Problem of the Week
Problem E and Solution
Luminous Detail

Problem

A *pixel* is the smallest unit of a digital image.

The number of pixels/cm in each of the horizontal and vertical directions of a digital image affects the quality of the image. The more pixels/cm, the sharper the image will be.

A small monitor has dimensions 15 cm by 10 cm and has 80 pixels/cm in each dimension. The total number of pixels is $(15 \times 80) \times (10 \times 80) = 960\,000$.

The manufacturer wants to build a new monitor with 2 145 624 pixels. To accomplish this, both the length and width of the screen will be increased by $n\%$ and the number of pixels/cm in each dimension will be increased by $2n\%$. Determine the dimensions of the new monitor and the new number of pixels/cm.

Solution

Let $a > 0$ represent the percentage increase, expressed as a decimal, in each dimension of the monitor. Then $2a$ represents the percentage increase, expressed as a decimal, in the number of pixels/cm. (So $n = 100a$ and $2n = 200a$.)

We know that

[(New Length)×(New pixels/cm)]×[(New Width)×(New pixels/cm)] = Total Number of Pixels

Thus,

$$\begin{aligned}
[15(1+a) \times 80(1+2a)] \times [10(1+a) \times 80(1+2a)] &= 2\,145\,624\\
(15)(80)(10)(80)(1+a)^2(1+2a)^2 &= 2\,145\,624\\
960\,000(1+a)^2(1+2a)^2 &= 2\,145\,624\\
(1+a)^2(1+2a)^2 &= 2.235\,025
\end{aligned}$$

Taking the square root of both sides, we have $(1+a)(1+2a) = \pm 1.495$.

Since $a > 0$, we have $(1+a)(1+2a) > 0$. Thus, $(1+a)(1+2a) = 1.495$.

Expanding and simplifying, we have $2a^2 + 3a + 1 = 1.495$, and so $2a^2 + 3a - 0.495 = 0$.

Using the quadratic formula, $a = \dfrac{-3 \pm \sqrt{9 - 4(2)(-0.495)}}{4} = \dfrac{-3 \pm \sqrt{12.96}}{4} = \dfrac{-3 \pm 3.6}{4}$.

It follows that $a = 0.15$ or $a = -1.65$. Since we are looking for a percentage increase, then we must have $a > 0$ and so $a = -1.65$ is inadmissible.

Therefore, the dimensions of the screen must each be increased by 15% and the numbers of pixels/cm must be increased by $2 \times 15\% = 30\%$.

The increased length is $15 \times 1.15 = 17.25$ cm and the increased width is $10 \times 1.15 = 11.5$ cm. The increased number of pixels/cm is $80 \times 1.3 = 104$ pixels/cm.