Problem of the Week
Problem C and Solution
Music Ensemble

Problem

Three students, Kaila, Octavia, and Sophia, formed a trio for a local music competition and won a cash award. They decided to split the award as follows:

- Kaila received \$100 plus $\frac{1}{5}$ of what then remained;
- Octavia then received \$160 plus $\frac{1}{4}$ of what then remained; and
- Sophia then received the rest, which was \$180.

How much was the original cash award? Which student received the most money?

Solution

We will start with Sophia, then determine what Octavia received, and then Kaila.

After Kaila got her share and Octavia received \$160, Octavia then received $\frac{1}{4}$ of what remained. Thus, what is left for Sophia is $\frac{3}{4}$ of what remained.
It follows that \$180 is $\frac{3}{4}$ of what remained.

If $\frac{3}{4}$ of the remainder is \$180, then $\frac{1}{4}$ of the remainder is $\$180 \div 3 = \$60$.

Therefore, just after Octavia received \$160, there is $\$60 + \$180 = \$240$ left.
Also, Octavia receives $\$160 + \$60 = \$220$.
Therefore, before Octavia receives any money there is $\$240 + \$160 = \$400$.

Kaila receives $\frac{1}{5}$ of the remainder, so what is left for Octavia is $\frac{4}{5}$ of the remainder.
It follows that \$400 is $\frac{4}{5}$ of the remainder.

If $\frac{4}{5}$ of the remainder is \$400, then $\frac{1}{5}$ of the remainder is $\$400 \div 4 = \$100$.
Therefore, Kaila received $\$100 + \$100 = \$200$.

So, just after Kaila received \$100, there is $\$100 + \$400 = \$500$ left.
Therefore, before Kaila receives any money there is $\$500 + \$100 = \$600$.

Therefore, the original monetary award was \$600, and Kaila received \$200, Octavia received \$220, and Sophia received \$180, and so Octavia received the largest share.

We can check our result by working through the given information one step at a time.