

Problem of the Week
Problem D and Solution
Random Number Threads 2

Problem

Travis creates t-shirts printed with three random numbers. The first number is a random integer from 1 to 22, inclusive, printed in green. The second number is a random integer from 1 to 15, inclusive, printed in pink. The third number is a random integer from 1 to 10, inclusive, printed in blue.

One of Travis' t-shirts is randomly chosen. What is the probability that at least two of the random numbers on this t-shirt are 5?

Solution

Solution 1

There are 22 different possibilities for the green number, 15 different possibilities for the pink number, and 10 different possibilities for the blue number. That means there is a total of $22 \times 15 \times 10 = 3300$ different t-shirts that could be printed.

To count the number of t-shirts with at least two numbers that are 5, we will consider cases.

- Case 1: The green, pink, and blue numbers are all 5.
  There is only 1 such t-shirt.
- Case 2: The green and pink numbers are 5 and the blue number is *not* 5.
  Since the blue number is a random integer from 1 to 10, there are 9 possibilities that are *not* 5. Thus, there are 9 possible t-shirts.
- Case 3: The green and blue numbers are 5 and the pink number is *not* 5.
  Since the pink number is a random integer from 1 to 15, there are 14 possibilities that are *not* 5. Thus, there are 14 possible t-shirts.
- Case 4: The blue and pink numbers are 5 and the green number is *not* 5.
  Since the green number is a random integer from 1 to 22, there are 21 possibilities that are *not* 5. Thus, there are 21 possible t-shirts.

The total number of possible t-shirts is then $1 + 9 + 14 + 21 = 45$. The probability of randomly choosing one of these t-shirts is $\frac{45}{3300} = \frac{3}{220} \approx 1.36\%$.

Therefore, the probability of randomly choosing a t-shirt with at least two numbers that are 5 is $\frac{3}{220}$, or approximately 1.36%.

Solution 2

This solution uses a known result from probability theory. If the probability of event A occurring is a, the probability of event B occurring is b, the probability of event C occurring is c, and the results are not dependent on each other, then the probability of all three events happening is $a \times b \times c$.

The probability of a specific green number being printed on a t-shirt is $\frac{1}{22}$ and the probability of any specific green number *not* being printed on a t-shirt is $\frac{21}{22}$.

The probability of a specific pink number being printed on a t-shirt is $\frac{1}{15}$ and the probability of any specific pink number *not* being printed on a t-shirt is $\frac{14}{15}$.

The probability of a specific blue number being printed on a t-shirt is $\frac{1}{10}$ and the probability of any specific blue number *not* being printed on a t-shirt is $\frac{9}{10}$.

In the following we will use $P(g, p, b)$ to mean the probability of the green number being g, the pink number being p, and the blue number being b on any randomly-selected t-shirt.

Probability of randomly-selecting a t-shirt with at least two numbers that are 5

$$\begin{aligned}
&= P(5,5,5) + P(5,5,\textit{not } 5) + P(5,\textit{not } 5,5) + P(\textit{not } 5,5,5) \\
&= \left(\frac{1}{22} \times \frac{1}{15} \times \frac{1}{10}\right) + \left(\frac{1}{22} \times \frac{1}{15} \times \frac{9}{10}\right) + \left(\frac{1}{22} \times \frac{14}{15} \times \frac{1}{10}\right) + \left(\frac{21}{22} \times \frac{1}{15} \times \frac{1}{10}\right) \\
&= \frac{1}{3300} + \frac{9}{3300} + \frac{14}{3300} + \frac{21}{3300} \\
&= \frac{45}{3300} \\
&= \frac{3}{220} \approx 1.36\%
\end{aligned}$$

Therefore, the probability of randomly choosing a t-shirt with at least two numbers that are 5 is $\frac{3}{220}$, or approximately 1.36%.