Problem of the Week
Problem C and Solution
Mapping the Beats 1

Problem

Streaming services often use artificial intelligence to recommend music. One way to do this is to represent song attributes and a listener's preferences using numbers or points on a grid. Recommendations for a listener can then be made by looking for songs represented by points that are close in distance to the point representing the listener's preferences.

A particular music app represents song attributes and listener preferences using two scores: Energy and Happiness, which are each integers from 1 to 10. Suppose that three songs are represented by the following points:

Song	(Energy, Happiness)
A	$(5, 5)$
B	$(4, 3)$
C	$(2, 8)$

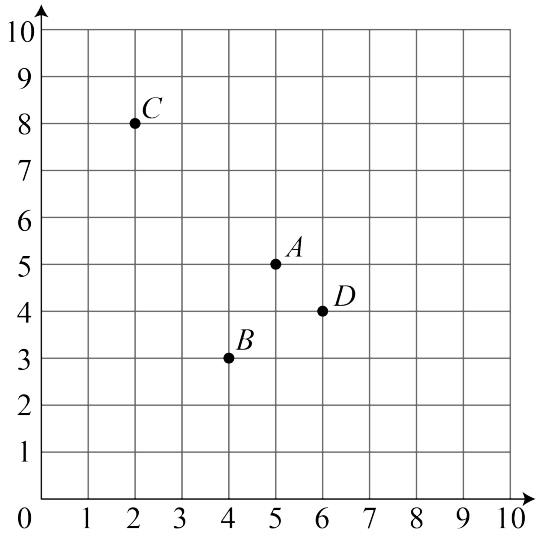
(a) Plot each song as a point on a coordinate grid. Denise's score is represented as $(6, 4)$. What song do you think the music app will recommend to Denise? Why?

(b) Philip's score is $(3, 9)$. Determine the Energy and Happiness scores for a song that both Denise and Philip might enjoy. Explain your reasoning.

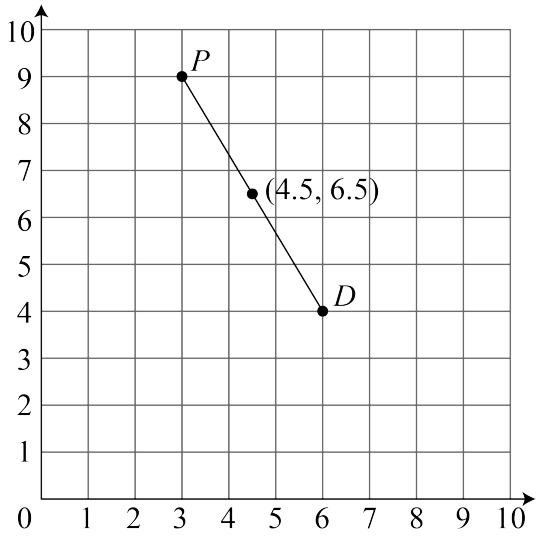
(c) After an app update, a song is recommended if both its Energy and Happiness scores differ from the listener's scores by at most 2. How many different scores of (E, H) could there be in the songs recommended to Denise?

Solution

(a) Songs A, B, and C, as well as Denise's score, D, are plotted on the coordinate grid. The music app will recommend the song that is closest to D on the grid. Point C is clearly the farthest away from D, so would not be recommended to Denise. From D to A we move up one unit and left one unit. From D to B we move down one unit and left two units. Thus, A is closer than B to D. So the music app would likely recommend song A to Denise.



(b) We start by plotting Philip's score, P, and Denise's score, D, on the grid. If Philip and Denise both enjoy a song, then it's likely close to each of them. We draw a line connecting points P and D and determine that the point halfway between P and D along this line is $(4.5, 6.5)$. However since the Energy and Happiness scores must be integers, this point cannot represent a song. Points with integer coordinates close to this point are $(4, 7)$, $(5, 7)$, $(5, 6)$, and $(4, 6)$. These are reasonable guesses for the scores of a song that both Denise and Philip might enjoy.



(c) Denise's score is $(6, 4)$. Since the Energy and Happiness scores of a recommended song can differ from the listener's score by at most 2, the Energy score for a recommended song could be 4, 5, 6, 7, or 8. The Happiness score for a recommended song could be 2, 3, 4, 5, or 6. So for each of the Energy scores, there are 5 possible Happiness scores. Since there are 5 possible Energy scores, it follows that there are $5 \times 5 = 25$ possible scores of (E, H) in the songs recommended to Denise.