Problem of the Week
Problem B and Solution
Funky Friday

Problem

Some people believe that 'Friday the 13th' brings bad luck. In 2026, the 13th of the month falls on a Friday three times: in February, March, and November.

The table below summarizes the occurrences of Friday the 13th each year for 2015 to 2026, and lists the day of the week that January 1st occurred on in that year.

Year	Months with a Friday the 13th	Day of week January 1 was on
2026	February, March, November	Thursday
2025	June	Wednesday
2024	September, December	Monday
2023	January, October	Sunday
2022	May	Saturday
2021	August	Friday
2020	March, November	Wednesday
2019	September, December	Tuesday
2018	April, July	Monday
2017	January, October	Sunday
2016	May	Friday
2015	February, March, November	Thursday

(a) In this twelve year sample, what is the average number of months in a year with a Friday the 13th? Write your answer as a decimal rounded to the nearest hundredth, and explain what that number means in terms of how these dates occur.

(b) The years 2016 and 2022 both have a Friday the 13th only in May, but those years did not start on the same day of the week. Explain how this can happen.

(c) Both 2020 and 2025 began on a Wednesday. Why do they have different months with a Friday the 13th?

(d) Do you think that every year with three months with a Friday the 13th must start on a Thursday? Why, or why not?
SUGGESTION: Try doing some research about this on the internet.

Solution

(a) First, we determine the total number of months with a Friday the 13^{th} in the 12 year sample:

$$3 + 1 + 2 + 2 + 1 + 1 + 2 + 2 + 2 + 2 + 1 + 3 = 22$$

Dividing by the number of years, the average is $22 \div 12 \approx 1.83$.

This means that most years have 1 or 2 months with a Friday the 13^{th}. Years with 3 months with a Friday the 13^{th} occur occasionally.

(b) The years 2016 and 2022 both have a Friday the 13^{th} only in May despite starting on different days. May 13^{th} is the 133^{rd} day of a normal year, which is exactly $133 \div 7 = 19$ weeks. Thus, since the year 2022 started on a Saturday, the last day of the 19^{th} week is Friday, May 13^{th}.

However, the year 2016 was a leap year, so May 13^{th} is the 134^{th} day of that year due to the extra day in February. Since 2016 started on a Friday, the 134^{th} day (which is equal to 19 weeks plus 1 day) was also Friday, May 13^{th}.

(c) This is because the year 2020 was a leap year, but the year 2025 was not.

To justify this, we will use the rule that a month has a Friday the 13^{th} exactly when the 1^{st} of that month falls on a Sunday (This rule is left as an exercise to the students to justify).

Since 2020 was a leap year, February had 29 days and the extra day pushed March 1^{st} to a Sunday. This is why March 2020 included a Friday the 13^{th}. However, the year 2025 was not a leap year. Thus, March 1^{st} was not on a Sunday (it was on a Saturday), and thus March 2025 did not include a Friday the 13^{th}.

(d) The years 2015 and 2026 both had three months with a Friday the 13^{th}, and both started on a Thursday. However, both 2015 and 2026 were not leap years. This pattern will always occur in a non-leap year if it starts on a Thursday.

The question now is whether it's possible to have three months with a Friday the 13^{th} for a non-leap starting on different day, or in a leap year starting on a particular day.

The evidence from the table tells us that non-leap years starting on Monday (2018), Tuesday (2019), and Sunday (2017 and 2023) have two months with a Friday the 13^{th}, while years starting on Wednesday (2025), Friday (2021), and Saturday (2022) have only one month with a Friday the 13^{th}. Thus, the only non-leap years with three months with a Friday the 13^{th} are those starting on a Thursday.

Some research on the internet revealed that for leap years starting on a Sunday, there are three months with a Friday the 13^{th}: January, April, July. This happens every 28 years, and last occurred in 2012.