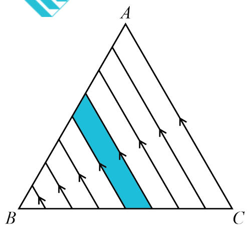


Problem of the Week
Problem C and Solution
Suncatcher

Problem

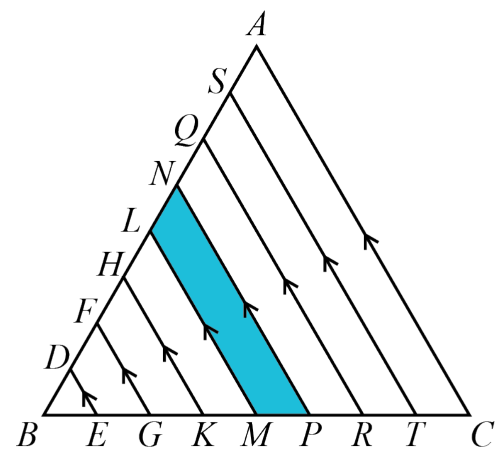
A glass suncatcher is in the shape of an equilateral triangle with sides of length 144 mm. The triangle is labeled ABC and divided into 8 smaller sections as follows.

- Sides AB and BC are each divided into 8 segments of equal length.
- Each point of division on AB is connected to its corresponding point of division on BC, creating 7 line segments.
- Each of the 7 line segments is parallel to the third side of the triangle, AC.

One of the sections is coloured blue, as shown. Determine the perimeter of this section.

Solution

Since AB and BC have each been divided into 8 equal segments, we will label the points of division as shown. Then, since $AB = BC = 144$ mm, and $144 \div 8 = 18$, it follows that $BD = DF = FH = HL = LN = NQ = QS = SA = 18$ mm, and $BE = EG = GK = KM = MP = PR = RT = TC = 18$ mm.



We now proceed with two different solutions.

Solution 1

The angles in an equilateral triangle are each 60°. Therefore, $\angle BAC = \angle BCA = \angle ABC = 60^\circ$.

Since $LM \parallel NP \parallel AC$, $\angle BLM = \angle BNP = \angle BAC = 60^\circ$ and $\angle BML = \angle BPN = \angle BCA = 60^\circ$.

In $\triangle BLM$, $\angle BLM = \angle BML = \angle LBM = 60^\circ$ and it follows that $\triangle BLM$ is equilateral. Since $BL = BD + DF + FH + HL = 18 + 18 + 18 + 18 = 72$, it follows that $LM = 72$ mm.

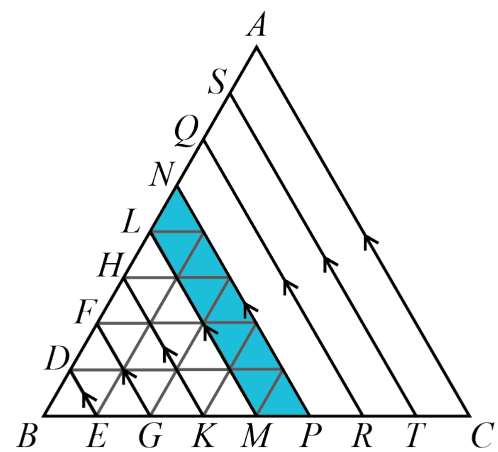
Similarly, in $\triangle BNP$, $\angle BNP = \angle BPN = \angle NBP = 60^\circ$ and it follows that $\triangle BNP$ is equilateral. Since $BN = BD + DF + FH + HL + LN = 5 \times 18 = 90$, it follows that $NP = 90$ mm.

Therefore the perimeter of the shaded region is $LM + MP + NP + LN = 72 + 18 + 90 + 18 = 198$ mm.

Solution 2

Observe that $\triangle ABC$ can be tiled with small equilateral triangles congruent to $\triangle BDE$. That is, equilateral triangles with side length 18 mm. A complete justification of this is not provided here but you may wish to verify this for yourself.

Three of the small equilateral triangles cover the entire area occupied by quadrilateral $DEGF$. Five of the small equilateral triangles cover the entire area occupied by quadrilateral $FGKH$. Seven of the small equilateral triangles cover the entire area occupied by quadrilateral $HKML$. Nine of the small equilateral triangles cover the entire area occupied by quadrilateral $LMPN$.



If we were to continue, $\triangle ABC$ would contain $1+3+5+7+9+11+13+15=64$ of the small equilateral triangles.

In quadrilateral $LMPN$, the smaller side, LM, contains the bases of 4 of the small equilateral triangles and therefore is $4 \times 18 = 72$ mm long. The larger side, NP, contains the bases of 5 of the small equilateral triangles and therefore is $5 \times 18 = 90$ mm long.

The perimeter is the sum of the lengths of the four sides of quadrilateral $LMPN$. Therefore, the perimeter of the shaded region is
$LM + MP + NP + LN = 72 + 18 + 90 + 18 = 198$ mm.