



Problem of the Week

Problem C and Solution

The Lucky Lineup

Problem

A middle school basketball team has two types of spare jerseys left in the equipment bag: some have the number 3 on them, and others have the number 6 on them. The bag contains more than five jerseys of each type. During practice, five players line up side-by-side for a photo. Each player puts on one jersey, creating a 5-digit number when read from left to right. The coach wants this 5-digit number to be a “lucky number”, which means it must be divisible by 9.

How many different 5-digit “lucky numbers” can the players form using only the spare jerseys from the equipment bag?

Solution

In this solution we look at cases based on the number of 3s and 6s in the five-digit number.

- **Case 1:** The number contains five 3s and no 6s.
In this case the digit sum is $5 \times 3 + 0 \times 6 = 15$. Since 15 is not divisible by 9, no five-digit number containing this combination of digits is divisible by 9.
- **Case 2:** The number contains four 3s and one 6.
In this case the digit sum is $4 \times 3 + 1 \times 6 = 18$. Since 18 is divisible by 9, any five-digit number containing this combination of digits is divisible by 9. The possible numbers are 33 336, 33 363, 33 633, 36 333, and 63 333. There are 5 such numbers.
- **Case 3:** The number contains three 3s and two 6s.
In this case the digit sum is $3 \times 3 + 2 \times 6 = 21$. Since 21 is not divisible by 9, no five-digit number containing this combination of digits is divisible by 9.
- **Case 4:** The number contains two 3s and three 6s.
In this case the digit sum is $2 \times 3 + 3 \times 6 = 24$. Since 24 is not divisible by 9, no five-digit number containing this combination of digits is divisible by 9.
- **Case 5:** The number contains one 3 and four 6s.
In this case the digit sum is $1 \times 3 + 4 \times 6 = 27$. Since 27 is divisible by 9, any five-digit number containing this combination of digits is divisible by 9. The possible numbers are 66 663, 66 636, 66 366, 63 666, and 36 666. There are 5 such numbers.
- **Case 6:** The number contains no 3s and five 6s.
In this case the digit sum is $0 \times 3 + 5 \times 6 = 30$. Since 30 is not divisible by 9, no five-digit number containing this combination of digits is divisible by 9.

Combining all of the above cases, there are $5 + 5 = 10$ five-digit numbers that can be formed from the given digits which are divisible by 9. Therefore, there are 10 different “lucky numbers” that the players can form. The 10 numbers are 33 336, 33 363, 33 633, 36 333, 63 333, 66 663, 66 636, 66 366, 63 666, and 36 666.