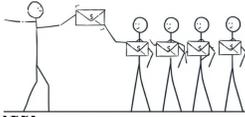




Problem of the Week

Problem D and Solution

Fair Split Collective



Problem

A team of employees completed a large project. They were recognized with a monetary bonus to share among themselves. The ages of the team members are consecutive integers and no one on the team has the same age. The oldest team member is 45.

The bonus was paid out as follows:

- (i) \$1000 to the oldest member of the project team plus $\frac{1}{10}$ of what remains, then
- (ii) \$2000 to the second oldest member of the project team plus $\frac{1}{10}$ of what then remains, then
- (iii) \$3000 to the third oldest member of the project team plus $\frac{1}{10}$ of what then remains, and so on.

After all of the bonus money had been distributed, each member of the project team had received the same amount and no money was left over. What is the age of the youngest member of the project team?

Solution

Solution 1

Let x represent the amount that each team member receives.

Let y represent the total amount of the bonus.

Then $y \div x$ is the number of team members.

The first team member gets \$1000 plus one-tenth of the remainder. Therefore,

$$x = 1000 + \frac{1}{10}(y - 1000)$$

Multiplying both sides by 10,

$$10x = 10\,000 + y - 1000$$

Simplifying and solving for y , we have

$$y = 10x - 9000$$

The second team member gets \$2000 plus one-tenth of the remainder after the first team member's share and \$2000 is removed. Therefore,

$$x = 2000 + \frac{1}{10}(y - x - 2000)$$

Multiplying both sides by 10,

$$10x = 20\,000 + y - x - 2000$$



Simplifying and solving for y , we have

$$y = 11x - 18\,000$$

Since $y = 10x - 9000$ and $y = 11x - 18\,000$, we have

$$\begin{aligned}10x - 9000 &= 11x - 18\,000 \\ x &= 9000\end{aligned}$$

Therefore, $y = 10x - 9000 = 10(9000) - 9000 = 81\,000$.

Therefore, each team member receives \$9000 and the total bonus is \$81 000. The number of team members is $y \div x = 81\,000 \div 9000 = 9$. The oldest team member is 45 and the ages of the team members are consecutive integers, so the youngest team member is 37. The ages of the team members are 37, 38, 39, 40, 41, 42, 43, 44, 45.

Solution 2

Let n represent the number of team members.

Team member n (the youngest team member) receives $n \times \$1000$ or $1000n$.

So $1000n$ is $\frac{9}{10}$ of what remains after team member $(n - 1)$ is given $(n - 1) \times 1000$.

Therefore, $\frac{1}{10}$ of what remains is $\frac{1000n}{9}$.

Then team member $(n - 1)$ receives $1000(n - 1) + \frac{1000n}{9}$.

But team member $(n - 1)$ and team member n each receive the same amount.

Therefore,

$$\begin{aligned}1000(n - 1) + \frac{1000n}{9} &= 1000n \\ 1000n - 1000 + \frac{1000n}{9} &= 1000n \\ \frac{1000n}{9} &= 1000 \\ 1000n &= 9000 \\ n &= 9\end{aligned}$$

Therefore, there are 9 team members and each receives $\$1000n = \$1000(9) = \$9000$. Since the oldest team member is 45 and the ages of the team members are consecutive integers, the youngest team member is 37.