Problem of the Week
Problem D and Solution
Coin Game

Problem

Subhash and Marijn take turns tossing a fair coin. Subhash goes first and each player has a total of two turns. The first player to toss a tail wins. If neither player tosses a tail, then neither player wins. What is the probability that Subhash wins?

Solution

Subhash can win on either his first or second turn. We will calculate the probability for each case. Note that the probability of a specific toss on any turn is $\frac{1}{2}$, since the coin is fair.

- **Case 1:** Subhash wins on his first turn.
  If Subhash wins on his first turn, then he must have tossed tails on this turn. The probability of this is $\frac{1}{2}$.
- **Case 2:** Subhash wins on his second turn.
  If Subhash wins on his second turn, then he must have tossed heads on his first turn, then Marijn must have tossed heads, and then Subhash must have tossed tails. We will call this sequence HHT. Note that this sequence is the only sequence that allows Subhash to win on his second turn. The probability of this sequence of tosses is $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$.

Therefore, the probability that Subhash wins is $\frac{1}{2} + \frac{1}{8} = \frac{4}{8} + \frac{1}{8} = \frac{5}{8} = 62.5\%$.