



Problem of the Week

Problem C and Solution

Pin Pals

Problem

There are 9 people in the Pin Pals bowling club. After a game one night, the scores for each player were 157, 153, 159, 161, 153, 151, 156, 157, and 154. During the game, the players were split between two lanes: Lane 1 and Lane 2. The following information is known about the scores of the players in each of the lanes.

- No player in Lane 1 scored higher than any player in Lane 2. However, one player in Lane 1 had the same score as a player in Lane 2.
- Two of the players in Lane 1 had the same score.

Determine the difference between the mean score of the players in Lane 1 and the mean score of the players in Lane 2.

This problem was inspired by a past [Beaver Computing Challenge \(BCC\)](#) problem.

Solution

We start by placing the scores in ascending order:

151, 153, 153, 154, 156, 157, 157, 159, 161

Since no player in Lane 1 scored higher than any player in Lane 2, the scores for the players in Lane 1 will be the numbers on the left of this list, and the scores for the players in Lane 2 will be the numbers on the right. We need to determine where in this list the scores change from Lane 1 to Lane 2.

We are told that one player in Lane 1 had the same score as a player in Lane 2. There are two possibilities for this. Either these players both had scores of 153 or they both had scores of 157.

Similarly, we are told two of the players in Lane 1 had the same score. Then either these players both had scores of 153 or they both had scores of 157. We will consider these two cases.

- If two players in Lane 1 had scores of 157, then since $153 < 157$, the two players with scores of 153 must also both have been in Lane 1. However this contradicts the fact that one player in Lane 1 had the same score as a player in Lane 2. So two players in Lane 1 could not have had scores of 157.
- If two players in Lane 1 had scores of 153, then it's possible one of the players with a score of 157 was in Lane 1 and the other was in Lane 2. In this case, the players in Lane 1 scored 151, 153, 153, 154, 156, and 157. Then, the players in Lane 2 scored 157, 159, and 161. Since there are no other cases, this must be the solution.

The mean score for a player in Lane 1 is $\frac{151+153+153+154+156+157}{6} = \frac{924}{6} = 154$.

The mean score for a player in Lane 2 is $\frac{157+159+161}{3} = \frac{477}{3} = 159$.

Therefore, the difference between the mean score of the players in Lane 1 and the mean score of the players in Lane 2 is $159 - 154 = 5$.