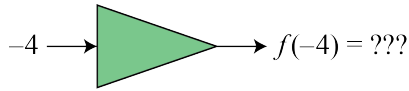




Problem of the Week



Problem E and Solution

How Does it Function?

Problem

The function $f(x) = x^5 - 4x^4 + ax^3 - x^2 + bx - 8$ has a value of 128 when $x = 4$. Determine the value of the function when $x = -4$.

Solution

We know that the function has a value of 128 when $x = 4$. That is, $f(4) = 128$. Then,

$$\begin{aligned} f(4) &= 128 \\ (4)^5 - 4(4)^4 + a(4)^3 - (4)^2 + b(4) - 8 &= 128 \\ 1024 - 1024 + 64a - 16 + 4b - 8 &= 128 \\ 64a + 4b &= 152 \end{aligned}$$

We want to determine the value of the function when $x = -4$. That is, we want to determine $f(-4)$. Then,

$$\begin{aligned} f(-4) &= (-4)^5 - 4(-4)^4 + a(-4)^3 - (-4)^2 + b(-4) - 8 \\ &= -1024 - 1024 - 64a - 16 - 4b - 8 \\ &= -64a - 4b - 2072 \end{aligned}$$

However, we know from earlier that $64a + 4b = 152$. Then,

$$f(-4) = -(64a + 4b) - 2072 = -152 - 2072 = -2224$$

Therefore, the value of the function is -2224 when $x = -4$.

Note that we are not given enough information to determine the values of a and b , however enough information is given to determine the value of $f(-4)$.