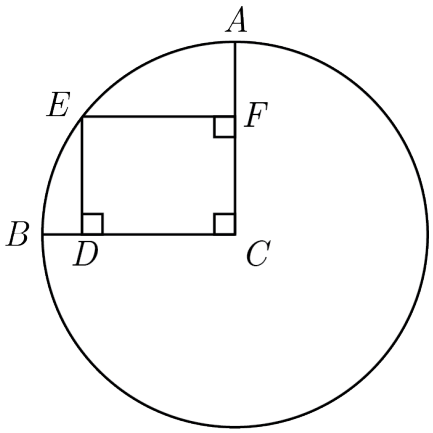
Problem of the Week
Problem C and Solution
A Rectangle in a Circle

Problem

Points A and B are on the circumference of a circle with radius 10 cm and centre C such that $AC \perp BC$. Point D lies inside the circle on BC such that $BD = 2$ cm. Point E lies on the circumference of the circle, on the minor arc AB, such that $DE \perp BC$. Point F lies inside the circle on AC such that $CDEF$ forms a rectangle.

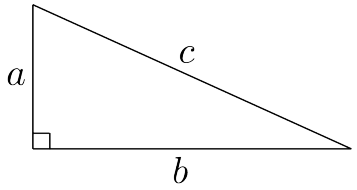


Determine the distance from A to F.

NOTE: You may find the following useful:

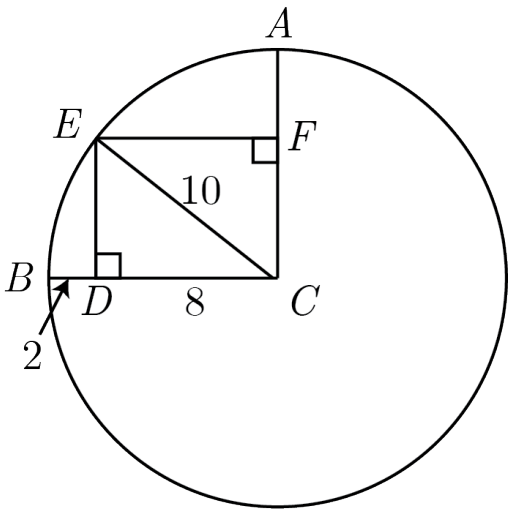
The *Pythagorean Theorem* states, "In a right-angled triangle, the square of the length of hypotenuse (the side opposite the right angle) equals the sum of the squares of the lengths of the other two sides."

In the right-angled triangle shown, c is the hypotenuse, a and b are the lengths of the other two sides, and $c^2 = a^2 + b^2$.



Solution

First we draw the radius EC. Since the radius of the circle is 10 cm, it follows that $AC = BC = EC = 10$. Then $DC = BC - BD = 10 - 2 = 8$.



Since $\angle CDE = 90°$, it follows that $\triangle CDE$ is a right-angled triangle. Using the Pythagorean Theorem in $\triangle CDE$,

$$\begin{aligned} DE^2 + CD^2 &= EC^2 \\ DE^2 &= EC^2 - CD^2 \\ &= 10^2 - 8^2 \\ &= 100 - 64 \\ &= 36 \end{aligned}$$

Then $DE = \sqrt{36} = 6$, since $DE > 0$.

Since $CDEF$ is a rectangle, $CF = DE = 6$. Then $AF = AC - CF = 10 - 6 = 4$. Thus, the distance from A to F is 4 cm.