

Problem of the Week

Problem B and Solution

Bird on a Barn

Problem

Esther Epp has decided to pay a company to repaint her barn quilt to brighten up the colours. The company charges the following rates for different colours:

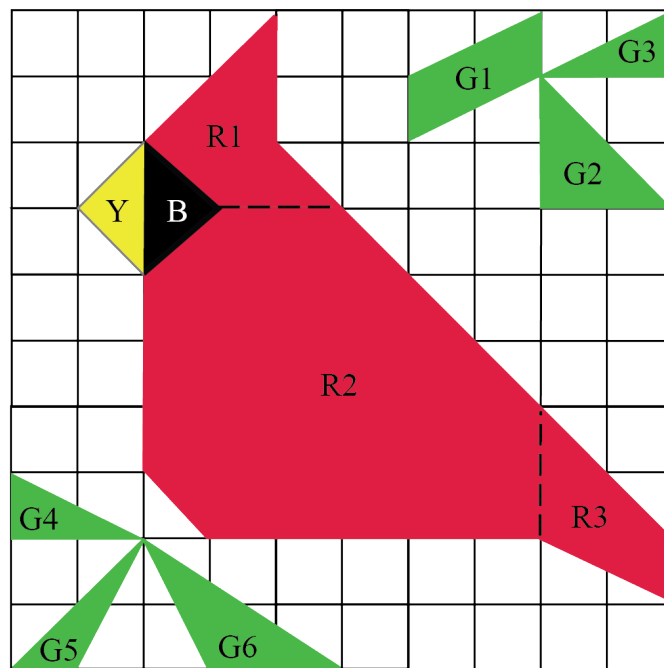
- White or black costs 27 cents per 10 cm^2 .
- Red, yellow, or green costs 32 cents per 10 cm^2 .

A pattern for Esther's 1 m by 1 m square barn quilt is shown. The background is white, and the letters Y, R, G, B designate yellow, red, green, and black, respectively. The grid lines are an aid only, you can assume they are painted white.

Determine the area of each colour in the quilt. Then determine the total cost, in dollars, of this paint job for Esther.

Solution

To calculate the areas, the red shape has been divided into three labelled regions, R1, R2, and R3, while the green shapes have been labelled individually as G1, G2, G3, G4, G5, and G6.



Since the quilt is 1 m by 1 m and there are $10 \times 10 = 100$ grid squares, each square is 10 cm by 10 cm with an area of 100 cm^2 .

The yellow region is composed of 2 half squares, which is equal in area to 1 whole square. Thus, the area painted yellow is 100 cm^2 .



The black region is composed of 2 half squares, which is equal in area to 1 whole square. Thus, the area painted black is 100 cm^2 .

The green area is composed of G1, G2, G3, G4, G5, G6.

G1 can be further divided into two triangles, each with base 2 and height 1. Thus, G1 can be divided into two triangles, each with area equal to $\frac{2 \times 1}{2} = 1$. Thus, G1 is equal in area to 2 whole squares.

G3 and G4 each have base 2 and height 1. Thus, each have area equal to $\frac{2 \times 1}{2} = 1$, so together they are equal in area to 2 whole squares.

G2 and G6 are triangles with base 2 and height 2. Thus, each have area equal to $\frac{2 \times 2}{2} = 2$, so together they are equal in area to 4 whole squares.

G5 is a triangle with base 1 and height 2, and hence has area equal to $\frac{1 \times 2}{2} = 1$. Thus, G5 is equal in area to 1 whole square.

The combined total number of green squares is equal to $2 + 2 + 4 + 1 = 9$ whole squares. Thus, the area painted green is 900 cm^2 .

The red area is composed of R1, R2, R3.

R1 is composed of 4 half squares and 2 whole squares, for a total area equal to the area of 4 whole squares.

R2 is composed of 5 half squares and 22 whole squares, for a total area equal to the area of 24.5 whole squares.

R3 is composed of 2 half squares and 1 whole square, plus one $\frac{1}{4}$ square and one $\frac{3}{4}$ square, for a total area equal to the area of 3 whole squares.

The combined total number of red squares is equal to $4 + 24.5 + 3 = 31.5$ whole squares. Thus, the area painted red is 3150 cm^2 .

The remaining squares are equal in area to $100 - (1 + 1 + 9 + 31.5) = 57.5$ squares. Thus, the area painted white is 5750 cm^2 .

Finally, we determine the total cost of the paint.

The total area that is white or black is $5750 \text{ cm}^2 + 100 \text{ cm}^2 = 5850 \text{ cm}^2$. This is painted at \$0.27 per 10 cm^2 , and so will cost $585 \times 0.27 = \$157.95$.

The total area that is yellow, green, or red is $100 \text{ cm}^2 + 900 \text{ cm}^2 + 3150 \text{ cm}^2 = 4150 \text{ cm}^2$. This is painted at \$0.32 per 10 cm^2 , and so will cost $415 \times 0.32 = \$132.80$.

Thus, the total cost of this paint job is $\$157.95 + \$132.80 = \$290.75$.