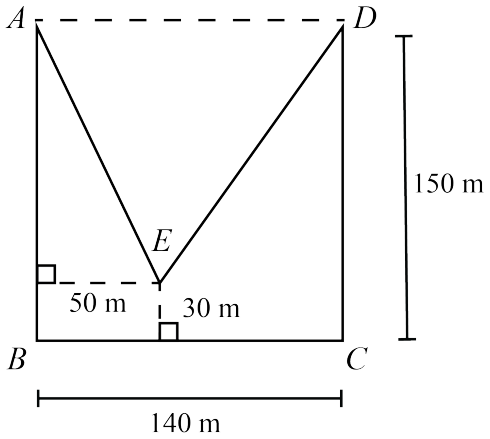
Problem of the Week
Problem C and Solution
Cache Circuit

Problem

Maria is setting up a geocaching course. She begins at point A, then walks south 150 m to point B, then walks east 140 m to point C, and then walks north 150 m to point D.

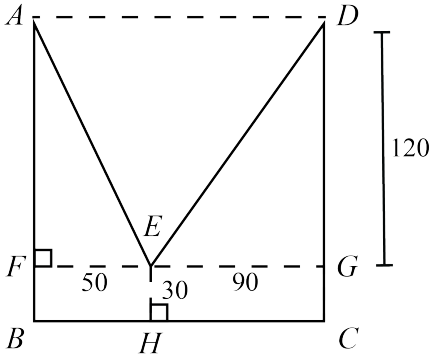
She hides a final cache at point E, which is located 50 m east of path AB and 30 m north of path BC, as shown in the diagram.

Determine the distance from A to E, the distance from D to E, and the total distance around the five-sided route $ABCDEA$.



Solution

Let F be the point on AB with $FE = 50$ m. Let H be the point on BC with $EH = 30$ m.
Extend EF to G on CD.
Since $ABCD$ is a rectangle and FG is perpendicular to AB, then FG is perpendicular to CD and $FGCB$ is a rectangle.
Therefore, $FB = EH = GC = 30$ m.
Also, $EG = BC - FE = 140 - 50 = 90$ m, and
$DG = AF = AB - FB = 150 - 30 = 120$ m.



Since $\triangle AFE$ and $\triangle DGE$ are right-angled triangles, we can use the Pythagorean Theorem to determine the lengths of AE and DE.

In $\triangle AFE$, $AE^2 = AF^2 + FE^2 = 120^2 + 50^2 = 14\,400 + 2500 = 16\,900$.
Thus, $AE = 130$, since $AE > 0$.

In $\triangle DGE$, $DE^2 = DG^2 + EG^2 = 120^2 + 90^2 = 14\,400 + 8100 = 22\,500$.
Thus, $DE = 150$, since $DE > 0$.

Therefore, $AE = 130$ m and $DE = 150$ m.

Also, the perimeter of pentagon $ABCDE$ is equal to

$AB + BC + CD + DE + AE = 150 + 140 + 150 + 150 + 130 = 720$ m.