



## Problem of the Week

### Problem E and Solution

### Coin Bags

#### Problem

Mara has a bag containing 2 gold coins and 2 silver coins. Sharad has a bag containing 2 gold coins, 2 silver coins, and  $b$  bronze coins (where  $b > 1$ ). Mara and Sharad each randomly draw two coins from their bag without replacement. Note that drawing two coins *without replacement* means drawing two coins, one after the other, without putting the first coin back in the bag.

If the probability that the two coins are the same colour is the same for Mara and Sharad, determine the value of  $b$ .

#### Solution

First we consider Mara's bag, which contains a total of  $2 + 2 = 4$  coins. There are 4 possible coins that can be drawn first, leaving 3 possible coins that can be drawn second. This gives a total of  $4 \times 3 = 12$  ways of drawing two coins.

For both coins to be gold, there are 2 possible coins (either gold coin) that can be drawn first, and 1 coin that must be drawn second (the remaining gold coin). This gives a total of  $2 \times 1 = 2$  ways of drawing two gold coins. For both coins to be silver, there are 2 possible coins that can be drawn first, and 1 coin that must be drawn second. This gives a total of  $2 \times 1 = 2$  ways of drawing two silver coins. Therefore, Mara has a total of  $2 + 2 = 4$  ways of drawing two coins of the same colour.

Thus, the probability of Mara drawing two coins of the same colour is  $\frac{4}{12} = \frac{1}{3}$ .

Next we consider Sharad's bag, which contains a total of  $2 + 2 + b = b + 4$  coins. There are  $(b + 4)$  possible coins that can be drawn first, leaving  $(b + 3)$  possible coins that can be drawn second. This gives a total of  $(b + 4)(b + 3)$  ways of drawing two coins.

As with Mara's bag, there are 2 ways of drawing two gold coins and 2 ways of drawing two silver coins. For both coins to be bronze, there are  $b$  possible coins that can be drawn first, and  $(b - 1)$  possible coins that can be drawn second. This gives a total of  $b(b - 1)$  ways of drawing two bronze coins. Therefore, Sharad has a total of  $2 + 2 + b(b - 1) = b^2 - b + 4$  ways of drawing two coins of the same colour.

Thus, the probability of Sharad drawing two coins of the same colour is  $\frac{b^2 - b + 4}{(b + 4)(b + 3)}$ .

Since these probabilities are equal,

$$\begin{aligned}\frac{1}{3} &= \frac{b^2 - b + 4}{(b + 4)(b + 3)} \\ (b + 4)(b + 3) &= 3(b^2 - b + 4) \\ b^2 + 7b + 12 &= 3b^2 - 3b + 12 \\ 0 &= 2b^2 - 10b \\ 0 &= b^2 - 5b \\ 0 &= b(b - 5)\end{aligned}$$

Since  $b > 1$ , it follows that  $b = 5$ .