



Problem of the Week

Problem C and Solution

Soccer Practice

Problem

Chris and Pat are practicing soccer. Standing 1 m apart, Pat first kicks the ball to Chris and then Chris kicks the ball back to Pat. Next, standing 2 m apart, Pat kicks the ball to Chris and Chris kicks it back to Pat. After each pair of kicks, Chris moves 1 m farther away from Pat. If they stop playing after the 29th kick, how far apart are they standing and how far did the ball travel in total?

Solution

At each distance, two kicks are made: the 1st and 2nd kicks are made when the pair is 1 m apart, the 3rd and 4th kicks are made when they are 2 m apart, and so on, with the 27th and 28th kicks being made when they are 14 m apart.

Therefore, the 29th kick is the first kick made when they are 15 m apart. At each distance, the first kick is made by Pat to Chris, so Pat does the 29th kick at a distance of 15 m.

The total distance travelled by the ball is equal to

$$\begin{aligned} & 1 + 1 + 2 + 2 + 3 + 3 + \cdots + 14 + 14 + 15 \\ &= (1 + 2 + 3 + \cdots + 14) + (1 + 2 + 3 + \cdots + 14 + 15) \\ &= \frac{14 \times 15}{2} + \frac{15 \times 16}{2} \\ &= 105 + 120 \\ &= 225 \end{aligned}$$

Therefore, the ball travelled a total distance of 225 m.

NOTE:

Did you know that the sum of the first n positive integers is equal to $\frac{n \times (n+1)}{2}$?

That is,

$$1 + 2 + 3 + \cdots + n = \frac{n \times (n + 1)}{2}$$

For example, $1 + 2 + 3 + 4 + 5 = 15$, and $\frac{5 \times 6}{2} = 15$.

Also, $1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 = 36$, and $\frac{8 \times 9}{2} = 36$.