

Problem of the Week
Problem C and Solution
Random Number Threads 1

Problem

Travis creates t-shirts printed with two random numbers. Each number is a random integer from 1 to 10, inclusive. The first number is printed in green and the second number is printed in pink.

One of Travis' t-shirts is randomly chosen and the two numbers are multiplied together. What is the probability that the ones digit of this product is 0?

Solution

If the ones digit of the product is 0, then the product must be both even and divisible by 5. Since the numbers are each random integers from 1 to 10, the only possibilities that are divisible by 5 are 5 and 10. This gives us two cases.

- Case 1: One of the numbers is 5.
  In this case, the other number must be even, since 5 is odd. The possible pairs of numbers are then $(5, 2)$, $(5, 4)$, $(5, 6)$, $(5, 8)$, $(5, 10)$, $(2, 5)$, $(4, 5)$, $(6, 5)$, $(8, 5)$, and $(10, 5)$, where the first number in each pair represents the green number and the second number represents the pink number. This is 10 possibilities.

- Case 2: One of the numbers is 10.
  In this case, the other number can be anything, since 10 is both even and divisible by 5. However we note that we already counted $(5, 10)$ and $(10, 5)$ in the previous case, so we won't count them again here. The possible pairs of numbers are then $(10, 1)$, $(10, 2)$, $(10, 3)$, $(10, 4)$, $(10, 6)$, $(10, 7)$, $(10, 8)$, $(10, 9)$, $(10, 10)$, $(1, 10)$, $(2, 10)$, $(3, 10)$, $(4, 10)$, $(6, 10)$, $(7, 10)$, $(8, 10)$, and $(9, 10)$. This is 17 possibilities.

Therefore, there are $10 + 17 = 27$ different t-shirts whose numbers have a product with a ones digit of 0.

In total, there are 10 possibilities for the green number and for each of these, there are 10 possibilities for the pink number. This is $10 \times 10 = 100$ different t-shirts in total that could be printed. Therefore, the probability that the numbers on the randomly-chosen t-shirt have a product with a ones digit of 0 is $\frac{27}{100} = 27\%$.