Problem of the Week
Problem C and Solution
Product Puzzle

Problem

Three cards are on a table. Each card has a letter on one side and a positive number on the other side. One card has an R on it, one card has a G on it, and one card has a B on it. The number side of each card is face down on the table. The following facts are known about the three concealed numbers:

(i) The product of the number on the card with an R and the number on the card with a G is equal to the number on the card with a B.

(ii) The product of the number on the card with a G and the number on the card with a B is 180.

(iii) Five times the number on the card with a B is equal to the number on the card with a G.

Determine the product of the numbers on the three cards.

Solution

Solution 1

Let the three numbers be represented by r, g, and b.

Since the product of the number on the card with an R and the number on the card with a G is equal to the number on the card with a B, $r \times g = b$. We are looking for $r \times g \times b = (r \times g) \times b = (b) \times b = b^2$. So when we find b^2 we have found the required product $r \times g \times b$.

We are also given that $g \times b = 180$ and $g = 5 \times b$. Substituting $g = 5 \times b$ into $g \times b = 180$, we have $(5 \times b) \times b = 180$ or $5 \times b^2 = 180$. Dividing by 5, we obtain $b^2 = 36$. This is exactly what we are looking for since $r \times g \times b = b^2$.

Therefore, the product of the three numbers is 36.

For those who would like to know the values of the three numbers, we can continue to solve. We know $b^2 = 36$, so $b = 6$, since b is a positive number. Therefore, $g = 5 \times b = 5 \times 6 = 30$. And finally, $r \times g = b$ so $r \times (30) = 6$. Dividing by 30, we get $r = \frac{6}{30} = \frac{1}{5}$.

We can verify the product $r \times g \times b = (\frac{1}{5}) \times (30) \times (6) = 6 \times 6 = 36$.

Solution 2

In this solution we try to determine the numbers by working with the factors of 180.

The product of the number on the card with a G and the number on the card with a B is 180, and the number on the card with a G is five times the number on the card with a B. The number 180 can be written as $2 \times 2 \times 3 \times 3 \times 5$. If G and B are integers, then G and B must share the same factors, along with G having one more factor, that being 5. By playing with the factors of 180, we find a possibility is that the number on the card with a G is $5 \times 2 \times 3$ and the number on the card with a B is 2×3. That is, the number on the card with a G could be 30 and the number on the card with a B could be 6.

Now using the fact that the number on the card with an R multiplied by the number on the card with a G is equal to the number on the card with a B, we determine that some number multiplied by 30 is equal to 6. It follows that the number on the card with an R would be $6 \div 30 = \frac{1}{5}$.

The product of the three numbers is $\frac{1}{5} \times 30 \times 6 = 6 \times 6 = 36$.

This solution works because the number on the card with a G and the number on the card with a B happen to be integers.