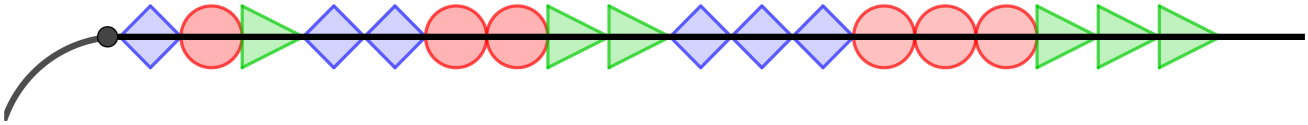
Problem of the Week
Problem C and Solution
Shape Sequence

Problem

A necklace is to be created that contains only square, circular, and triangular beads. A total of 180 beads will be strung on the necklace as follows. First, 1 square, then 1 circular, and then 1 triangular bead are strung on the necklace. Then, 2 square, then 2 circular, and then 2 triangular beads are strung on the necklace. This pattern continues, with each time there being one more of each type of bead strung on the necklace. So, next there are 3 square, then 3 circular, and then 3 triangular beads strung on the necklace, then 4 square, then 4 circular, and then 4 triangular beads, and so on.

Once the necklace is completed, how many of each bead shape will be on the necklace?



Solution

An equal number of square, circular and triangular beads occur after $3(1) = 3$ beads are strung, then after $3(1) + 3(2) = 9$ beads are strung, then after $3(1) + 3(2) + 3(3) = 3 + 6 + 9 = 18$ beads are strung, and so on.

The greatest total number of beads that can be strung while maintaining equal numbers of squares, circles and triangles is:

$$\begin{aligned} &3(1) + 3(2) + 3(3) + 3(4) + 3(5) + 3(6) + 3(7) + 3(8) + 3(9) + 3(10) \\ &\quad = 3 + 6 + 9 + 12 + 15 + 18 + 21 + 24 + 27 + 30 \\ &\quad = 165 \end{aligned}$$

At this point, there are $165 \div 3 = 55$ of each of the three shapes. The final group of 30 beads strung are 10 squares, 10 circles and 10 triangles, in that order.

The next group would contain 11 of each shape if all of the beads could be strung. However, there are only $180 - 165 = 15$ beads remaining to be strung. The first 11 beads in this group are squares, leaving 4 beads to be strung. At this point there are

$$55 + 11 = 66$$

square beads on the necklace. The final 4 beads strung are circles, giving a total of

$$55 + 4 = 59$$

circular beads on the necklace. No more triangular beads can be added, so the total number of triangular beads remains at 55.

Therefore, the completed necklace will contain 66 square beads, 59 circular beads, and 55 triangular beads.