz + 2xz = 13$.

Since we have squared terms and pair factor terms it might be helpful to expand $(x + y + z)^2$.

$$\begin{aligned}(x + y + z)^2 &= (x + (y + z))^2 \\ &= x^2 + 2x(y + z) + (y + z)^2 \\ &= x^2 + 2xy + 2xz + y^2 + 2xy + z^2 \\ &= (x^2 + y^2 + z^2) + (2xy + 2xz + 2yz)\end{aligned}$$

Since $x + y + z = 7$, $d^2 = x^2 + y^2 + z^2$, and $2xy + 2yz + 2xz = 13$, we have

$$\begin{aligned}7^2 &= d^2 + 13 \\ d^2 &= 49 - 13 = 36\end{aligned}$$

Since $d > 0$, we have $d = 6$. Therefore, the length of EC is 6 cm.