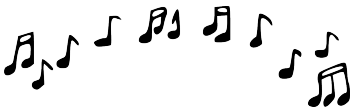

Problem of the Week
Problem C and Solution
Choir Sections

Problem

There are 30 students in the school choir at Pascal Middle School. Their conductor split them into two singing groups: altos and sopranos. In total there are 13 altos and 17 sopranos in the choir.

One day there were some students absent from choir practice. Their conductor noticed that more than half the students in the choir were present, and two-thirds of the students present were altos. How many students were absent from choir practice that day?

Solution

Solution 1

Since more than half the students in the choir were present that day, that means there were at least 16 students present. Since there were some students absent, that means there were at most 29 students present. Since two-thirds of the students present were altos, that tells us the number of students present must be a multiple of 3. Thus, the number of students present is a multiple of 3 between 16 and 29. The possibilities are 18, 21, 24, and 27.

If there were 18 students present, then $\frac{2}{3} \times 18 = 12$ of them were altos and $18 - 12 = 6$ of them were sopranos. This is a possibility since there are 13 altos and 17 sopranos in the choir.

If there were 21 students present, then $\frac{2}{3} \times 21 = 14$ of them were altos and $21 - 14 = 7$ of them were sopranos. However there are only 13 altos in the choir, so it's not possible that 14 of the students were altos. Thus, this is not a possibility.

If there were more than 21 students present, then the number of altos would be more than 14. Thus, 24 and 27 are also not possibilities for the number of students present.

Therefore, there were 18 students present that day, so $30 - 18 = 12$ students were absent.

Solution 2

This solution uses algebra to solve the problem. Note that the algebra used may be beyond what students have done so far in their math classes.

Let a be the number of altos absent and let s be the number of sopranos absent. Since some students were absent, but more than half the class was present, it follows that the total number of students absent is greater than 0 but less than 15. Thus, $a + s > 0$ and $a + s < 15$.

The number of altos present that day was $13 - a$ and the number of sopranos present was $17 - s$. Since two-thirds of the students present were altos, it follows that one-third of the students present were sopranos. So there were twice as many altos as sopranos. Thus,

$$\begin{aligned} 13 - a &= 2(17 - s) \\ 13 - a &= 34 - 2s \\ 0 &= 21 + a - 2s \\ a &= 2s - 21 \end{aligned}$$

Since a represents the number of altos absent, it follows that $a \geq 0$. So the smallest possible value for s is 11, since $2(10) - 21 = 20 - 21 = -1 < 0$ but $2(11) - 21 = 22 - 21 = 1$. Thus, if $s = 11$ then $a = 1$. In this case, the total number of students absent would be $11 + 1 = 12$.

If $s = 12$, then $a = 2(12) - 21 = 24 - 21 = 3$. Then the total number of students absent would be $12 + 3 = 15$. However this is too many, since the number of students absent must be less than 15. If s is any other number larger than 12 then there will be too many students absent. Thus $s = 11$, $a = 1$, and the total number of students absent that day was 12.