



Problem of the Week

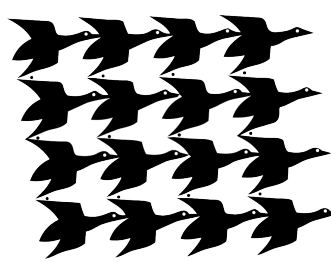
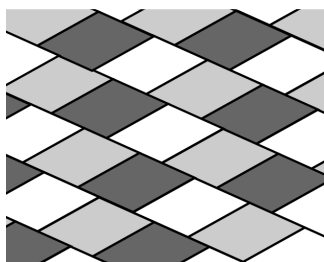
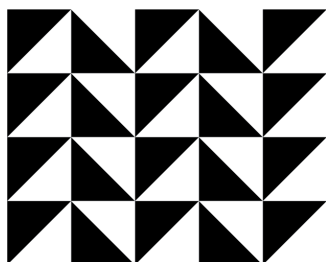
Problem B and Solution

Tessellation Consternation

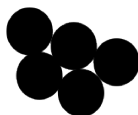
Problem

A *tessellation* is a tiling pattern in which two-dimensional shapes fit together with no overlaps or gaps. A *regular tessellation* uses congruent shapes.

Below are three examples of tessellations, using right triangles, parallelograms, and two slightly different birds, respectively. The first two are regular tessellations, the third is not.

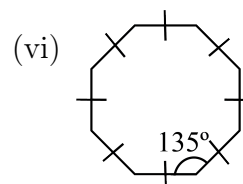
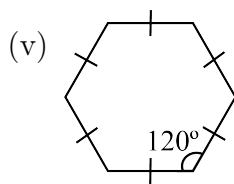
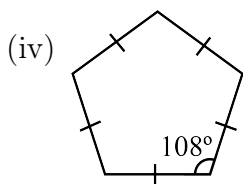
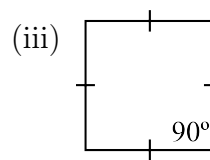
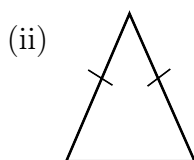
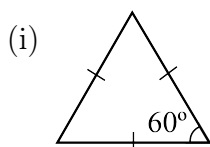


A circle is an example of a shape that does not tessellate, since circles cannot be tiled without creating overlap or gaps:



- (a) For each shape below, name the shape and then decide whether or not it can be tessellated. Note that shapes may be rotated, reflected, and/or translated in order to make them tessellate.

HINT: For no gaps, what must the total angle be where vertices meet?



- (b) On a piece of cardboard or sturdy construction paper, draw an irregular quadrilateral (that is, a quadrilateral with all four sides of different lengths) and cut it out. Trace it on a piece of paper, then continue to move and retrace it to see whether it will tessellate or not.

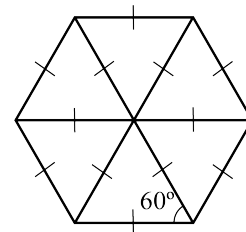
SUGGESTION: If desired, use a digital tool instead of tracing by hand.



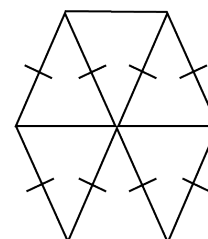
Solution

- (a) A key requirement for a tessellation is that the total angle where vertices meet must be 360° . Otherwise there will either be a gap (if it is less) or an overlap (if it is more).

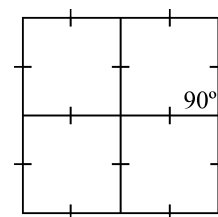
- (i) The shape is an equilateral triangle. An equilateral triangle tessellates. A tessellation of six equilateral triangles is shown. The total angle where the six vertices meet is $6 \times 60^\circ = 360^\circ$. The tessellation can be extended by adding copies of the tessellation shown.



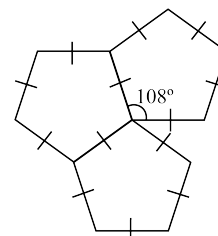
- (ii) The shape is an isosceles triangle. An isosceles triangle tessellates. By rotating three copies of the triangle 180° , these three rotated triangles along with three other copies of the triangle can be arranged so that they meet at a common vertex, as shown. Since the three angles in an isosceles triangle sum to 180° , the total angle at this vertex is $2 \times 180^\circ = 360^\circ$. The tessellation can be extended by adding copies of the tessellation shown.



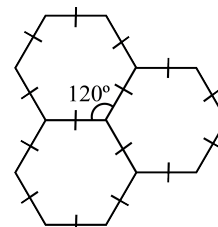
- (iii) The shape is a square. A square tessellates. A tessellation of four squares is shown. The total angle where the four vertices meet is $4 \times 90^\circ = 360^\circ$. The tessellation can be extended by adding copies of the tessellation shown.



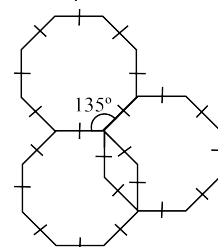
- (iv) The shape is a regular pentagon. A regular pentagon does not tessellate. This is due to the fact that each angle in a regular pentagon is 108° , and $3 \times 108^\circ = 324^\circ$, which is less than 360° , and $4 \times 108^\circ = 432^\circ$, which is greater than 360° .



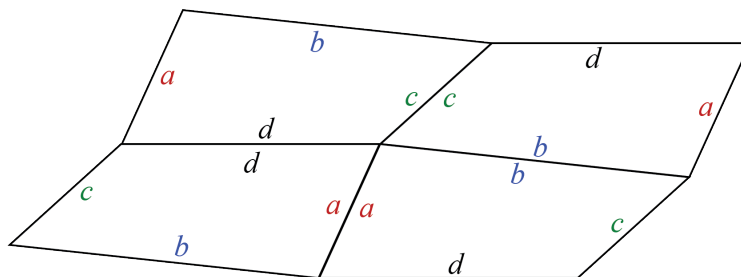
- (v) The shape is a regular hexagon. A regular hexagon tessellates. A tessellation of a regular hexagon is shown. The total angle where the three vertices meet is $3 \times 120^\circ = 360^\circ$. The tessellation can be extended by adding copies of the tessellation shown.



- (vi) The shape is a regular octagon. A regular octagon does not tessellate. This is due to the fact that each angle in the octagon is 135° , and $2 \times 135^\circ = 270^\circ$, which is less than 360° , and $3 \times 135^\circ = 405^\circ$, which is greater than 360° .



- (b) Any quadrilateral will tessellate. To understand why, consider a quadrilateral with sides of length a , b , c , and d . By rotating two copies of the quadrilateral 180° , these rotated quadrilaterals along with other copies of the quadrilateral can be arranged to meet at a common vertex, as shown.



Since the interior angles in a quadrilateral sum to 360° , the total angle where the four vertices meet in the tiling shown is 360° . The tessellation can be extended by adding copies of the tessellation shown.

EXTENSION: Can you use your solution to part (b) to show that any scalene triangle will tessellate?