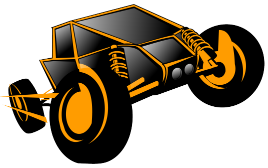


Problem of the Week
Problem E and Solution
Straight Ahead

Problem

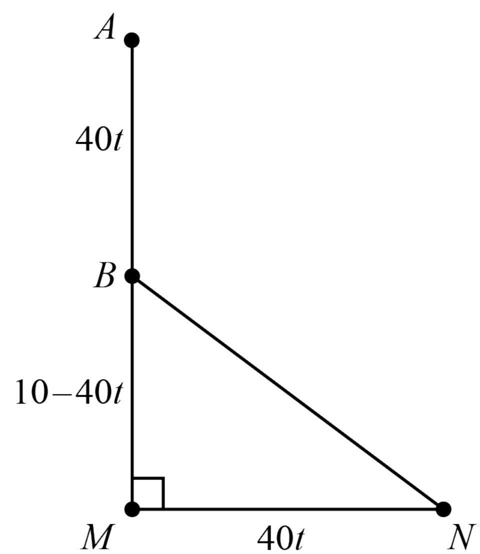
Albertino and Mara are driving their remote control cars in a large field. Albertino's car starts 10 km north of Mara's car. At the same time, Albertino starts driving his car south and Mara start's driving her car east. Both cars travel at a constant speed of 40 km/hr.

After how many minutes of driving will the **total** distance travelled by the cars be equal to the distance between the cars?

Solution

Let t be the time in hours until the total distance travelled by the cars is equal to the distance between the cars. Since each car is travelling at 40 km/hr, the distance traveled by each car in t hours is $40t$ km. We will now consider three cases: first when $40t < 10$, then when $40t = 10$, and finally when $40t > 10$.

- **Case 1:** $40t < 10$
  In this case, at time t, Albertino's car had not yet reached the initial position of Mara's car. The diagram shows the initial positions of Albertino's car, A, and Mara's car, M, as well as their positions after t hours of driving, which are represented as B and N, respectively. Point B is $40t$ km south of point A, and point N is $40t$ km east of point M. Since the initial distance between the cars, AM, is 10 km, it follows that $BM = (10 - 40t)$ km.



  Since each car travelled $40t$ km in t hours, the total distance travelled by the cars is $2 \times 40t = 80t$ km. Thus, we want to determine the value of t when $BN = 80t$. Since $\triangle BMN$ is a right-angled triangle, we can use the Pythagorean Theorem.

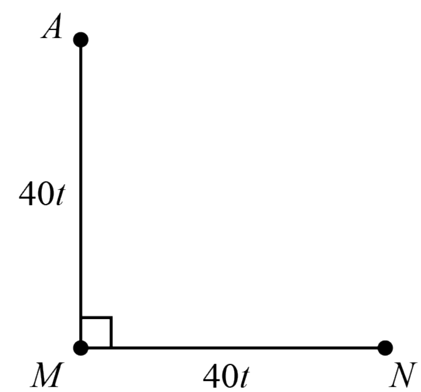
$$BN^2 = BM^2 + MN^2$$
$$(80t)^2 = (10 - 40t)^2 + (40t)^2$$
$$6400t^2 = 100 - 800t + 1600t^2 + 1600t^2$$
$$3200t^2 + 800t - 100 = 0$$
$$32t^2 + 8t - 1 = 0$$

Using the quadratic formula,

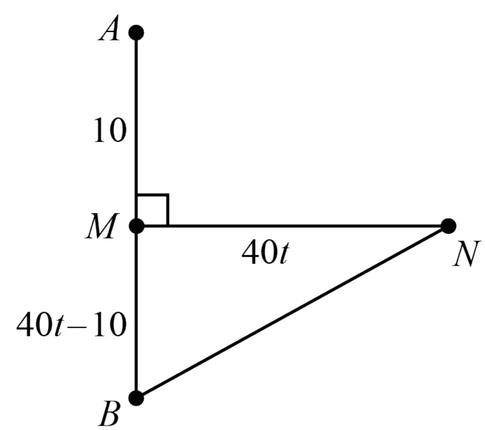
$$t = \frac{-8 \pm \sqrt{8^2 - 4(32)(-1)}}{2(32)} = \frac{-8 \pm \sqrt{192}}{64} = \frac{-1 \pm \sqrt{3}}{8}$$

Since $t > 0$, $t = \frac{-1+\sqrt{3}}{8} \approx 0.09150635$ hours. This is equivalent to approximately 5.49 minutes, or 5 minutes and 29 seconds.

- **Case 2:** $40t = 10$
  In this case, at time t, Albertino's car was at the initial position of Mara's car. So the distance between the cars is the distance that Mara's car travelled, which is $40t$ km. As before, the total distance travelled by the cars is $80t$ km. If these values are equal, then $40t = 80t$. The only solution to this is $t = 0$, but since $t > 0$, this solution is inadmissable. Thus, it is not possible that $40t = 10$.



- **Case 3:** $40t > 10$
  In this case, at time t, Albertino's car had travelled past the initial position of Mara's car. Since the distance between the initial positions of the cars is 10 km, and Albertino's car travelled $40t$ km, it follows that his car travelled $40t - 10$ km past the initial position of Mara's car.



  As before, the total distance travelled by the cars is $80t$ km. If this is equal to the distance between the cars, then $BN = 80t$. Then, in $\triangle BMN$, $BM + MN = 40t - 10 + 40t = 80t - 10 < 80t = BN$. Therefore, this does not satisfy the triangle inequality, so there is no solution for t. Thus, it is not possible that $40t > 10$.

Therefore there is only one solution. After approximately 5.49 minutes, or 5 minutes and 29 seconds, the total distance travelled by the cars will be equal to the distance between the cars.