Problem of the Week

Problem E

Check Please 3

Debit and credit cards contain account numbers which consist of many digits. When shopping online, customers are often asked to type in their account number. Because there are so many digits, it is easy to make a mistake. The last digit of the number is a specially generated check digit which can be used to verify whether or not the number is valid. A common algorithm used for verifying numbers is called the *Luhn Algorithm*. The steps performed in the Luhn Algorithm are outlined in the flowchart below. Two examples are provided.

Example 1

- Number: 135792
- Reversal: 297531
- Digits in odd positions are underlined: $\underline{2}9\underline{7}5\underline{3}1$

$$\begin{aligned} A &= 2 + 7 + 3 \\ &= 12 \end{aligned}$$

- Double remaining digits:

$$\begin{aligned} 2 \times 9 &= 18 \\ 2 \times 5 &= 10 \\ 2 \times 1 &= 2 \end{aligned}$$

- Calculate B:

$$\begin{aligned} B &= (1 + 8) + (1 + 0) + 2 \\ &= 9 + 1 + 2 \\ &= 12 \end{aligned}$$

- $C = 12 + 12 = 24$
- C does not end in zero, so the number is not valid.

Example 2

- Number: 1357987
- Reversal: 7897531
- Digits in odd positions are underlined: $\underline{7}8\underline{9}7\underline{5}3\underline{1}$

$$\begin{aligned} A &= 7 + 9 + 5 + 1 \\ &= 22 \end{aligned}$$

- Double remaining digits:

$$\begin{aligned} 2 \times 8 &= 16 \\ 2 \times 7 &= 14 \\ 2 \times 3 &= 6 \end{aligned}$$

- Calculate B:

$$\begin{aligned} B &= (1 + 6) + (1 + 4) + 6 \\ &= 7 + 5 + 6 \\ &= 18 \end{aligned}$$

- $C = 22 + 18 = 40$
- C ends in zero, so the number is valid.

The number 2643 $XY5X$ 081 is a valid card number when verified by the Luhn Algorithm, where X and Y are single digits and $X \leq Y$. Determine all possible values of X and Y.