Problem of the Week
## Problem D and Solution
Walking Away

Problem

Anna, Biljana, and Cees stand in a row in a large field. Anna is 180 m west of Biljana and Cees is 450 m east of Biljana. At the same time, Anna and Cees begin walking. Anna walks north at a constant rate of 75 m/min and Cees walks south at a constant rate of 90 m/min. Biljana does not move.

After how many minutes of walking will the distance between Biljana and Cees be twice the distance between Anna and Biljana?

Solution

Solution 1

Let t represent the number of minutes until the distance between Biljana and Cees is twice the distance between Anna and Biljana. In t minutes Anna will walk $75t$ m north and Cees will walk $90t$ m south. The following diagram shows Anna's position, A, Biljana's position, B, and Cees's position, C, in metres, at time $t > 0$.
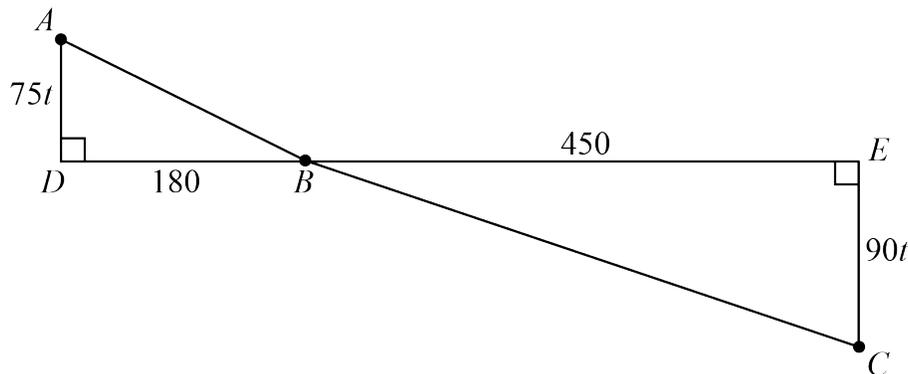
Since both triangles in the diagram are right-angled triangles, we can use the Pythagorean Theorem to determine the value of t when $BC = 2AB$.
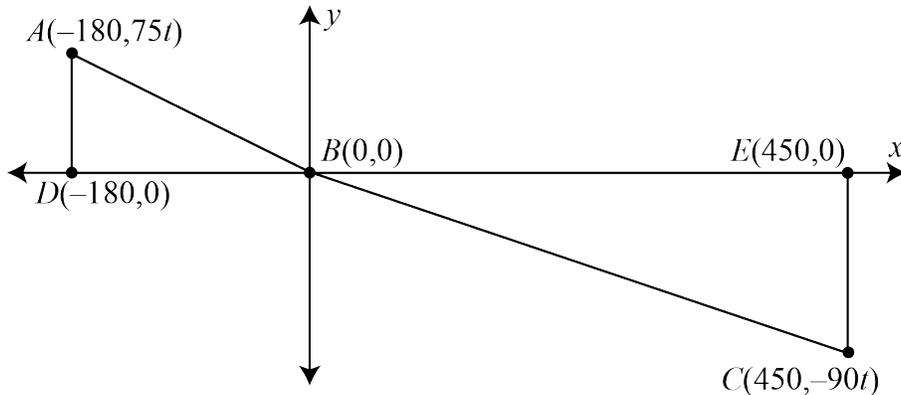
$$BC = 2AB$$
$$(BC)^2 = (2AB)^2$$
$$BE^2 + EC^2 = 4(AD^2 + DB^2)$$
$$450^2 + (90t)^2 = 4((75t)^2 + 180^2)$$
$$202\,500 + 8100t^2 = 4(5625t^2 + 32\,400)$$
$$202\,500 + 8100t^2 = 22\,500t^2 + 129\,600$$
$$72\,900 = 14\,400t^2$$
$$\frac{81}{16} = t^2$$

Since $t > 0$, it follows that $t = \frac{9}{4} = 2\frac{1}{4}$ min. Therefore, after $2\frac{1}{4}$ minutes of walking, the distance between Biljana and Cees will be twice the distance between Anna and Biljana.

Solution 2

In this solution, we represent Anna, Biljana and Cees's original positions as points on the x-axis so that Biljana is positioned at the origin $B(0,0)$, Anna is positioned 180 units left of Biljana at $D(-180,0)$ and Cees is positioned 450 units right of Biljana at $E(450,0)$.

Let t represent the number of minutes until the distance between Biljana and Cees is twice the distance between Anna and Biljana. In t minutes Anna will walk $75t$ m north to the point $A(-180, 75t)$, and Cees will walk $90t$ m south to the point $C(450, -90t)$.

The distance from a point $P(x, y)$ to the origin can be found using the formula $d = \sqrt{x^2 + y^2}$.

Then $AB = \sqrt{(-180)^2 + (75t)^2} = \sqrt{32\,400 + 5625t^2}$ and
$BC = \sqrt{(450)^2 + (-90t)^2} = \sqrt{202\,500 + 8100t^2}$.

$$BC = 2AB$$
$$\sqrt{202\,500 + 8100t^2} = 2\sqrt{32\,400 + 5625t^2}$$
$$(\sqrt{202\,500 + 8100t^2})^2 = (2\sqrt{32\,400 + 5625t^2})^2$$
$$202\,500 + 8100t^2 = 4(32\,400 + 5625t^2)$$
$$202\,500 + 8100t^2 = 129\,600 + 22\,500t^2$$
$$72\,900 = 14\,400t^2$$
$$\frac{81}{16} = t^2$$

Since $t > 0$, it follows that $t = \frac{9}{4} = 2\frac{1}{4}$ min. Therefore, after $2\frac{1}{4}$ minutes of walking, the distance between Biljana and Cees will be twice the distance between Anna and Biljana.