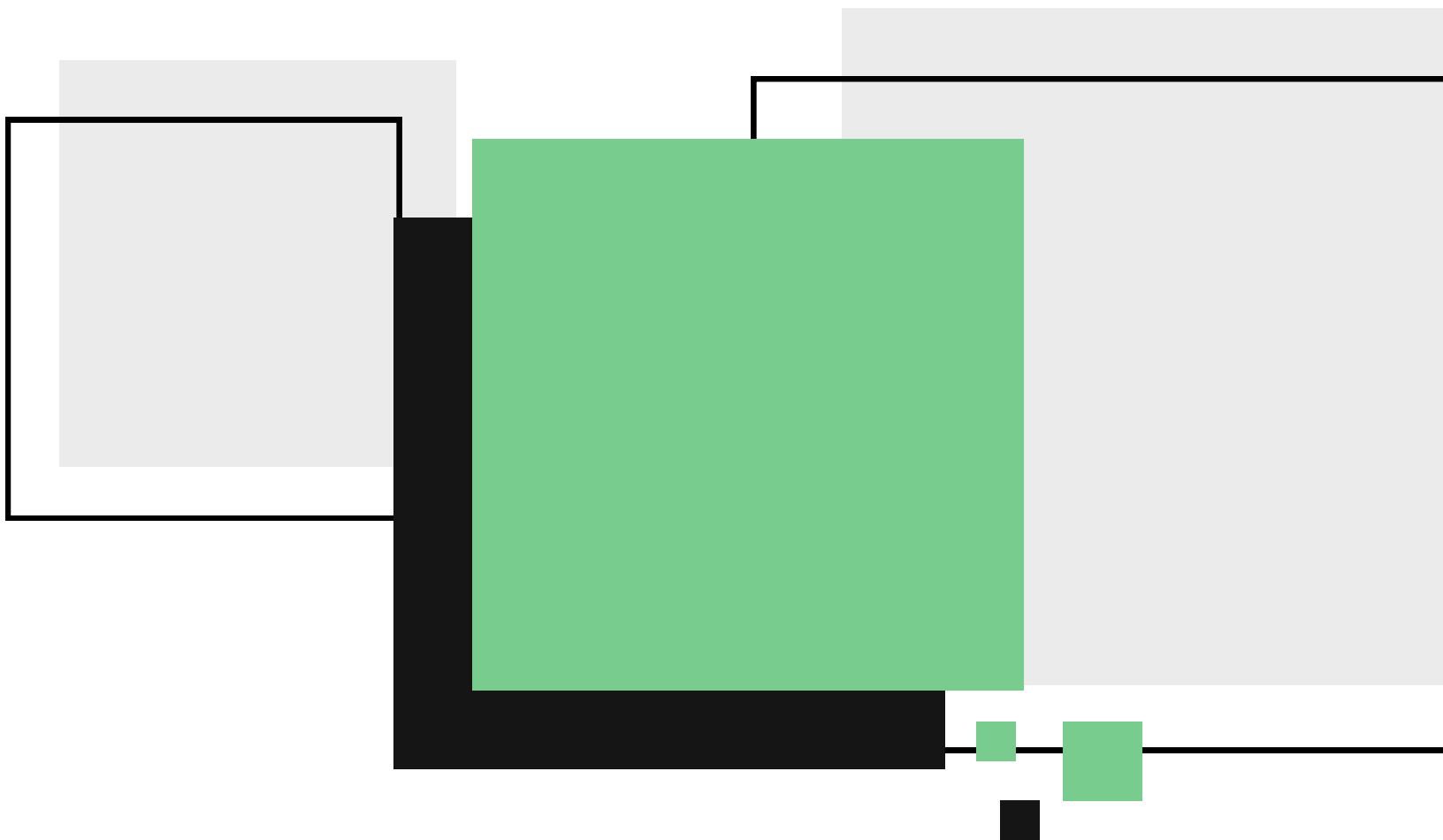


Problème de la Semaine

Problèmes et Solutions 2025-2026



Problème B

5e/6e année



Le CENTRE d'ÉDUCATION
en MATHÉMATIQUES et en INFORMATIQUE
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[Raisonnement informatiques \(C\)](#)

[Gestion des données \(D\)](#)

[Géométrie et mesure \(G\)](#)

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Algèbre (A)



**Amène-moi à la
couverture**



Problème de la semaine

Problème B

Choix de smoothies

Justin prépare avec ses frères et sœurs des smoothies chaque matin. Leur recette de smoothie utilise 3 tasses de fruits. Parfois, ils utilisent une seule sorte de fruit et parfois ils en utilisent plusieurs, mais ils emploient toujours exactement 1, 2, ou 3 tasses de chaque fruit.

Les smoothies ont des goûts différents selon les quantités de chaque fruit. Par exemple, un smoothie avec 2 tasses de mangue et 1 tasse de bleuets est différent d'un smoothie avec 1 tasse de mangue et 2 tasses de bleuets.

- (a) S'ils n'ont que des fraises et des bananes, combien de smoothies différents peuvent-ils préparer ?
- (b) S'ils ont des fraises, des bananes et des framboises, combien de smoothies différents peuvent-ils préparer ?
- (c) S'ils ont des fraises, des bananes, des framboises et des pêches, combien de smoothies différents peuvent-ils préparer ?





Problem of the Week

Problem B and Solution

Smoothie Choices

Problem

Justin and his siblings make smoothies every morning. Their smoothie recipe uses 3 cups of fruit. Sometimes they use only one type of fruit and sometimes they use more than one type of fruit, but they always use exactly 1, 2, or 3 cups of each fruit.

Smoothies taste different when they change the amounts of each fruit. For example, a smoothie with 2 cups of mango and 1 cup of blueberries is different than a smoothie with 1 cup of mango and 2 cups of blueberries.

- (a) If they have only strawberries and bananas, how many different smoothies can they make?
- (b) If they have strawberries, bananas, and raspberries, how many different smoothies can they make?
- (c) If they have strawberries, bananas, raspberries, and peaches, how many different smoothies can they make?

Solution

- (a) To determine the number of smoothies they can make using only bananas and/or strawberries, we will write out all the options. Let S represent 1 cup of strawberries and B represent 1 cup of bananas. The smoothies they can make are as follows:

$$SSS, SSB, SBB, BBB$$

Thus, they can make 4 different smoothies if they have only bananas and strawberries.

- (b) Let R represent 1 cup of raspberries. The smoothies they can make using strawberries, bananas, and/or raspberries are as follows:

$$SSS, BBB, RRR, SSB, SBB, SSR, SRR, BBR, BRR, SBR$$

Thus, they can make 10 different smoothies if they have strawberries, bananas, and raspberries.



(c) Let P represent 1 cup of peaches. Since there's more to count, we'll organize the smoothies by the number of cups of peaches.

- **Smoothies with 0 cups of peaches:** From part (b), we know that they can make 10 different smoothies if they have strawberries, bananas, and raspberries.
- **Smoothies with 1 cup of peaches:** The possibilities for the remaining 2 cups of fruit are SS , BB , RR , SB , SR , and BR . Therefore, they can make 6 different smoothies if 1 cup is peaches.
- **Smoothies with 2 cups of peaches:** The possibilities for the remaining 1 cup of fruit are S , B , and R . Therefore, they can make 3 different smoothies if 2 cups are peaches.
- **Smoothies with 3 cups of peaches:** This can be done in only one way as PPP .

Thus, in total they can make $10 + 6 + 3 + 1 = 20$ different smoothies if they have strawberries, bananas, raspberries, and peaches.



Problème de la semaine

Problème B

Le défi des grains de riz

La classe de Caden veut gagner un prix pour l'estimation la plus proche du nombre de grains de riz dans un sac de riz de 1 kg.



On leur a donné trois indices :

Indice (1): Le nombre de grains est un nombre pair à 5 chiffres.

Indice (2): La somme des cinq chiffres du nombre est 15.

Indice (3): Le nombre, arrondi à la centaine près, est 51 200.

- (a) Aide la classe de Caden à gagner le prix. Utilise les indices pour déterminer toutes les estimations possibles du nombre de grains de riz dans le sac.
- (b) Kendala, une camarade de classe de Caden, a découvert que la masse d'un seul grain de riz dépend du type de riz et varie d'environ 20 à 45 milligrammes. Lorsqu'elle a utilisé cette information pour calculer combien de grains se trouvaient dans le sac de 1 kg, sa réponse a-t-elle suggéré que le riz était du type plus léger ou du type plus lourd ?



Problem of the Week

Problem B and Solution

Grain Guess

Problem

Caden's class wants to win a prize for the closest estimate to the number of grains of rice in a 1 kg bag of rice.

They have been given three clues:

Clue (1): The number of grains is a 5-digit even number.

Clue (2): The sum of the five digits in the number is 15.

Clue (3): The number, rounded to the nearest hundred, is 51 200.

- (a) Help Caden's class win the prize. Use the given clues to determine all possible estimates of the number of grains of rice in the bag.
- (b) Kendala, one of Caden's classmates, found that the mass of a single grain of rice depends on the type, ranging from about 20 to 45 milligrams. When she used this to calculate how many grains were in the 1 kg bag, did her answer suggest the rice was of the lighter or heavier type?

Solution

- (a) From Clues (1) and (3), we know that the number is an even number between 51 150 and 51 249.

Therefore, the number is of the form 51 1XY or 51 2XY, where X and Y are digits.

Suppose the number is of the form 51 1XY. We know that Y is even, and $X \geq 5$. Since $5 + 1 + 1 = 7$, then from Clue (2), $X + Y = 8$. Therefore, we could have $X = 6$ and $Y = 2$, or $X = 8$ and $Y = 0$.

Suppose the number is of the form 51 2XY. We know that Y is even and $X \leq 4$. Since $5 + 1 + 2 = 8$, then from Clue (2), $X + Y = 7$. Since Y is an even digit, we could have $X = 3$ and $Y = 4$, or $X = 1$ and $Y = 6$.

Therefore, the estimates that satisfy all three clues are 51 162, 51 180, 51 234, and 51 216.

- (b) Since $1 \text{ kg} = 1000 \text{ g} = 1\,000\,000 \text{ mg}$, for the lighter rice, Kendala would calculate $1\,000\,000 \div 20 = 50\,000$ grains of rice. For the heavier rice, she would calculate $1\,000\,000 \div 45 \approx 22\,222$ grains of rice. Thus, her answers indicate that the class's rice was of the lighter type, since that answer is closest to their estimate.



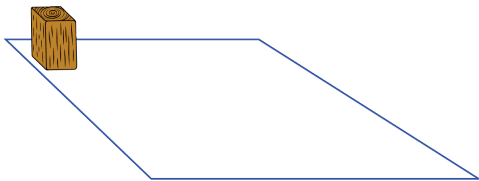
Problème de la semaine

Problème B

Le bloc-tampon

Evelina utilise un bloc de bois comme tampon pour créer du papier d'emballage décoratif. Le bloc de bois mesure 3 cm par 3 cm par 4 cm et Evelina utilise l'une de ses faces carrées de 3 cm par 3 cm pour tamponner.

- (a) Evelina crée une rangée de 10 carrés tamponnés avec un écart de 0,5 cm entre deux carrés consécutifs. Détermine la longueur de cette rangée.
- (b) Evelina dispose d'un morceau de papier d'emballage carré dont le côté mesure 77,5 cm. Elle laisse une marge de 0,5 cm à partir de chaque bord, puis crée des rangées de carrés tamponnés avec un écart de 0,5 cm entre les carrés, ainsi qu'entre les rangées. Combien de carrés tamponnés au total seront placés sur ce morceau de papier d'emballage ?





Problem of the Week

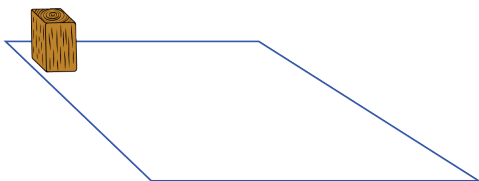
Problem B and Solution

Stamp it Out

Problem

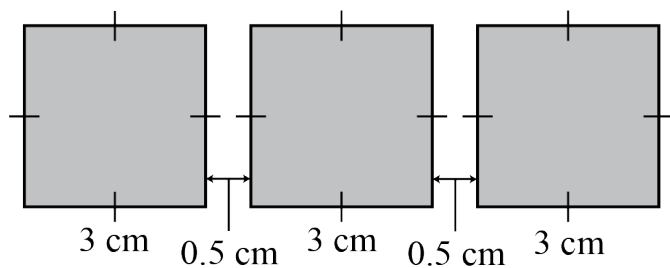
Evelina is using a wooden block as a stamp to make decorative wrapping paper. The wooden block measures 3 cm by 3 cm by 4 cm and Evelina is stamping with one of the 3 cm by 3 cm square faces of the block.

- (a) Evelina creates a row of 10 stamped squares with a gap of 0.5 cm in between each square. Determine the length of this row.
- (b) Evelina has a square piece of wrapping paper with side length 77.5 cm. She leaves a gap of 0.5 cm around each edge and then creates rows of stamped squares with a gap of 0.5 cm in between each square as well as between each row. How many stamped squares in total will be on this piece of wrapping paper?



Solution

- (a) We start by drawing a diagram of the first three stamped squares in the row.



From here we show two different ways to solve the problem.

Solution 1

In this solution we use a table to help us determine the length of the row. The first stamped square has a length of 3 cm, and each square after that adds $3 + 0.5 = 3.5$ cm to the length.



Number of Stamped Squares	Length of Row (cm)
1	3
2	$3 + 3.5 = 6.5$
3	$6.5 + 3.5 = 10$
4	$10 + 3.5 = 13.5$
5	$13.5 + 3.5 = 17$
6	$17 + 3.5 = 20.5$
7	$20.5 + 3.5 = 24$
8	$24 + 3.5 = 27.5$
9	$27.5 + 3.5 = 31$
10	$31 + 3.5 = 34.5$

Thus, the length of the row is 34.5 cm.

Solution 2

In this solution we use an algebraic expression help us determine the length of the row. This solution is more efficient, especially as the row gets longer. We know the length of each stamped square is 3 cm and the length of each gap between the squares is 0.5 cm. If we let s represent the total number of squares and g represent the total number of gaps between the squares, then the length of the row, in cm, is $3 \times s + 0.5 \times g$.

If there are 10 squares in the row then there are 9 gaps between them. Then we can use our equation to determine the length of the row.

$$\begin{aligned} 3 \times s + 0.5 \times g &= 3 \times 10 + 0.5 \times 9 \\ &= 30 + 4.5 \\ &= 34.5 \end{aligned}$$

Thus, the length of the row is 34.5 cm.

- (b) We will start by determining the number of stamped squares in each row. Since the piece of wrapping paper is a square, this will also be the number of rows of stamped squares.

If we subtract the gap at the start of the row, then the length of the row will be $77.5 - 0.5 = 77$ cm. This is helpful because each square is followed by a gap, so the total length of the square and its gap is $3 + 0.5 = 3.5$ cm. Then we can divide to determine the number of squares in the row. Since $77 \div 3.5 = 22$, it follows that there are 22 squares in the row.

Since there are also 22 rows of squares, the total number of squares is $22 \times 22 = 484$.

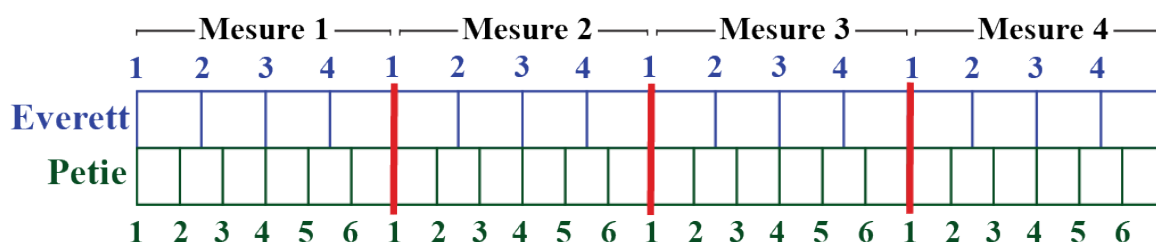


Problème de la semaine

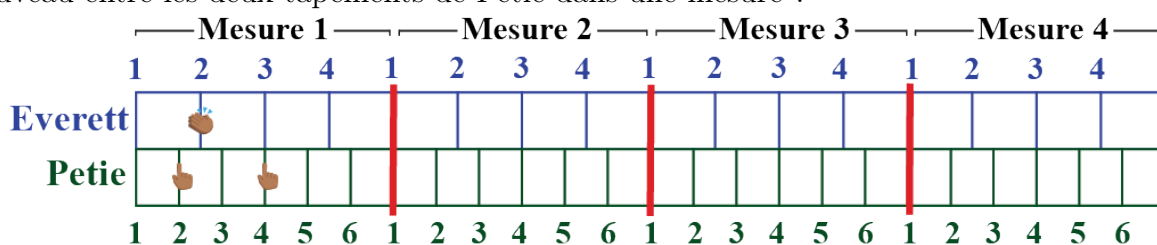
Problème B

Jouer en rythme

Une « mesure » est un segment de musique divisé en « temps » uniformément espacés. En classe de musique, Everett frappe des mains selon un rythme de quatre temps par mesure. Petie tape sur son pupitre selon un rythme de six temps par mesure. Les quatre premières mesures d'Everett et de Petie sont illustrées dans le diagramme. Les nombres au-dessus des lignes verticales indiquent le numéro du temps dans chaque mesure pour Everett et les nombres en dessous des lignes verticales indiquent le numéro du temps dans chaque mesure pour Petie.



- (a) Dans la mesure 1, Everett frappe des mains au deuxième temps. Par la suite, il frappe des mains tous les trois temps (ainsi, son prochain battement a lieu au premier temps de la mesure 2). Petie tape sur son pupitre aux deuxième et quatrième temps de chaque mesure. Le diagramme ci-dessous illustre les battements de mains d'Everett et les tapements de Petie dans la mesure 1. Complète le diagramme pour montrer leurs battements de mains et leurs tapements au cours des trois mesures suivantes. Dans la mesure 1, Everett frappe des mains entre les deux tapements de Petie. Quand est-ce qu'Everett frappe des mains de nouveau entre les deux tapements de Petie dans une mesure ?



- (b) Si Petie et Everett continuent de taper et de frapper des mains de cette façon, est-ce qu'un motif répétitif de sons se produit ? Si oui, combien de mesures comporte ce motif avant de se répéter ?
- (c) Comment Petie pourrait-il modifier son approche pour que, dans chaque mesure, au moins un des battements de mains d'Everett se produise entre les deux tapements de Petie ?

SUGGESTION : Travaille avec un camarade de classe pour taper et frapper des mains selon les rythmes d'Everett et de Petie.



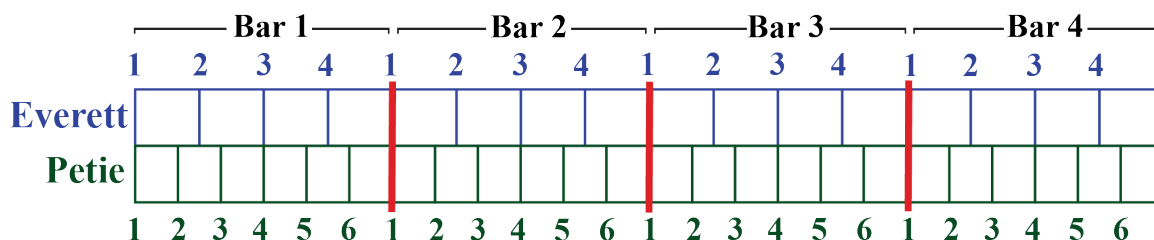
Problem of the Week

Problem B and Solution

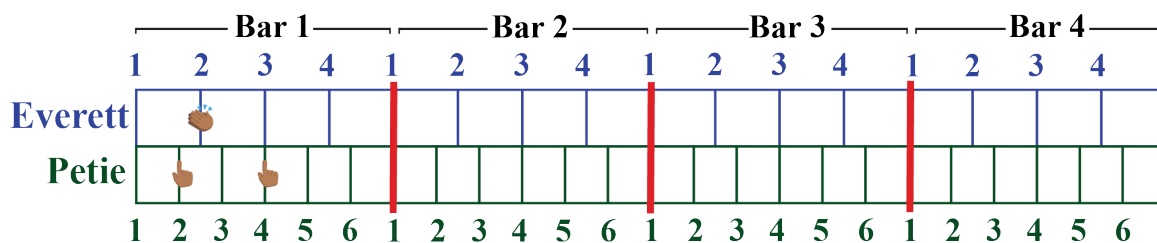
Playing to the Beat

Problem

A “bar” is a segment of music, broken up into evenly spaced “beats”. In music class, Everett claps his hands in a rhythm with four beats per bar. Petie taps on his desk in a rhythm with six beats per bar. The first four bars for Everett and Petie are shown in the diagram. The numbers above the vertical lines indicate the beat number in each bar for Everett and the numbers below the vertical lines indicate the beat number in each bar for Petie.



- (a) In Bar 1, Everett claps on the second beat. After that, Everett claps on every third beat (so his next clap is on the first beat in Bar 2). Petie taps on the second and fourth beats in every bar. The diagram below shows Everett's clap and Petie's taps in Bar 1. Complete the diagram to show their taps and claps over the next three bars. In Bar 1, Everett claps between Petie's two taps. When does Everett next clap between Petie's two taps in a bar?



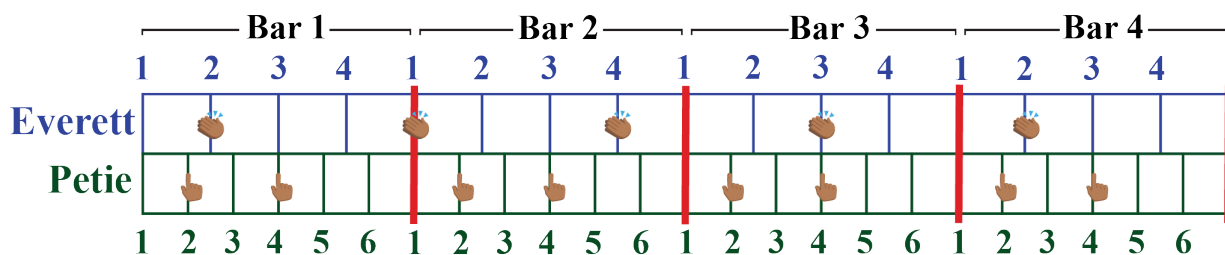
- (b) If Petie and Everett continue tapping and clapping this way, does a repeated pattern of sounds occur? If so, how many bars are there in this pattern before it repeats?
- (c) How could Petie change his approach so that, in every bar, at least one of Everett's claps happens between Petie's two taps?

SUGGESTION: Work with a classmate to tap and clap in Everett's and Petie's rhythms.



Solution

- (a) The completed diagram is shown.



Everett next claps between Petie's two taps in Bar 4.

- (b) Notice that Bar 4 matches Bar 1. Thus, a repeated pattern of sounds occurs after there are three bars in the pattern. That is, the pattern repeats every three bars.
- (c) Answers will vary. Petie could tap on the first and sixth beats in each bar. Alternatively, Petie could tap on the second and sixth beats in each bar.



Problème de la semaine

Problème B

Les formes symboliques

Dans chacune des questions, chaque forme ($\square$, $\bigcirc$, $\triangle$) représente le même nombre entier. Par exemple, $\square + \square = 8$ signifie que $2 \times \square = 8$, donc $\square = 4$. (La valeur de chaque forme peut changer d'une question à l'autre.)

Détermine la valeur de chaque forme dans chaque question.

(a)

$$\square + \square = 8$$

$$\bigcirc + \square = 10$$

(b)

$$\square + \square + \square = 18$$

$$\square + \bigcirc + \bigcirc = 14$$

$$\bigcirc + \square + \triangle = 18$$

(c)

$$\bigcirc + \bigcirc + \bigcirc = 24$$

$$\square \times \bigcirc + \square = 36$$

$$\triangle \times \square + \triangle = 25$$

(d)

$$2 \times \triangle = 28$$

$$\triangle - 10 = \bigcirc$$

$$\bigcirc + \square + \square = 10$$

(e)

$$\square + \triangle + \triangle = 21$$

$$\triangle + \bigcirc - \square = 9$$

$$\square + \square = 18$$



EXTENSION: Crée un problème semblable et mets un ou une camarade de classe au défi de le résoudre.



Problem of the Week

Problem B and Solution

Symbolic Shapes

Problem

In each question, each shape ($\square$, $\bigcirc$, $\triangle$) is equal to the same whole number. For example, $\square + \square = 8$ implies that $2 \times \square = 8$, so $\square = 4$. (The value of each shape can change from question to question.)

Determine the value of each shape in each question.

(a)

$$\square + \square = 8$$

$$\bigcirc + \square = 10$$

(b)

$$\square + \square + \square = 18$$

$$\square + \bigcirc + \bigcirc = 14$$

$$\bigcirc + \square + \triangle = 18$$

(c)

$$\bigcirc + \bigcirc + \bigcirc = 24$$

$$\square \times \bigcirc + \square = 36$$

$$\triangle \times \square + \triangle = 25$$

(d)

$$2 \times \triangle = 28$$

$$\triangle - 10 = \bigcirc$$

$$\bigcirc + \square + \square = 10$$

(e)

$$\square + \triangle + \triangle = 21$$

$$\triangle + \bigcirc - \square = 9$$

$$\square + \square = 18$$



EXTENSION: Create a similar problem and challenge a classmate to solve it.



Solution

- (a) Since $\square + \square = 8$, we know that $2 \times \square = 8$, and so $\square = 8 \div 2 = 4$.
Then, since $\bigcirc + \square = 10$, we know that $\bigcirc + 4 = 10$, and so $\bigcirc = 10 - 4 = 6$.
Thus, $\square = 4$ and $\bigcirc = 6$.
- (b) Since $\square + \square + \square = 18$, we know that $3 \times \square = 18$, and so $\square = 18 \div 3 = 6$.
Then, since $\square + \bigcirc + \bigcirc = 14$, we know that $6 + \bigcirc + \bigcirc = 14$, and so
 $\bigcirc + \bigcirc = 14 - 6 = 8$ and $\bigcirc = 8 \div 2 = 4$.
Then, since $\bigcirc + \square + \triangle = 18$, we have $4 + 6 + \triangle = 18$, and so
 $\triangle = 18 - 10 = 8$.
Thus, $\square = 6$, $\bigcirc = 4$, and $\triangle = 8$.
- (c) Since $\bigcirc + \bigcirc + \bigcirc = 24$, we know that $3 \times \bigcirc = 24$, and so $\bigcirc = 24 \div 3 = 8$.
Then, since $\square \times \bigcirc + \square = 36$, we have $\square \times 8 + \square = 36$, and so $9 \times \square = 36$.
Therefore, $\square = 36 \div 9 = 4$.
Now, since $\triangle \times \square + \triangle = 25$, we have $\triangle \times 4 + \triangle = 25$, and so $5 \times \triangle = 25$
and $\triangle = 25 \div 5 = 5$.
Thus, $\square = 4$, $\bigcirc = 8$, and $\triangle = 5$.
- (d) Since $2 \times \triangle = 28$, we have $\triangle = 28 \div 2 = 14$.
Then, since $\triangle - 10 = \bigcirc$, we have $14 - 10 = \bigcirc$, and so $\bigcirc = 4$.
Then, since $\bigcirc + \square + \square = 10$, we have $4 + \square + \square = 10$, and so
 $\square + \square = 10 - 4 = 6$. Therefore, $\square = 6 \div 2 = 3$.
Thus, $\square = 3$, $\bigcirc = 4$, and $\triangle = 14$.
- (e) For this problem, start with the third equation. Since $\square + \square = 18$, we have
 $2 \times \square = 18$, and so $\square = 18 \div 2 = 9$.
Then, since $\square + \triangle + \triangle = 21$, we have $9 + \triangle + \triangle = 21$, and so
 $\triangle + \triangle = 21 - 9 = 12$. Thus, $2 \times \triangle = 12$, and so $\triangle = 12 \div 2 = 6$.
Now, since $\triangle + \bigcirc - \square = 9$, we have $6 + \bigcirc - 9 = 9$, and so
 $\bigcirc = 9 - 6 + 9 = 12$.
Thus, $\square = 9$, $\bigcirc = 12$, and $\triangle = 6$.



Problème de la semaine

Problème B

Des fleurs pour Sarah

Sarah a aménagé 8 nouvelles plates-bandes dans un endroit ensoleillé et souhaite y planter un jardin fleuri. Les sachets de graines coûtent 2,69 \$ par variété de fleur et elle doit payer des frais d'expédition fixes de 9,00 \$ pour sa commande. Elle aimerait planter plusieurs variétés différentes sans dépenser plus de 53,00 \$.

- (a) Écris une inégalité qui représente le problème de Sarah sous forme mathématique, en utilisant les coûts donnés, le nombre de variétés qu'elle achète et les fonds dont elle dispose.
- (b) Quelles sont toutes les possibilités pour le nombre de sachets de graines que Sarah peut acheter ?
- (c) Si Sarah achète le nombre maximal de sachets de graines qu'elle peut se permettre et qu'elle plante le même nombre de sachets dans chaque plate-bande, combien de sachets plante-t-elle dans chaque plate-bande ?
- (d) En fin de compte, Sarah achète le plus grand nombre possible de sachets de graines, en choisissant quatre variétés au total : des zinnias, des sauges, des marguerites et des cosmos. Elle achète trois fois plus de sachets de zinnias que de sauges et un nombre égal de sachets de marguerites et de cosmos. Si elle achète deux sachets de sauges, combien de sachets de chacune des trois autres variétés a-t-elle achetés ?





Problem of the Week

Problem B and Solution

Flowers for Sarah

Problem

Sarah has constructed 8 new garden beds in a sunny location and is looking to plant a flower garden. Seed packets cost \$2.69 per flower variety, and she must pay a \$9.00 flat rate shipping fee for her order. She would like to plant quite a few different varieties, but not spend more than \$53.00.

- (a) Write an inequality which states Sarah's problem mathematically, using the given costs, the number of varieties she buys, and her available funds.
- (b) What are all possibilities for the number of seed packets Sarah can purchase?
- (c) If Sarah purchases the maximum number of seed packets that she can, and plants the same number of seed packets in each garden bed, then how many packets of seeds does she plant in each garden bed?
- (d) In the end, Sarah buys as many seed packets as she can, purchasing in total four varieties: zinnias, salvia, daisies, and cosmos. She purchases three times as many packets of zinnias as salvia, and equal numbers of packets of daisies and cosmos. If she buys two seed packets of salvia, how many did she buy of each of the other three flower types?

Solution

- (a) Let x represent the number of seed packets Sarah purchases. Then the total cost of her purchase will be $\$2.69 \times x + \9.00 . Since this must be no greater than \$53.00, an appropriate inequality is $\$2.69 \times x + \$9.00 \leq \$53.00$.
- (b) The most Sarah could purchase would occur if $\$2.69 \times x + \$9.00 = \$53.00$. Solving for x gives $2.69 \times x = 53 - 9$, or $x = \frac{44}{2.69} \approx 16.36$. Thus, she can buy at most 16 seed packets. That is, she could purchase any number from 0 to 16 packets.
- (c) If she maximizes her seed purchase, then she will buy 16 packets of seeds. So she will plant $16 \div 8 = 2$ packets of seeds in each of the eight garden beds.
- (d) Since she bought 2 packets of salvia, she bought $3 \times 2 = 6$ packets of zinnias. This leaves $16 - 2 - 6 = 8$ packets to be divided evenly between daisies and cosmos, so she bought $8 \div 2 = 4$ packets of each of those.

The background features a complex arrangement of 3D cubes in various shades of blue and black, creating a sense of depth and perspective. A dark, textured banner with a white border is positioned horizontally across the middle of the image. The text is centered on this banner.

Raisonnement informatiques (C)

A dark, rounded rectangular box is centered below the main title. It contains a black triangle pointing upwards towards the text. The text is in a bold, white, sans-serif font.

**Amène-moi à la
couverture**



Problème de la semaine

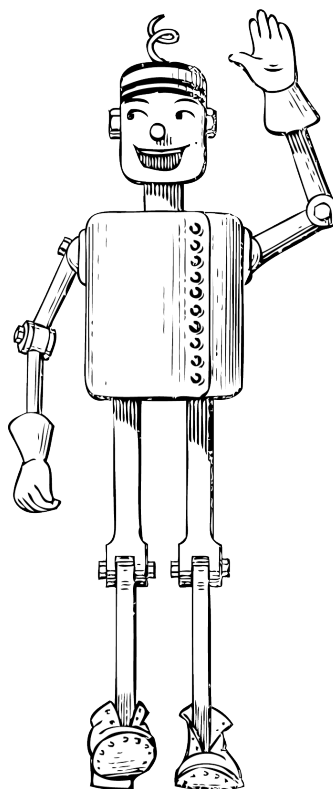
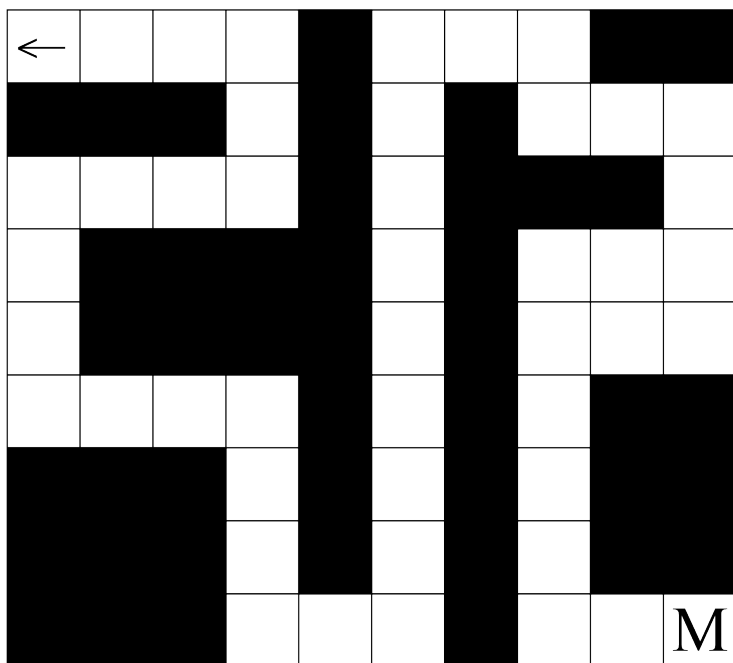
Problème B

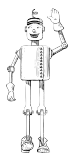
Aide Freddy à rentrer chez lui

Freddy le robot démarre dans la case du coin supérieur gauche du labyrinthe ci-dessous, orienté vers la gauche (symbolisé par la flèche $\leftarrow$). Freddy veut se rendre à sa station de recharge à la maison située dans le coin inférieur droit du labyrinthe et marquée d'un **M**. Freddy peut seulement tourner à droite (dans le sens des aiguilles d'une montre dans le diagramme) et ne tournera que du nombre de degrés que tu lui indiques. De plus, il avancera exactement du nombre de cases que tu lui commandes dans la direction indiquée par la flèche actuelle.

Par exemple, si Freddy démarre orienté vers $\leftarrow$ et que ta commande est « Tourne de 180 degrés et avance de 3 cases », il tournera de 180° vers la droite (dans le sens des aiguilles d'une montre) et avancera ensuite de 3 cases (vers la droite).

Écris un programme de commandes qui permettra à Freddy d'atteindre sa station de recharge à la maison.





Problem of the Week

Problem B and Solution

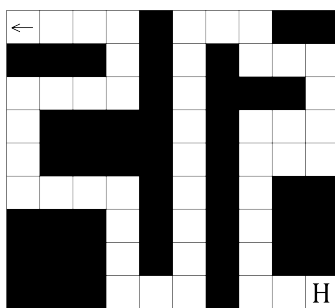
Help Send Freddy Home

Problem

Freddy the Robot starts in the top left square of the maze below, facing left (symbolized by $\leftarrow$). Freddy wants to get to his Home Charging Station in the bottom right corner of the maze, marked with an **H**. Freddy can only turn right (clockwise on the diagram), and will only turn the number of degrees which you tell him. As well, he will go exactly the number of squares you tell him and in the direction that the arrow is currently pointing.

For example, if he starts facing $\leftarrow$, and your command said, “Turn 180 degrees and go 3 spaces”, he would turn 180° right (clockwise) and then move 3 spaces forward (to the right).

Write a program of commands that gets Freddy to his Home Charging Station.



Solution

A program of commands is below. Note that there two options for commands 12 and 14. The second option is in parentheses.

1. Turn 180° and go 3 spaces.
2. Turn 90° and go 2 spaces.
3. Turn 90° and go 3 spaces.
4. Turn 270° and go 3 spaces.
5. Turn 270° and go 3 spaces.
6. Turn 90° and go 3 spaces.
7. Turn 270° and go 2 spaces.
8. Turn 270° and go 8 spaces.
9. Turn 90° and go 2 spaces.
10. Turn 90° and go 1 space.
11. Turn 270° and go 2 spaces.
12. Turn 90° and go 3 (or 2) spaces.
13. Turn 90° and go 2 spaces.
14. Turn 270° and go 4 (or 5) spaces.
15. Turn 270° and go 2 spaces.

Now Freddy is at his Home Charging Station!



Problème de la semaine

Problème B

Le codage de figures

Ayanna écrit des programmes en utilisant les commandes suivantes qui dirigent son robot pour dessiner des formes.

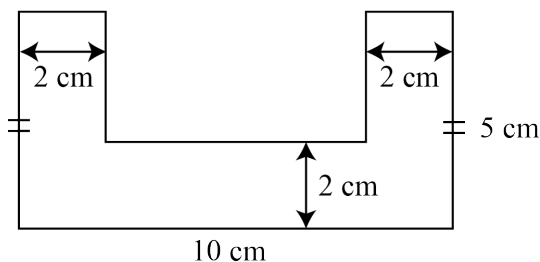
Commande	Explication
Crayon en bas	Place le crayon en position pour dessiner lorsque le robot se déplace
Avancer	Fait avancer le robot d'une distance donnée
Tourner SH	Fait pivoter le robot d'un quart de tour (90°) dans le sens horaire
Tourner SAH	Fait pivoter le robot d'un quart de tour (90°) dans le sens antihoraire

(a) Pour chaque programme, suis le code afin de tracer la forme. L'utilisation de papier quadrillé pourrait t'être utile.

(i) Crayon en bas
Répéter 4 fois
 Avancer 5 cm
 Tourner SH
Fin

(ii) Crayon en bas
Avancer 4 cm
Tourner SAH
Avancer 2 cm
Tourner SH
Avancer 2 cm
Tourner SH
Avancer 6 cm
Tourner SH
Avancer 6 cm
Tourner SH
Avancer 4 cm
Fin

(b) Écris un programme qui dirigerait le robot d'Ayanna pour dessiner la forme en U rectangulaire donnée.





Problem of the Week

Problem B and Solution

Coding Figures

Problem

Ayanna writes programs using the following commands that direct her robot to draw shapes.

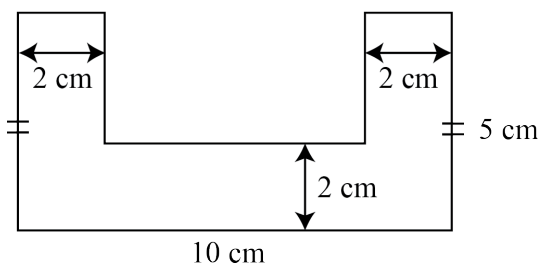
Command	Explanation
Pencil down	Puts the pencil in position to draw as the robot moves
Move	Moves the robot a given distance forward
Turn CW	Rotates the robot one quarter turn (90°) clockwise
Turn CCW	Rotates the robot one quarter turn (90°) counter clockwise

(a) For each program, follow the code to draw the shape. You may find grid paper helpful.

(i) Pencil down
Repeat 4 times
 Move 5 cm
 Turn CW
End

(ii) Pencil down
Move 4 cm
Turn CCW
Move 2 cm
Turn CW
Move 2 cm
Turn CW
Move 6 cm
Turn CW
Move 6 cm
Turn CW
Move 4 cm
End

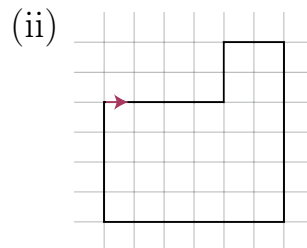
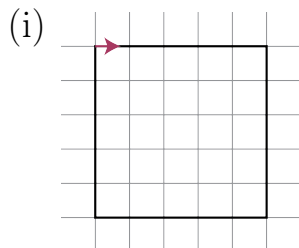
(b) Write a program that would direct Ayanna's robot to draw the given rectangular U-shape.



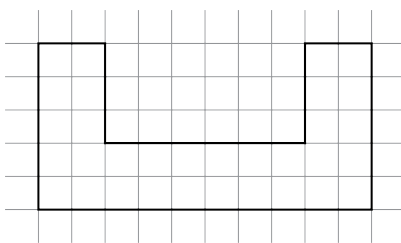


Solution

- (a) For each program, the shape is shown on 1 cm grid paper with an arrow indicating the initial direction of the robot. Note that for part (i), the shape will look the same regardless of the initial direction. However for part (ii), the shape will be rotated depending on the initial direction.



- (b) We start by determining the length of each side of the shape. We do this by drawing the shape on grid paper, as shown.



There are several different programs that can be written to direct Ayanna's robot to draw the given shape, depending on the robot's starting position and direction. In the following program, the robot starts in the bottom-left corner, facing to the right.

```
Pencil down
Move 10 cm
Turn CCW
Move 5 cm
Turn CCW
Move 2 cm
Turn CCW
Move 3 cm
Turn CW
Move 6 cm
Turn CW
Move 3 cm
Turn CCW
Move 2 cm
Turn CCW
Move 5 cm
End
```



Problème de la semaine

Problème B

Les fractions cachées

Zoé tente de trouver des fractions qui satisfont à certaines des conditions suivantes :

- (1) Le dénominateur de la fraction est divisible par 3.
 - (2) Tous les chiffres de la fraction sont pairs.
 - (3) La valeur décimale de la fraction est supérieure à 1 mais inférieure à 1,5.
 - (4) Le dénominateur de la fraction est un multiple de 2.
 - (5) La valeur décimale de la fraction est comprise entre 0,25 et 0,6.
 - (6) La valeur décimale de la fraction ne contient pas le chiffre 9.
 - (7) Le numérateur de la fraction est pair.
 - (8) La valeur décimale de la fraction est plus proche de 0,75 que de 0,55.
- (a) Trouve une fraction qui satisfait à au moins trois de ces conditions.
 - (b) Peux-tu trouver une fraction qui satisfait à toutes ces conditions ? Si oui, donne une telle fraction. Sinon, explique pourquoi une telle fraction n'existe pas.
 - (c) Quel est le nombre minimal de fractions que Zoé doit trouver de sorte que ces fractions satisfassent ensemble aux huit conditions ?





Problem of the Week

Problem B and Solution

Hidden Fractions

Problem

Zoe is trying to find fractions that satisfy some of the following conditions:

- (1) The denominator of the fraction is divisible by 3.
 - (2) All the digits in the fraction are even.
 - (3) The decimal value of the fraction is greater than 1 but less than 1.5.
 - (4) The denominator of the fraction is a multiple of 2.
 - (5) The decimal value of the fraction is between 0.25 and 0.6.
 - (6) The decimal value of the fraction does not include a 9.
 - (7) The numerator of the fraction is even.
 - (8) The decimal value of the fraction is closer to 0.75 than to 0.55.
- (a) Find a fraction that satisfies at least three of the conditions.
 - (b) Can you find a fraction that satisfies all of these conditions? If yes, give such a fraction. If no, explain why no such fraction exists.
 - (c) What is the least number of fractions Zoe needs to find such that together they meet all eight conditions?

Solution

- (a) Answers will vary. Some examples:
 - $\frac{2}{10}$ satisfies conditions (4), (6), and (7).
 - $\frac{4}{9}$ satisfies conditions (1), (5), (6), and (7).
 - $\frac{7}{9}$ satisfies conditions (1), (6), and (8).
- (b) No such fraction exists. This is because conditions (3) and (5) are contradictory, as a number cannot be both between 0.25 and 0.6 **and** be greater than 1. Similarly, conditions (5) and (8) are also contradictory.
- (c) From part (b), we know that we will need at least 2 fractions. It turns out that it is possible to satisfy all eight conditions together with just two fractions. Consider the fractions $\frac{1}{2}$ and $\frac{8}{6}$.
The fraction $\frac{1}{2} = 0.5$ satisfies conditions (4), (5), and (6).
The fraction $\frac{8}{6} = 1.333\dots$ satisfies conditions (1), (2), (3), (4), (6), (7), (8).
Answers will vary. Many other fraction pairs exist.



Gestion des données (D)



**Amène-moi à la
couverture**



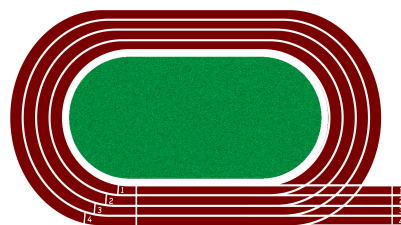
Problème de la semaine

Problème B

Qui gagne le meilleur couloir ?

L'école élémentaire Donovan Bailey a organisé une course à relais dans le cadre de sa journée d'athlétisme. Quatre équipes, A , B , C et D y participent.

Pour attribuer à chaque équipe un couloir sur la piste, la direction tire au hasard les noms des équipes. La première équipe tirée courra dans le couloir 1, la suivante dans le couloir 2, la suivante dans le couloir 3 et la dernière équipe courra dans le couloir 4.



- (a) Quelle est la probabilité théorique que l'équipe A se voie attribuer le couloir 1 ? Exprime ta réponse sous forme de fraction, de nombre décimal et de pourcentage.
- (b) La direction décide d'effectuer plusieurs tirages et d'attribuer le couloir 1 à l'équipe qui a été tirée en premier le plus souvent. Les résultats sont présentés dans le tableau.

Équipe	A	B	C	D
Nombre de fois tirée en premier	20	6	12	10

Utilise ces résultats pour calculer la probabilité expérimentale que l'équipe A obtienne le couloir 1. Comment cette probabilité se compare-t-elle à la probabilité théorique de la partie a) ?

- (c) Utilise la probabilité théorique de la partie a) pour calculer le nombre de fois où tu t'attendrais à ce que chaque équipe gagne le couloir 1. Est-ce que certains des résultats expérimentaux de la partie b) correspondent à ce résultat prévu ?

EXTENSION :

Dessine une roulette à quatre secteurs de sorte que la probabilité théorique de s'arrêter sur chaque secteur soit égale à la probabilité expérimentale calculée à partir du tableau.



Problem of the Week

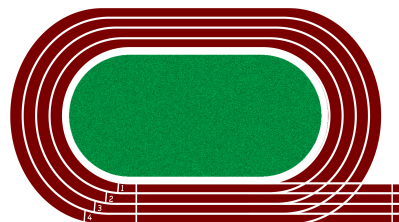
Problem B and Solution

Who Wins the Best Lane?

Problem

Donovan Bailey Elementary School has organized a relay race as part of its track and field day. Four teams, A , B , C , and D , are participating.

To assign each team to a lane on the track, the principal draws the team names randomly. The first team drawn will run in Lane 1, the next in Lane 2, the next in Lane 3, and the remaining team will run in Lane 4.



- (a) What is the theoretical probability that Team A will be assigned to Lane 1? Express your answer as a fraction, a decimal, and a percentage.
- (b) The principal decides to draw teams several times, and assign Lane 1 to the team that was drawn first most often. The results are in the table.

Team	A	B	C	D
Number of Times First	20	6	12	10

Use these results to calculate the experimental probability of Team A being assigned to Lane 1. How does this compare to the theoretical probability from part (a)?

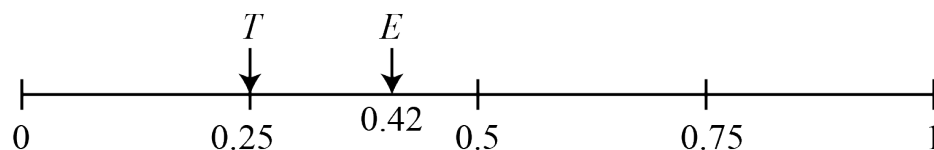
- (c) Use the theoretical probability from part (a) to calculate the number of times you would expect each team to win Lane 1. Do any of the experimental results in part (b) equal this expected result?

EXTENSION:

Draw a spinner with four sections so that the theoretical probability of landing on each section is the same as the experimental probability calculated from the table.

Solution

- (a) Since there are 4 teams, and each team has an equal chance of being assigned to Lane 1, the theoretical probability that Team A will be assigned to Lane 1 is $\frac{1}{4}$, or 0.25, or 25%.
- (b) The principal drew teams $20 + 6 + 12 + 10 = 48$ times. Thus, the experimental probability of Team A being assigned to Lane 1 was $\frac{20}{48}$, or $\frac{5}{12}$, which is approximately 0.42, or 42%. This is significantly higher than the theoretical probability from part (a). They are shown on the following probability number line, where E represents the experimental probability and T represents the theoretical probability.



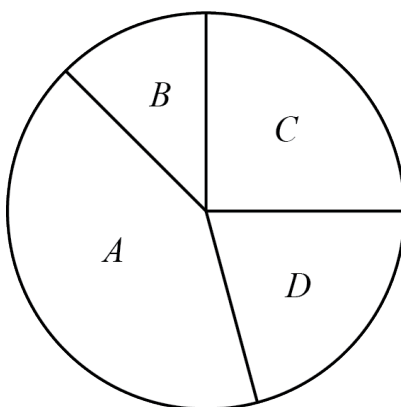
- (c) Since the principal drew teams 48 times, and each team had an equal chance of getting drawn each time, we would expect that the number of times each team would win Lane 1 would be $48 \div 4 = 12$. The experimental result for Team *C* is equal to this expected result.

SOLUTION TO EXTENSION:

When creating the spinner, we note that the probability of landing on section *C* should be $\frac{1}{4}$. Since a circle has 360° , this means that the centre angle for section *C* should be $\frac{1}{4}$ of 360° , which is 90° .

We notice that the number of times first for Team *B* is half the number of times first for Team *C*. Thus, the centre angle for section *B* should be half of 90° , which is 45° . This leaves a section with a centre angle of $360^\circ - 90^\circ - 45^\circ = 225^\circ$ that will be split between sections *A* and *D*.

We notice that the number of times first for Team *A* is twice the number of times first for Team *D*. Thus, section *A* should be twice as large as section *D*. Since they're sharing a section with a centre angle of 225° , we can determine that $\frac{2}{3}$ of this section will be section *A* and $\frac{1}{3}$ will be section *D*. Therefore, the centre angle for section *A* will be $\frac{2}{3}$ of 225° , which is 150° , and the centre angle for section *D* will be $\frac{1}{3}$ of 225° , which is 75° . Now we use our protractor to draw the spinner as shown.



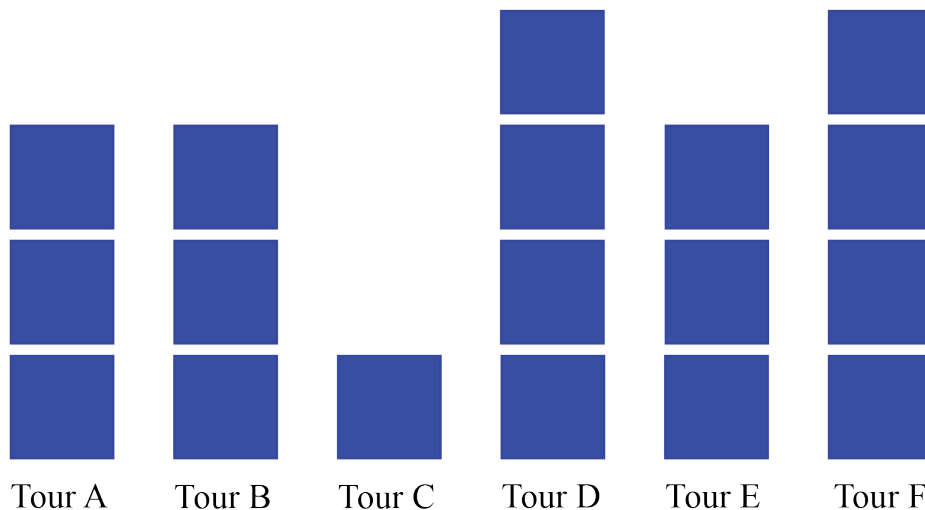


Problème de la semaine

Problème B

Moyennes à la mode

Octavia possède un grand bac rempli de blocs. En utilisant 18 blocs bleus, elle construit exactement six tours. La hauteur d'une tour correspond au nombre total de blocs utilisés pour la construire.



- (a) Détermine la moyenne, la médiane et le mode des hauteurs des tours construites par Octavia.
- (b) Crée un ensemble différent de six tours utilisant 18 blocs, de sorte que la moyenne des hauteurs soit supérieure au mode des hauteurs.
- (c) Crée un ensemble différent de six tours utilisant 18 blocs, de sorte que la moyenne des hauteurs soit supérieure à la médiane des hauteurs.
- (d) Crée un ensemble différent de six tours utilisant 18 blocs, de sorte que la médiane des hauteurs soit supérieure à la moyenne des hauteurs.



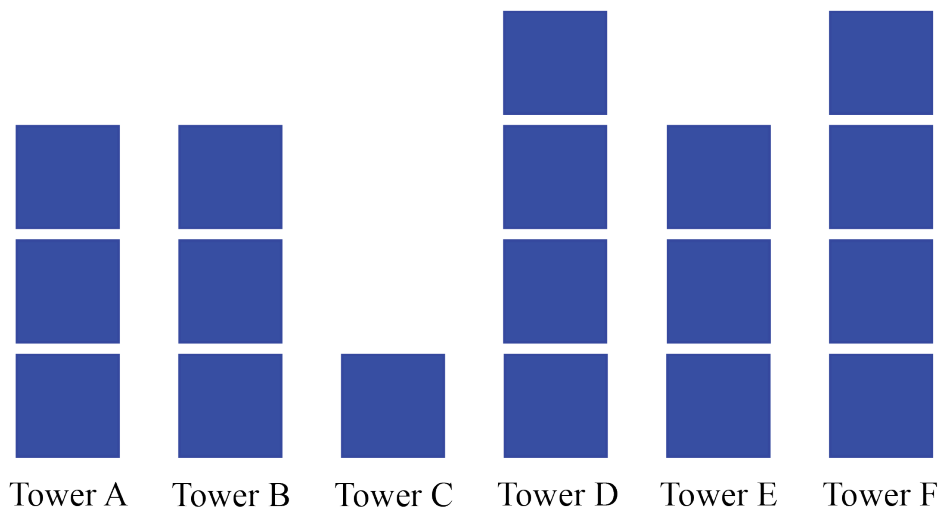
Problem of the Week

Problem B and Solution

Means à la Mode

Problem

Octavia has a large bin of blocks. Using 18 blue blocks she makes exactly six towers. The height of the tower is the total number of blocks used in that tower.



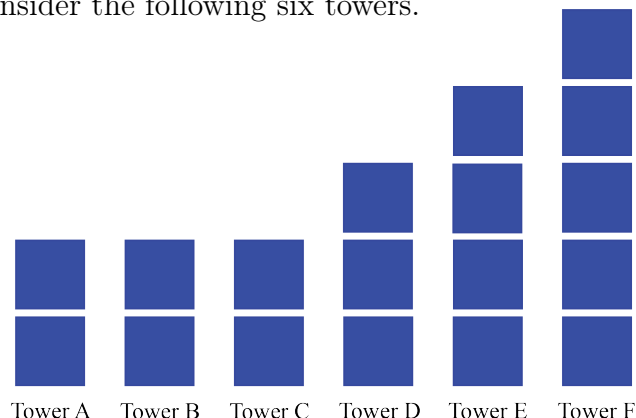
- (a) Determine the mean, the median, and the mode of the heights of the towers made by Octavia.
- (b) Create a different set of six towers using 18 blocks where the mean height is greater than the mode height.
- (c) Create a different set of six towers using 18 blocks where the mean height is greater than the median height.
- (d) Create a different set of six towers using 18 blocks where the median height is greater than the mean height.



Solution

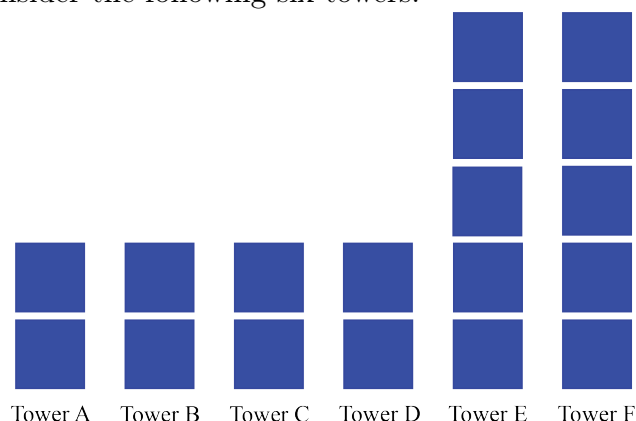
(a) Written in increasing order, the tower heights are 1, 3, 3, 3, 4, 4. Thus the mode is 3, the median is $(3 + 3) \div 2 = 3$, and the mean is $(1 + 3 + 3 + 3 + 4 + 4) \div 6 = 3$.

(b) Answers will vary. Consider the following six towers.



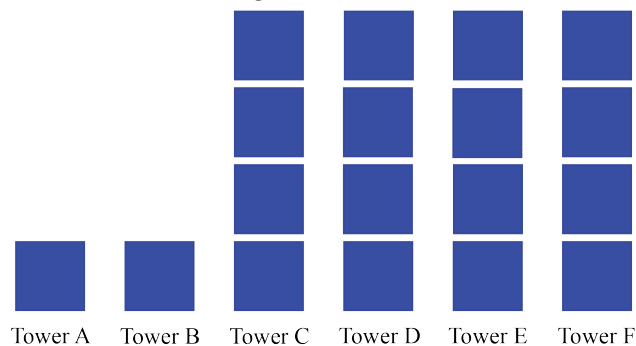
The tower heights are 2, 2, 2, 3, 4, 5. The mode is 2 and the mean is $(2 + 2 + 2 + 3 + 4 + 5) \div 6 = 3$. Thus, the mean is greater than the mode.

(c) Answers will vary. Consider the following six towers.

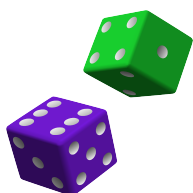


The tower heights are 2, 2, 2, 2, 5, 5. The median is $(2 + 2) \div 2 = 2$ and the mean is $(2 + 2 + 2 + 2 + 5 + 5) \div 6 = 3$. Thus, the mean is greater than the median.

(d) Answers will vary. Consider the following six towers.



The tower heights are 1, 1, 4, 4, 4, 4. The median is $(4 + 4) \div 2 = 4$, and the mean is $(1 + 1 + 4 + 4 + 4 + 4) \div 6 = 3$. Thus, the median is greater than the mean.



Problème de la semaine

Problème B

Des dés risqués

Jordyn et Langdon ont deux dés, un violet et un vert. Lorsqu'ils lancent les dés, ils savent qu'il y a un total de $6 \times 6 = 36$ résultats possibles pour les deux nombres qui apparaissent sur le dessus des dés.

Violet	Vert
1 •	• 1
2 •	• 2
3 •	• 3
4 •	• 4
5 •	• 5
6 •	• 6

- (a) Dans le diagramme, relie n'importe quel point à gauche à un point à droite de façon que le produit des deux nombres soit impair. Combien y a-t-il de façons d'obtenir un produit impair ?
- (b) Que t'indique ton résultat de la partie (a) sur le nombre de façons d'obtenir un produit pair ?
- (c) Est-il plus probable d'obtenir un produit de 4 ou un produit de 6 ? Pourquoi ?
- (d) Dans le tableau, inscrie les produits impairs possibles dans la première colonne, le nombre de façons d'obtenir chacun de ces produits dans la colonne du milieu et la probabilité théorique de chacun de ces résultats dans la troisième colonne. (Le travail est déjà fait pour le produit impair 1.)

Produit impair	Nombre de façons	Probabilité théorique
1	1	$\frac{1}{36}$

- (e) Lance deux dés 36 fois et note le produit des deux nombres obtenus pour chaque lancer. Compte le nombre de fois que chaque produit impair s'est produit, puis calcule la probabilité expérimentale de lancer ce nombre. Compare la probabilité expérimentale à la probabilité théorique calculée dans la partie (d).



Problem of the Week

Problem B and Solution

Dicey Products

Problem

Jordyn and Langdon have two dice, one purple and one green. When they roll the dice, they know there are a total of $6 \times 6 = 36$ possible outcomes for the top two numbers on the dice.

- (a) In the diagram, join any dot on the left with one on the right such that the product of those two numbers is odd. How many ways are there to obtain an odd product?
- (b) What does your result in part (a) tell you about how many ways there are to obtain an even product?
- (c) Is the product 4 or 6 more likely to occur? Why?
- (d) In the table, enter the possible odd products in the first column, the number of ways that product could occur in the middle column, and the theoretical probability of that outcome in the third column. (This has been done for odd product 1.)



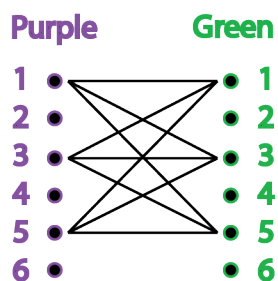
Odd Product	Number of Ways	Theoretical Probability
1	1	$\frac{1}{36}$

- (e) Roll two dice 36 times and record the product of the top two numbers for each roll. Count the number of each odd product that occurred, and then calculate the experimental probability of rolling that number. Compare the experimental probability to the theoretical probability calculated in part (d).



Solution

- (a) Odd products will occur exactly when the top two numbers are odd. We connect each odd number on the left with each odd number on the right to obtain the completed diagram shown. Counting the number of connections, we determine that there are 9 ways to obtain an odd product.



- (b) Since there are 36 possible products in total, and a product must be either even or odd, there must be $36 - 9 = 27$ ways to obtain an even product.
- (c) The product 6 is more likely to occur. This is because $6 = 1 \times 6$, $6 = 6 \times 1$, $6 = 2 \times 3$, and $6 = 3 \times 2$, and so can be formed by 4 different rolls. However, since $4 = 1 \times 4$, $4 = 4 \times 1$, and $4 = 2 \times 2$, it can only be formed by 3 different rolls.
- (d) Noting that there is only one way to get each of 1, 9, and 25, and two ways to get each of 3, 5, and 15, the appropriate data and the theoretical probabilities are calculated.

Odd Product	Number of Ways	Theoretical Probability
1	1	$\frac{1}{36}$
3	2	$\frac{2}{36}$
5	2	$\frac{2}{36}$
9	1	$\frac{1}{36}$
15	2	$\frac{2}{36}$
25	1	$\frac{1}{36}$

- (e) Answers will vary. Note that the result of a single experiment won't necessarily agree very closely with the theoretical probabilities.

SUGGESTION: Average the experimental probabilities obtained by all students in your class for each of the six odd products. Do these average probabilities agree more closely with the theoretical probabilities for each odd product than those of individual students?



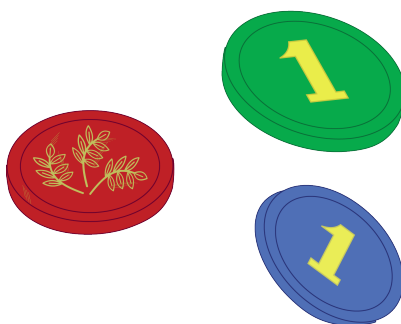
Problème de la semaine

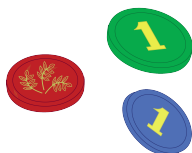
Problème B

Trois à la fois

Jack et Jill ont inventé un jeu où ils lancent à tour de rôle une pièce rouge, une pièce bleue et une pièce verte. Chaque pièce a un côté face (F) et un autre pile (P). Ils accumulent des points selon les règles suivantes.

1. Si le lancer donne trois faces (FFF) ou trois piles (PPP), alors Jack obtient un point.
 2. Si le lancer donne toute autre combinaison de faces et de piles (par exemple, PPF), alors Jill obtient un point.
 3. Le gagnant est le joueur qui a le plus de points après 10 lancers des trois pièces.
- (a) Joue à ce jeu avec un partenaire. Désigne un joueur pour être Jack et l'autre pour être Jill.
- (b) Un jeu est considéré comme *équitable* si chaque joueur a la même probabilité de gagner. Explique pourquoi ce jeu **n'est pas** équitable.
- (c) Explique comment tu pourrais modifier les règles pour rendre ce jeu équitable.





Problem of the Week

Problem B and Solution

Three Coins in a Toss

Problem

Jack and Jill have made up a game where they take turns tossing a red coin, a blue coin, and a green coin. Each coin has heads (H) on one side and tails (T) on the other side. They earn points according to the following rules.

1. If the toss results in all three coins being heads (HHH), or all three being tails (TTT), then Jack gets one point.
 2. If the toss results in any other combination of heads and tails (e.g., TTH), then Jill gets one point.
 3. The winner of the game is the player with the most points after 10 tosses of all three coins.
- (a) Play this game with a partner. Assign one player to be Jack and the other to be Jill.
- (b) A game is considered a *fair game* if each player is equally likely to win the game. Explain why this game is **not** a fair game.
- (c) Explain how you could change the rules to make this a fair game.

Solution

- (a) After playing the game, check how many pairs had Jack win and how many had Jill win.
- (b) First we determine the possible outcomes of each toss of the three coins. In total there are eight possible outcomes:

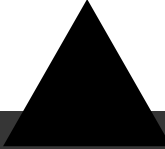
TTT, TTH, THT, HTT, THH, HTH, HHT, HHH

From here we see that Jack gets a point for only 2 out of the 8 outcomes, while Jill gets a point for 6 out of the 8 outcomes. Thus, the theoretical probability of Jack getting a point is $\frac{2}{8} = \frac{1}{4}$, while the theoretical probability of Jill getting a point is $\frac{6}{8} = \frac{3}{4}$. Since Jill is more likely to get a point on any toss, she is more likely to win the game. Thus, it is not a fair game.

- (c) There are many different ways to change the rules to make it a fair game. One way is to give Jack a point if there are at least 2 heads, and give Jill a point if there are at least 2 tails. Then Jack would get a point for four outcomes (HHH, THH, HTH, and HHT), and Jill would get a point for the remaining four outcomes (TTT, TTH, THT, and HTT). This would be a fair game because each player is equally likely to earn a point on any toss. Thus, each player is equally likely to win the game.



Géométrie et mesure (G)



**Amène-moi à la
couverture**

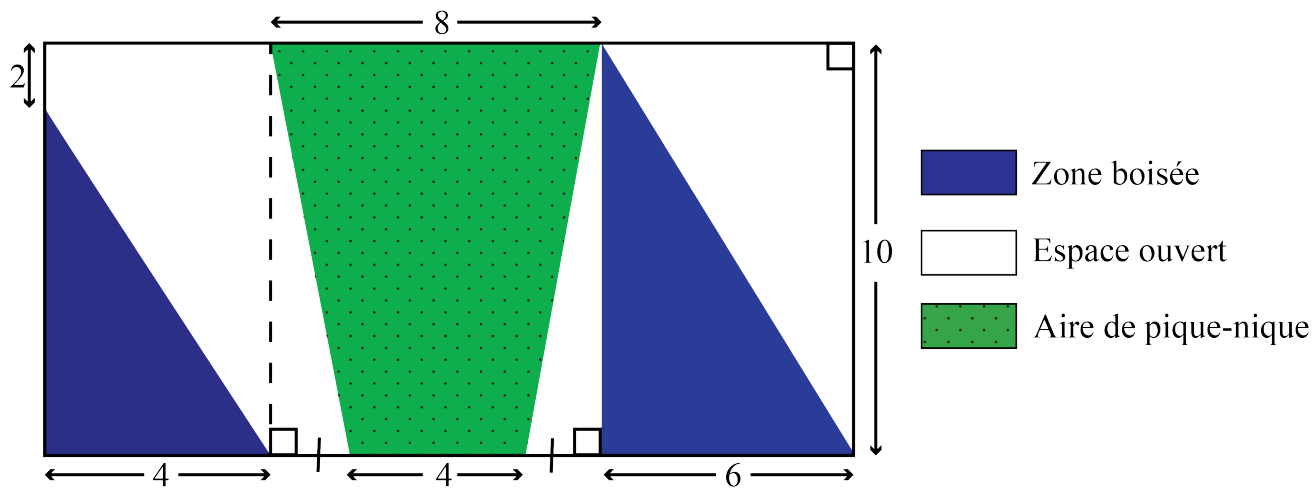


Problème de la semaine

Problème B

Bienvenue au parc de la ville des Formes

La ville des Formes a aménagé un nouveau parc. Le parc est de forme rectangulaire et il est divisé en zones boisées, espaces ouverts et une aire de pique-nique, comme illustré.



Quelle est l'aire totale de chaque type d'espace ?



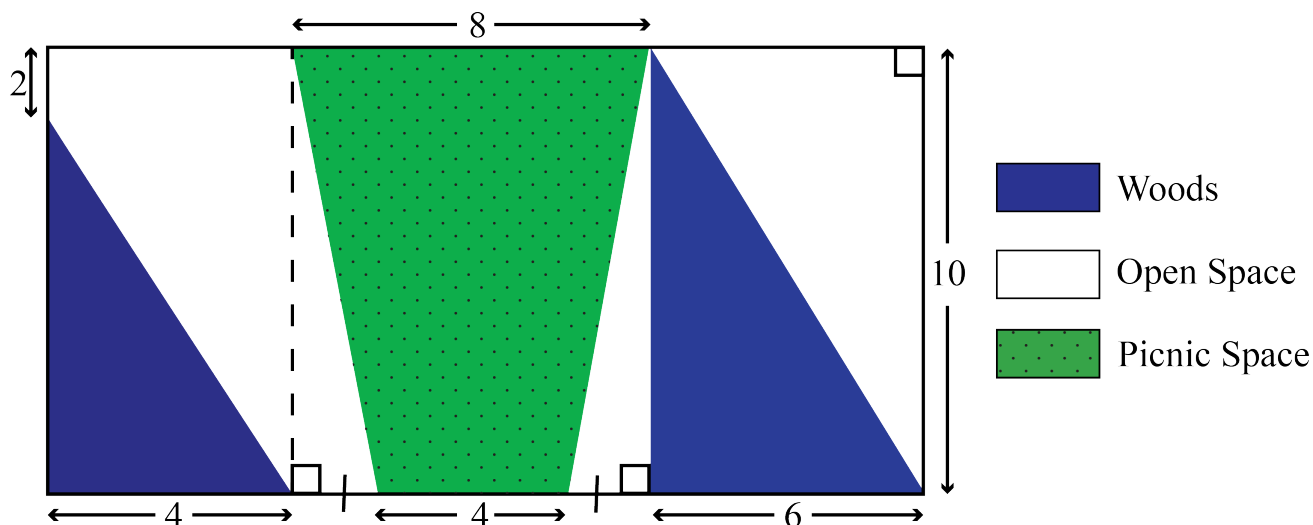
Problem of the Week

Problem B and Solution

Welcome to Shape City Park

Problem

Shape City has built a new park. The park is in the shape of a rectangle, and is divided into woods, open space, and picnic space, as shown.



What is the total area of each type of space?

Solution

Both woods regions are right-angled triangles. One triangle has base 4 and height $10 - 2 = 8$, and thus has area equal to $\frac{1}{2}(4 \times 8) = 16$ square units. The other triangle has base 6 and height 10, and thus has area equal to $\frac{1}{2}(6 \times 10) = 30$ square units. Thus, the area of the woods is $16 + 30 = 46$ square units.

The picnic space is in the shape of a trapezoid with height 10 and parallel sides of length 8 and 4. Thus, the area of the picnic space is $\frac{1}{2}(8 + 4) \times 10 = 60$ square units.

Alternatively, the area of the trapezoidal picnic space can be determined by calculating the area of the rectangular strip containing the trapezoid, and subtracting the two triangular regions of open space. That is, the area of the picnic space is $8 \times 10 - \frac{1}{2}(2 \times 10) - \frac{1}{2}(2 \times 10) = 60$ square units.

The entire park has vertical length 10 and width equal to $4 + 8 + 6 = 18$. Hence the total area of the park is $10 \times 18 = 180$ square units.

The area of the open space is equal to the total area of the park minus the areas of the picnic space and woods, or $180 - 46 - 60 = 74$ square units.



Problème de la semaine

Problème B

Frank mélange les choses

Frank veut préparer un punch aux fruits pour une fête de famille. La recette de son punch aux fruits est la suivante :

500 mL de jus de canneberge 1,8 L de jus d'orange 1450 mL de jus d'ananas 350 mL de soda au gingembre 2,4 L d'eau

- (a) Frank dispose de trois récipients de capacités différentes : une cruche de 6 L, une cruche de 5700 mL et une cruche de 6,8 L. Quelle(s) cruche(s) Frank pourrait-il utiliser pour contenir son punch aux fruits ?
- (b) Si le coût moyen de ses quatre boissons aromatisées est de 4,56 \$ par litre, environ combien cela coûtera-t-il à Frank pour préparer son punch aux fruits ? Suppose que Frank n'a pas besoin de payer pour l'eau.
- (c) Il y aura 31 membres de la famille à la fête et Frank veut préparer suffisamment de punch pour que chaque personne puisse boire une tasse (250 mL). Actuellement, sa recette ne produit pas assez de punch pour tout le monde. Ajuste les quantités de chaque ingrédient dans la recette afin que Frank ait assez de punch pour tous les invités, sans trop de surplus.





Problem of the Week

Problem B and Solution

Frank Mixes Things Up

Problem

Frank wants to make a fruit punch drink for a family party. The recipe for his fruit punch drink is:

500 mL cranberry juice
1.8 L orange juice
1450 mL pineapple juice
350 mL ginger ale
2.4 L water

- (a) Frank has three different-sized containers: a 6 L jug, a 5700 mL jug, and a 6.8 L jug. Which jug(s) could Frank use to hold his fruit punch?
- (b) If the costs of his four flavoured drinks average to \$4.56 per litre, approximately how much will it cost Frank to make his fruit punch? Assume Frank does not need to pay for water.
- (c) There will be 31 family members at the party and Frank wants to make enough fruit punch so that each person can drink one cup (250 mL). Currently his recipe will not make enough fruit punch for everyone. Adjust the amounts of each ingredient in the recipe so that Frank has enough of his fruit punch drink for everyone, but not too much extra.



Solution

- (a) Since there are 1000 mL in one litre, we can convert all volumes in the recipe to litres. This gives 0.5 L of cranberry juice, 1.45 L of pineapple juice, and 0.35 L of ginger ale. Now we can find the total volume of the fruit punch by adding up the volumes of each of the five ingredients. This gives
 $0.5 + 1.8 + 1.45 + 0.35 + 2.4 = 6.5$ L.

In litres, the capacity of the jugs are 6 L, 5.7 L, and 6.8 L. Therefore, the only jug large enough to hold 6.5 L of fruit punch is the 6.8 L jug.



- (b) First we find the total volume of the four flavoured drinks. We could add up the volumes of the four flavoured drinks, or simply subtract the volume of water from the total we found in part (a). This gives $6.5 - 2.4 = 4.1$ L. Since the costs of the four flavoured drinks average to \$4.56 per litre, and their total volume is 4.1 L, then it will cost Frank approximately $\$4.56 \times 4.1 \approx \18.70 to make the fruit punch.
- (c) One recipe makes a total of 6.5 L or 6500 mL of fruit punch. In order to make enough for everybody, Frank should make $31 \times 250 = 7750$ mL. So his recipe is short by $7750 - 6500 = 1250$ mL.

We could just double the recipe, but since the difference is only 1250 mL, that would make much more fruit punch than Frank actually wants. Notice that $7750 \div 6500 \approx 1.1923$. So if we multiply each of the volumes in the recipe by 1.2, then there will be enough fruit punch, without having too much extra. This gives the following volumes, in mL:

- Cranberry juice: $500 \text{ mL} \times 1.2 = 600 \text{ mL}$
- Orange juice: $1800 \text{ mL} \times 1.2 = 2160 \text{ mL}$
- Pineapple juice: $1450 \text{ mL} \times 1.2 = 1740 \text{ mL}$
- Ginger ale: $350 \text{ mL} \times 1.2 = 420 \text{ mL}$
- Water: $2400 \text{ mL} \times 1.2 = 2880 \text{ mL}$

The total volume of the fruit punch will then be $600 + 2160 + 1740 + 420 + 2880 = 7800$ mL, which is enough for everyone, with only $7800 - 7750 = 50$ mL extra.



Problème de la semaine

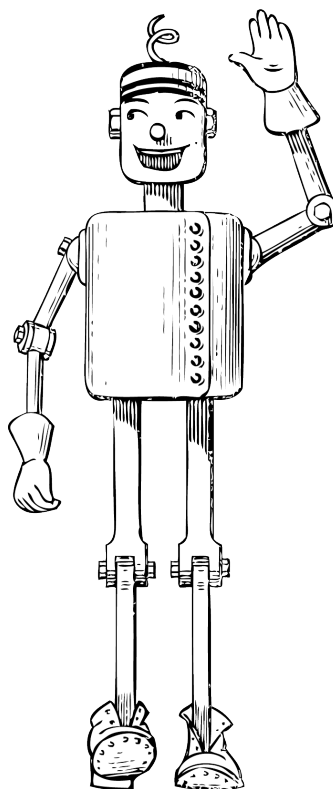
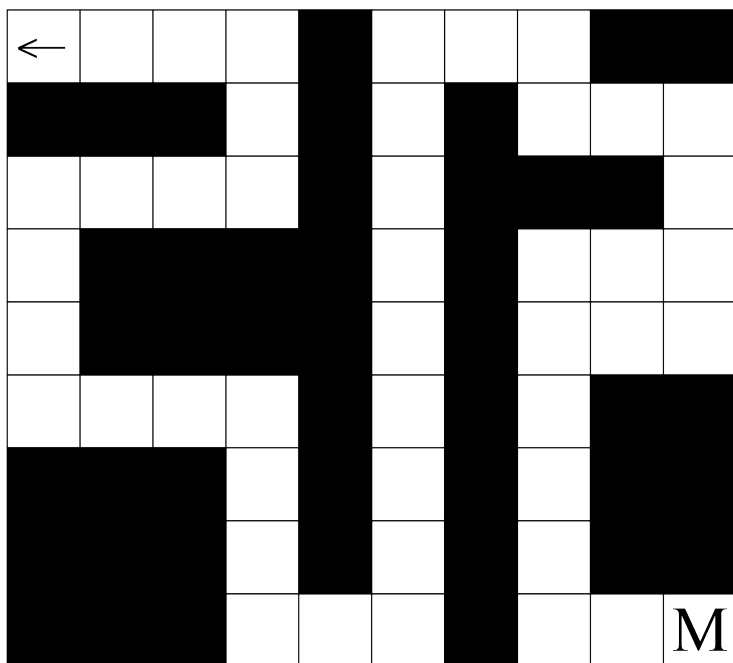
Problème B

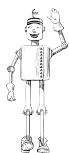
Aide Freddy à rentrer chez lui

Freddy le robot démarre dans la case du coin supérieur gauche du labyrinthe ci-dessous, orienté vers la gauche (symbolisé par la flèche $\leftarrow$). Freddy veut se rendre à sa station de recharge à la maison située dans le coin inférieur droit du labyrinthe et marquée d'un **M**. Freddy peut seulement tourner à droite (dans le sens des aiguilles d'une montre dans le diagramme) et ne tournera que du nombre de degrés que tu lui indiques. De plus, il avancera exactement du nombre de cases que tu lui commandes dans la direction indiquée par la flèche actuelle.

Par exemple, si Freddy démarre orienté vers $\leftarrow$ et que ta commande est « Tourne de 180 degrés et avance de 3 cases », il tournera de 180° vers la droite (dans le sens des aiguilles d'une montre) et avancera ensuite de 3 cases (vers la droite).

Écris un programme de commandes qui permettra à Freddy d'atteindre sa station de recharge à la maison.





Problem of the Week

Problem B and Solution

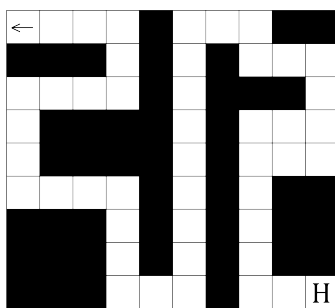
Help Send Freddy Home

Problem

Freddy the Robot starts in the top left square of the maze below, facing left (symbolized by $\leftarrow$). Freddy wants to get to his Home Charging Station in the bottom right corner of the maze, marked with an **H**. Freddy can only turn right (clockwise on the diagram), and will only turn the number of degrees which you tell him. As well, he will go exactly the number of squares you tell him and in the direction that the arrow is currently pointing.

For example, if he starts facing $\leftarrow$, and your command said, “Turn 180 degrees and go 3 spaces”, he would turn 180° right (clockwise) and then move 3 spaces forward (to the right).

Write a program of commands that gets Freddy to his Home Charging Station.



Solution

A program of commands is below. Note that there two options for commands 12 and 14. The second option is in parentheses.

1. Turn 180° and go 3 spaces.
2. Turn 90° and go 2 spaces.
3. Turn 90° and go 3 spaces.
4. Turn 270° and go 3 spaces.
5. Turn 270° and go 3 spaces.
6. Turn 90° and go 3 spaces.
7. Turn 270° and go 2 spaces.
8. Turn 270° and go 8 spaces.
9. Turn 90° and go 2 spaces.
10. Turn 90° and go 1 space.
11. Turn 270° and go 2 spaces.
12. Turn 90° and go 3 (or 2) spaces.
13. Turn 90° and go 2 spaces.
14. Turn 270° and go 4 (or 5) spaces.
15. Turn 270° and go 2 spaces.

Now Freddy is at his Home Charging Station!



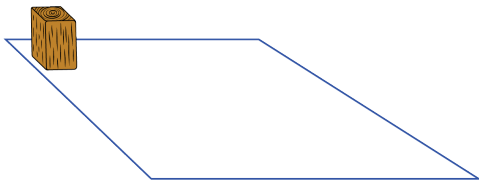
Problème de la semaine

Problème B

Le bloc-tampon

Evelina utilise un bloc de bois comme tampon pour créer du papier d'emballage décoratif. Le bloc de bois mesure 3 cm par 3 cm par 4 cm et Evelina utilise l'une de ses faces carrées de 3 cm par 3 cm pour tamponner.

- (a) Evelina crée une rangée de 10 carrés tamponnés avec un écart de 0,5 cm entre deux carrés consécutifs. Détermine la longueur de cette rangée.
- (b) Evelina dispose d'un morceau de papier d'emballage carré dont le côté mesure 77,5 cm. Elle laisse une marge de 0,5 cm à partir de chaque bord, puis crée des rangées de carrés tamponnés avec un écart de 0,5 cm entre les carrés, ainsi qu'entre les rangées. Combien de carrés tamponnés au total seront placés sur ce morceau de papier d'emballage ?





Problem of the Week

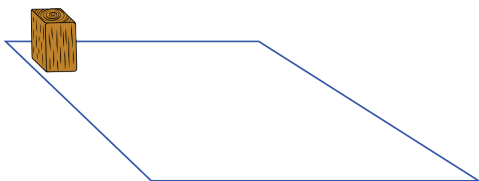
Problem B and Solution

Stamp it Out

Problem

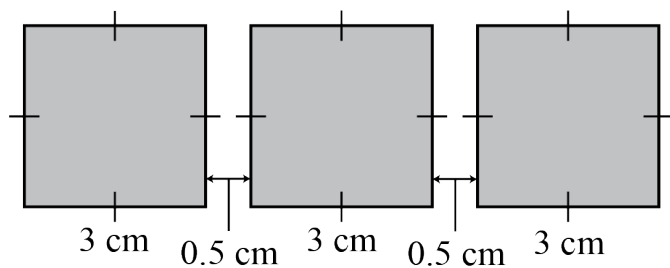
Evelina is using a wooden block as a stamp to make decorative wrapping paper. The wooden block measures 3 cm by 3 cm by 4 cm and Evelina is stamping with one of the 3 cm by 3 cm square faces of the block.

- (a) Evelina creates a row of 10 stamped squares with a gap of 0.5 cm in between each square. Determine the length of this row.
- (b) Evelina has a square piece of wrapping paper with side length 77.5 cm. She leaves a gap of 0.5 cm around each edge and then creates rows of stamped squares with a gap of 0.5 cm in between each square as well as between each row. How many stamped squares in total will be on this piece of wrapping paper?



Solution

- (a) We start by drawing a diagram of the first three stamped squares in the row.



From here we show two different ways to solve the problem.

Solution 1

In this solution we use a table to help us determine the length of the row. The first stamped square has a length of 3 cm, and each square after that adds $3 + 0.5 = 3.5$ cm to the length.



Number of Stamped Squares	Length of Row (cm)
1	3
2	$3 + 3.5 = 6.5$
3	$6.5 + 3.5 = 10$
4	$10 + 3.5 = 13.5$
5	$13.5 + 3.5 = 17$
6	$17 + 3.5 = 20.5$
7	$20.5 + 3.5 = 24$
8	$24 + 3.5 = 27.5$
9	$27.5 + 3.5 = 31$
10	$31 + 3.5 = 34.5$

Thus, the length of the row is 34.5 cm.

Solution 2

In this solution we use an algebraic expression help us determine the length of the row. This solution is more efficient, especially as the row gets longer. We know the length of each stamped square is 3 cm and the length of each gap between the squares is 0.5 cm. If we let s represent the total number of squares and g represent the total number of gaps between the squares, then the length of the row, in cm, is $3 \times s + 0.5 \times g$.

If there are 10 squares in the row then there are 9 gaps between them. Then we can use our equation to determine the length of the row.

$$\begin{aligned}3 \times s + 0.5 \times g &= 3 \times 10 + 0.5 \times 9 \\&= 30 + 4.5 \\&= 34.5\end{aligned}$$

Thus, the length of the row is 34.5 cm.

- (b) We will start by determining the number of stamped squares in each row. Since the piece of wrapping paper is a square, this will also be the number of rows of stamped squares.

If we subtract the gap at the start of the row, then the length of the row will be $77.5 - 0.5 = 77$ cm. This is helpful because each square is followed by a gap, so the total length of the square and its gap is $3 + 0.5 = 3.5$ cm. Then we can divide to determine the number of squares in the row. Since $77 \div 3.5 = 22$, it follows that there are 22 squares in the row.

Since there are also 22 rows of squares, the total number of squares is $22 \times 22 = 484$.

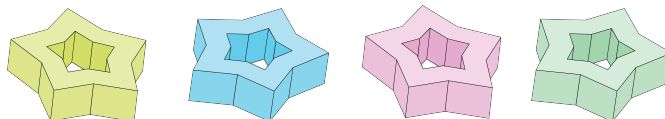


Problème de la semaine

Problème B

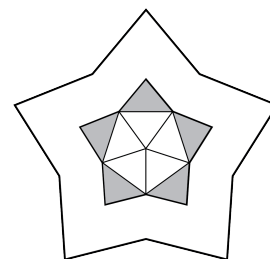
Des anneaux glacés en forme d'étoile

Michel fabrique des anneaux de glace colorée en forme d'étoile en versant de l'eau colorée dans un moule spécial, puis en la congelant. Quelques-unes de ses étoiles glacées sont illustrées.

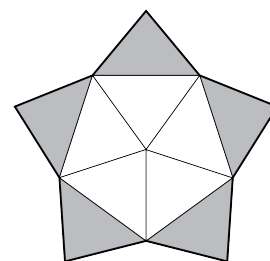


Le moule de Michel est composé d'une étoile extérieure et d'une étoile intérieure, et l'eau est versée entre les parois de l'étoile intérieure et de l'étoile extérieure.

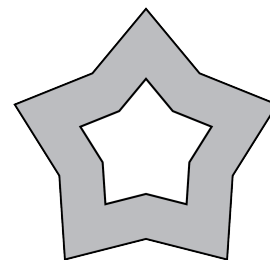
- (a) Le diagramme montre l'étoile intérieure et l'étoile extérieure du moule de Michel. L'étoile intérieure est divisée en cinq triangles externes, illustrés en gris, et cinq triangles internes, illustrés en blanc. Chaque triangle externe a une base de 3 cm et une hauteur de 1,7 cm. Chaque triangle interne a une base de 3 cm et une hauteur de 2 cm. Détermine l'aire de l'étoile intérieure.



- (b) Si on ignore l'étoile intérieure, on peut alors diviser l'étoile extérieure en cinq triangles externes et cinq triangles internes, comme on l'a fait pour l'étoile intérieure dans la partie (a). La base et la hauteur de chaque triangle de l'étoile extérieure sont le double de la base et de la hauteur du triangle correspondant dans l'étoile intérieure. Détermine l'aire de l'étoile extérieure.



- (c) La région ombrée représente l'aire comprise entre les parois de l'étoile intérieure et de l'étoile extérieure. Utilise tes calculs des parties (a) et (b) pour déterminer l'aire de cette région.





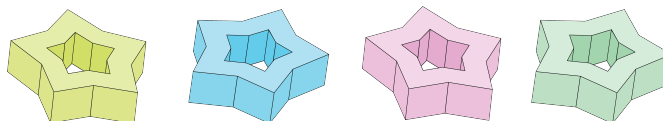
Problem of the Week

Problem B and Solution

Icy Star Rings

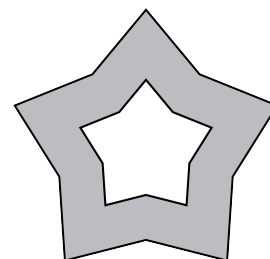
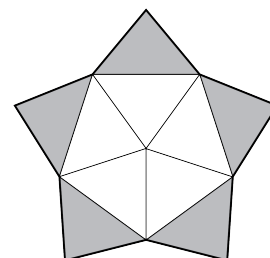
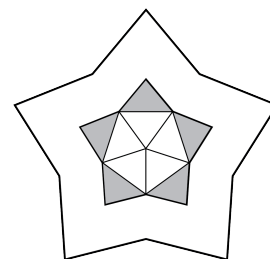
Problem

Michel makes coloured ice in the shape of a star ring by pouring coloured water into a special mold and freezing it. Some of his ice stars are shown.



Michel's mold consists of an outer star and an inner star, and water is poured between the inner and outer star walls.

- (a) The diagram shows the inner and outer stars from Michel's mold. The inner star is divided into five outer triangles, shown in grey, and five inner triangles, shown in white. The outer triangles each have a base of 3 cm and a height of 1.7 cm. The inner triangles each have a base of 3 cm and a height of 2 cm. Determine the area of the inner star.
- (b) If we ignore the inner star, then we can divide the outer star into five outer triangles and five inner triangles, like we did with the inner star in part (a). The base and height of each of the triangles in the outer star are twice the base and height of their corresponding triangle in the inner star. Determine the area of the outer star.
- (c) The shaded region represents the area between the inner and outer star walls. Use your calculations from parts (a) and (b) to determine the area of this region.





Solution

- (a) The area of each outer triangle is $3 \times 1.7 \div 2 = 5.1 \div 2 = 2.55 \text{ cm}^2$.

Since there are 5 outer triangles, their total area is $5 \times 2.55 = 12.75 \text{ cm}^2$.

The area of each inner triangle is $3 \times 2 \div 2 = 3 \text{ cm}^2$.

Since there are 5 inner triangles, their total area is $5 \times 3 = 15 \text{ cm}^2$.

Therefore, the area of the inner star is $12.75 + 15 = 27.75 \text{ cm}^2$.

- (b) The base of each outer triangle is $2 \times 3 = 6 \text{ cm}$ and the height is $2 \times 1.7 = 3.4 \text{ cm}$.

The area of each outer triangle is then $6 \times 3.4 \div 2 = 20.4 \div 2 = 10.2 \text{ cm}^2$.

Since there are 5 outer triangles, their total area is $5 \times 10.2 = 51 \text{ cm}^2$.

The base of each inner triangle is $2 \times 3 = 6 \text{ cm}$ and the height is $2 \times 2 = 4 \text{ cm}$.

The area of each inner triangle is then $6 \times 4 \div 2 = 24 \div 2 = 12 \text{ cm}^2$.

Since there are 5 inner triangles, their total area is $5 \times 12 = 60 \text{ cm}^2$.

Therefore, the area of the outer star is $51 + 60 = 111 \text{ cm}^2$.

Notice that this is 4 times the area of the inner star. Does it make sense why the area was multiplied by 4 when we doubled the dimensions?

- (c) The area of the shaded region is the area of the outer star minus the area of the inner star. This is $111 - 27.75 = 83.25 \text{ cm}^2$.



Problème de la semaine

Problème B

Calendriers grégoriens

Le calendrier grégorien est utilisé dans la majorité des régions du monde aujourd'hui. Afin de garder ce calendrier synchronisé avec l'année solaire (le temps nécessaire à la Terre pour effectuer une révolution complète autour du Soleil), on y inclut des années bissextiles comportant un jour supplémentaire en février. Les années bissextiles surviennent généralement tous les quatre ans, lors des années divisibles par 4. Cependant, les années divisibles par 100 sont exclues, SAUF si elles sont divisibles par 400. Par exemple, l'année 2000 était bissextile, alors que l'année 1900 ne l'était pas.

L'année 2025 a un calendrier différent de celui de 2024, puisqu'elle a commencé un jour différent de la semaine. Le 1^{er} janvier 2024 était un lundi, tandis que le 1^{er} janvier 2025 était un mercredi.

- (a) Le 1^{er} janvier 2023 était un dimanche. Explique pourquoi le 1^{er} janvier 2024 est tombé un jour de la semaine plus tard qu'en 2023, alors que le 1^{er} janvier 2025 est tombé deux jours de la semaine plus tard qu'en 2024.
- (b) Le 1^{er} janvier 1992 et le 1^{er} janvier 2025 sont tous les deux tombés un mercredi. L'année 1992 avait-elle le même calendrier annuel que celui de l'année 2025 ?
- (c) Combien de calendriers annuels différents existe-t-il au total ? Deux calendriers annuels sont considérés identiques si chaque date tombe le même jour de la semaine.





Problem of the Week

Problem B and Solution

Gregorian Calendars

Problem

The Gregorian Calendar is used in most parts of the world today. In order to keep this calendar in synch with the solar year (the time for the Earth to complete one orbit around the sun), it has leap years with an extra day in February. Leap years generally occur every four years, in years that are divisible by 4. However, years divisible by 100 are excluded, UNLESS they are divisible by 400. For example, the year 2000 was a leap year, but 1900 was not.

The year 2025 has a different calendar than the year 2024 since it started on a different day of the week. January 1, 2024 was on a Monday, while January 1, 2025 was on a Wednesday.

- (a) January 1, 2023 was on a Sunday. Explain why January 1, 2024 was one day of the week later than in 2023, while January 1, 2025 was two days of the week later than in 2024.
- (b) January 1, 1992 and January 1, 2025 were both on a Wednesday. Did 1992 have the same yearly calendar as 2025?
- (c) How many different yearly calendars are there in total? Two yearly calendars are considered the same if each date occurred on the same day of the week.



Solution

- (a) Since the year 2023 had 365 days, and a week has 7 days, there were $365 \div 7 = 52 \frac{1}{7}$ weeks in 2023. That is, there were 52 full weeks plus 1 day. Thus, all of the next year's dates fall one day of the week later than the previous year. This is why January 1, 2024 was one day of the week later than January 1, 2023.
However, 2024 was a leap year with 366 days, so it had 52 full weeks plus 2 days. Thus, January 1, 2025 was two days later in the week (a Wednesday) than January 1, 2024 (a Monday).
- (b) Despite the fact that January 1 in 1992 and in 2025 were both on a Wednesday, the years won't have the same yearly calendar because 1992 was a leap year. This is since 1992 is divisible by 4, but not 100.
- (c) There are 7 possible weekdays on which January 1 can occur. For each of these possibilities, there is a calendar that includes February 29 and a calendar that does not. Thus, there are $7 \times 2 = 14$ different yearly calendars.



Problème de la semaine

Problème B

Le codage de figures

Ayanna écrit des programmes en utilisant les commandes suivantes qui dirigent son robot pour dessiner des formes.

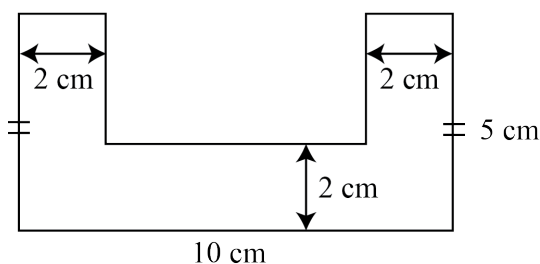
Commande	Explication
Crayon en bas	Place le crayon en position pour dessiner lorsque le robot se déplace
Avancer	Fait avancer le robot d'une distance donnée
Tourner SH	Fait pivoter le robot d'un quart de tour (90°) dans le sens horaire
Tourner SAH	Fait pivoter le robot d'un quart de tour (90°) dans le sens antihoraire

(a) Pour chaque programme, suis le code afin de tracer la forme. L'utilisation de papier quadrillé pourrait t'être utile.

(i) Crayon en bas
Répéter 4 fois
 Avancer 5 cm
 Tourner SH
Fin

(ii) Crayon en bas
Avancer 4 cm
Tourner SAH
Avancer 2 cm
Tourner SH
Avancer 2 cm
Tourner SH
Avancer 6 cm
Tourner SH
Avancer 6 cm
Tourner SH
Avancer 4 cm
Fin

(b) Écris un programme qui dirigerait le robot d'Ayanna pour dessiner la forme en U rectangulaire donnée.





Problem of the Week

Problem B and Solution

Coding Figures

Problem

Ayanna writes programs using the following commands that direct her robot to draw shapes.

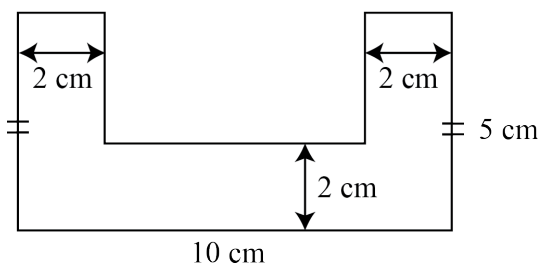
Command	Explanation
Pencil down	Puts the pencil in position to draw as the robot moves
Move	Moves the robot a given distance forward
Turn CW	Rotates the robot one quarter turn (90°) clockwise
Turn CCW	Rotates the robot one quarter turn (90°) counter clockwise

(a) For each program, follow the code to draw the shape. You may find grid paper helpful.

(i) Pencil down
Repeat 4 times
 Move 5 cm
 Turn CW
End

(ii) Pencil down
Move 4 cm
Turn CCW
Move 2 cm
Turn CW
Move 2 cm
Turn CW
Move 6 cm
Turn CW
Move 6 cm
Turn CW
Move 4 cm
End

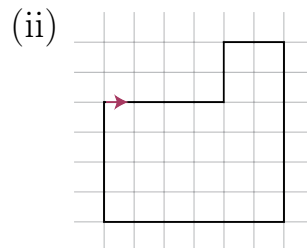
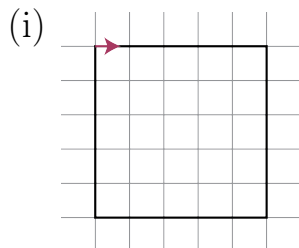
(b) Write a program that would direct Ayanna's robot to draw the given rectangular U-shape.



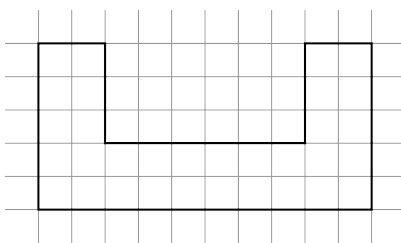


Solution

- (a) For each program, the shape is shown on 1 cm grid paper with an arrow indicating the initial direction of the robot. Note that for part (i), the shape will look the same regardless of the initial direction. However for part (ii), the shape will be rotated depending on the initial direction.



- (b) We start by determining the length of each side of the shape. We do this by drawing the shape on grid paper, as shown.



There are several different programs that can be written to direct Ayanna's robot to draw the given shape, depending on the robot's starting position and direction. In the following program, the robot starts in the bottom-left corner, facing to the right.

```
Pencil down
Move 10 cm
Turn CCW
Move 5 cm
Turn CCW
Move 2 cm
Turn CCW
Move 3 cm
Turn CW
Move 6 cm
Turn CW
Move 3 cm
Turn CCW
Move 2 cm
Turn CCW
Move 5 cm
End
```



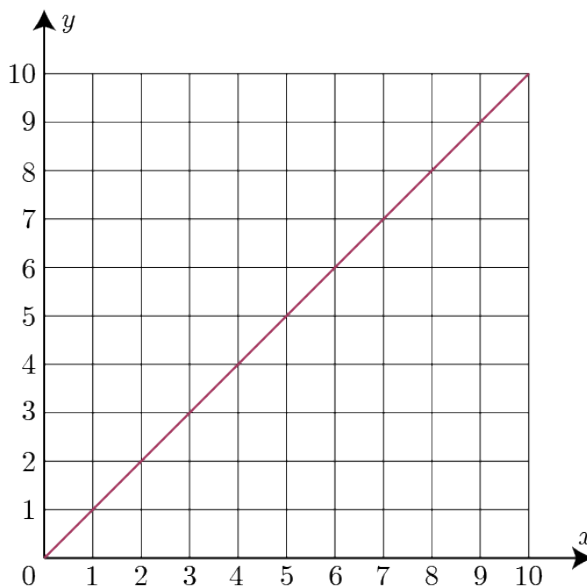
Problème de la semaine

Problème B

Une transformation capitale

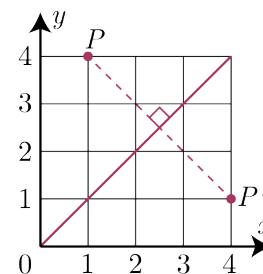
- (a) Place les points donnés dans le plan cartésien, puis relie les points selon leur ordre dans le tableau. Termine en reliant le dernier point au premier.

x	y
1	10
6	10
6	9
4	9
4	5
3	5
3	9
1	9



- (b) Effectue la réflexion de chacun des points placés dans la partie (a) par rapport à la droite diagonale dans le plan cartésien ci-dessus, puis relie les points réfléchis comme tu l'as fait dans la partie (a).

ASTUCE : Lorsque tu effectues la réflexion d'un point par rapport à une droite, le point réfléchi (ou le point image) se trouve du côté opposé de la droite, à la même distance perpendiculaire que le point d'origine. Ceci est illustré dans le diagramme où P est le point d'origine et P' est son point image par réflexion par rapport à la droite diagonale.



- (c) Comment les coordonnées de chaque point image de la partie (b) se comparent-elles aux coordonnées de leur point d'origine ?
- (d) En partant des points d'origine, peux-tu utiliser 2 ou 3 transformations géométriques différentes successives pour obtenir la même image finale que tu as tracée dans la partie (b) ? Si oui, explique les transformations que tu as utilisées.



Problem of the Week

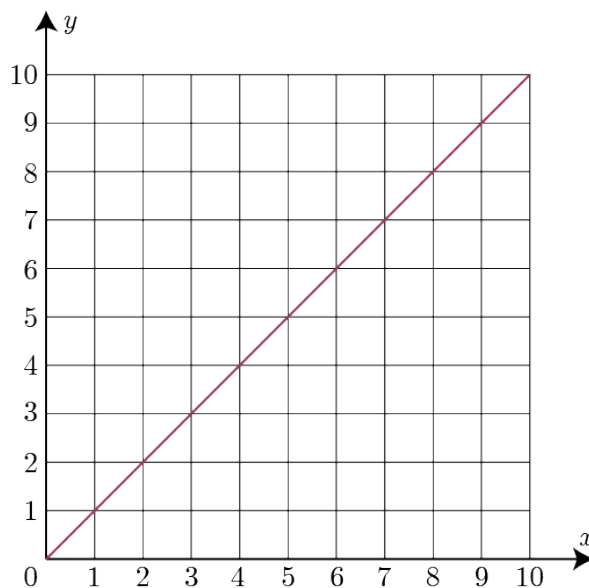
Problem B and Solution

A Capital Transformation

Problem

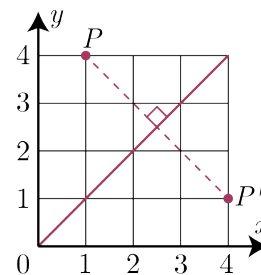
- (a) Plot the given points on the grid, then connect the points in the order they appear in the table. Finish by connecting the last point with the first.

x	y
1	10
6	10
6	9
4	9
4	5
3	5
3	9
1	9



- (b) Reflect each of the points you plotted in part (a) across the diagonal line in the grid above, then connect the reflected points like you did in part (a).

TIP: When you reflect a point across a line, the reflected point lies on the opposite side of the line, the same perpendicular distance away as the original point. This is shown in the diagram where P is the original point and P' is its reflection across the diagonal line.

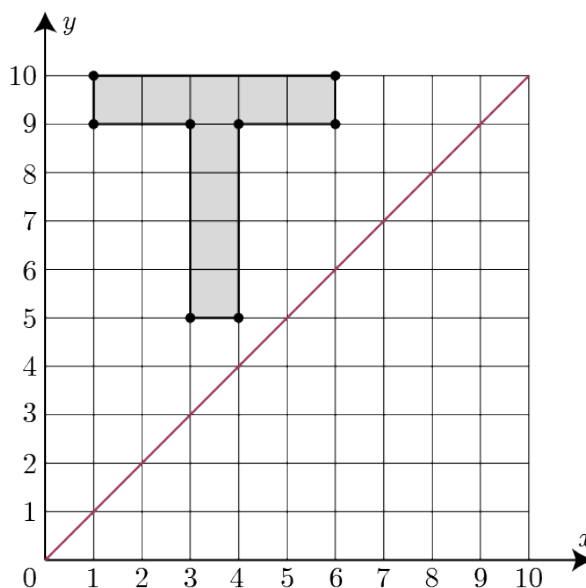


- (c) How do the coordinates of each reflected point from part (b) compare with the coordinates of their original point?
- (d) Starting with the original points, can you use 2 or 3 different transformations in a row to get the same final image you drew in part (b)? If so, explain the transformations you used.

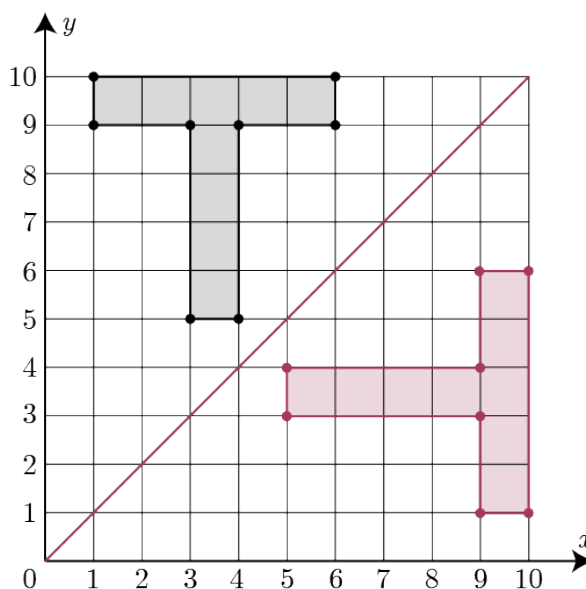


Solution

- (a) The plotted points are shown on the grid. When joined in order, they form the letter T.



- (b) The reflected points are shown on the grid. When joined in order, they form a sideways letter T.



- (c) The coordinates for each point are reversed in its reflected point. For example, the reflection of the point $(1, 10)$ is $(10, 1)$.
- (d) There are several different ways to get the same final image using multiple transformations. One way is as follows:
- Step 1: Rotate each point 90° clockwise about the point $(3, 5)$, which is the bottom-left corner of the T.
- Step 2: Translate each point 2 units to the right.
- Step 3: Translate each point 1 unit down.



Problème de la semaine

Problème B

De long en large

Geordie a tracé un triangle équilatéral de côté 2 et un rectangle de longueur 2 et de largeur 1. Il a alors remarqué que chacune des deux formes avait un périmètre de 6.



- (a) Geordie a tracé un triangle équilatéral de côté 3. Peut-il tracer un rectangle ayant le même périmètre que celui de ce triangle équilatéral, si les dimensions du rectangle doivent être des nombres entiers ? Explique ta réponse.
- (b) Geordie a tracé des triangles équilatéraux de côtés 4, 5, 6 et 7. Pour lesquels de ces triangles peut-il tracer un rectangle ayant le même périmètre, si les dimensions du rectangle doivent être des nombres entiers ? Explique ta réponse.
- (c) Si les mesures des côtés d'un triangle équilatéral et d'un rectangle sont toutes des nombres entiers, alors :
 - (i) Quelles valeurs peut prendre le périmètre du triangle ?
 - (ii) Quelles valeurs peut prendre le périmètre du rectangle ?
 - (iii) Quelles valeurs peuvent représenter à la fois le périmètre du triangle et celui du rectangle ?
 - (iv) Si le triangle et le rectangle ont le même périmètre, quelle condition la mesure du côté du triangle doit-elle satisfaire ?



Problem of the Week

Problem B and Solution

The Long and the Short of It

Problem

Geordie drew an equilateral triangle with side length 2 and a rectangle with length 2 and width 1. He then noticed that each shape had a perimeter of 6.



- (a) Geordie drew an equilateral triangle with side length 3. Can Geordie draw a rectangle that has the same perimeter as the equilateral triangle, if the side lengths of the rectangle must be whole numbers? Explain.
- (b) Geordie drew equilateral triangles with side lengths 4, 5, 6, and 7. For which of these can he draw a rectangle with the same perimeter, if the side lengths of the rectangle must be whole numbers? Explain.
- (c) If the side lengths of an equilateral triangle and a rectangle are all whole numbers, then:
 - (i) What numbers could be the perimeter of the triangle?
 - (ii) What numbers could be the perimeter of the rectangle?
 - (iii) What numbers could be the perimeter of both the triangle and the rectangle?
 - (iv) If the triangle and the rectangle have the same perimeter, what must be true about the side length of the triangle?



Solution

(a) If the side length of the equilateral triangle is 3, then its perimeter is $3 \times 3 = 9$. We will try different lengths and widths for the rectangle to see if we can find ones that give us a perimeter of 9. We will use the formula for the perimeter of a rectangle, which is $P = 2l + 2w$.

- Suppose the rectangle has length 1. Then $9 = 2 \times 1 + 2w$, so $9 = 2 + 2w$. Then $7 = 2w$, so $w = 7 \div 2 = 3.5$. However this is not a whole number, so this is not a possibility.
- Suppose the rectangle has length 2. Then $9 = 2 \times 2 + 2w$, so $9 = 4 + 2w$. Then $5 = 2w$, so $w = 5 \div 2 = 2.5$. However this is not a whole number, so this is not a possibility.
- Suppose the rectangle has length 3. Then $9 = 2 \times 3 + 2w$, so $9 = 6 + 2w$. Then $3 = 2w$, so $w = 3 \div 2 = 1.5$. However this is not a whole number, so this is not a possibility.
- Suppose the rectangle has length 4. Then $9 = 2 \times 4 + 2w$, so $9 = 8 + 2w$. Then $1 = 2w$, so $w = 1 \div 2 = 0.5$. However this is not a whole number, so this is not a possibility.
- Suppose the rectangle has length 5. Then since $5 + 5 = 10$ and $10 > 9$, it is not possible for this rectangle to have a perimeter of 9.

Therefore, Geordie cannot draw a rectangle with a perimeter of 9 if its side lengths must be whole numbers.

In general, if the length and width of a rectangle are whole numbers, then $2l$ and $2w$ will always be even numbers because any whole number multiplied by 2 results in an even number. So $2l + 2w$ will also be an even number, since an even number plus an even number results in an even number. Since $P = 2l + 2w$, it follows that P also must be an even number. This explains why we cannot draw a rectangle with a perimeter of 9, since 9 is an odd number.

(b) Using our results from part (a), we know that if the side lengths of a rectangle are whole numbers, then its perimeter must be even. In other words, if the perimeter is not even, then the side lengths of the rectangle cannot all be whole numbers.

- An equilateral triangle with side length 4 has perimeter $4 \times 3 = 12$, which is even. Geordie can draw a rectangle with perimeter 12; one example is a rectangle with length 5 and width 1.



- An equilateral triangle with side length 5 has perimeter $5 \times 3 = 15$. Since this is odd, Geordie cannot draw a rectangle with perimeter 15.
 - An equilateral triangle with side length 6 has perimeter $6 \times 3 = 18$, which is even. Geordie can draw a rectangle with perimeter 18; one example is a rectangle with length 6 and width 3.
 - An equilateral triangle with side length 7 has perimeter $7 \times 3 = 21$. Since this is odd, Geordie cannot draw a rectangle with perimeter 21.
- (c) Based on our results from parts (a) and (b), we can conclude the following:
- (i) The perimeter of the equilateral triangle must be a multiple of 3, since the perimeter is equal to 3 times its side length.
 - (ii) The perimeter of the rectangle must be an even number.
 - (iii) If a number is the perimeter of both the equilateral triangle and the rectangle, then it must be both a multiple of 3 and even. This is equivalent to saying it must be a multiple of 6.
 - (iv) If the equilateral triangle and the rectangle have the same perimeter, then this perimeter must be even. Therefore, the side length of the triangle must be even.



Problème de la semaine

Problème B

Une soif de chien

Lorsqu'un chien boit, il lape l'eau en plongeant sa langue dans l'eau, puis en la retirant avec un peu d'eau qui y adhère. Certains chiens sont plus habiles que d'autres. Chase est un buveur plutôt maladroit et il laisse généralement une grande flaque d'eau sur le plancher.

- (a) Chase a une masse de 40 kg. Chaque jour, il doit boire 50 mL d'eau par kilogramme de masse corporelle. Combien de litres d'eau Chase doit-il boire par jour ?
- (b) Si 70% de la masse corporelle de Chase est composée d'eau, quelle est sa masse qui n'est PAS de l'eau ?
- (c) Pour chaque 3,5 mL d'eau que Chase lape avec sa langue, 1,5 mL se retrouve sur le plancher. S'il prend une lapée toutes les 5 secondes pendant une minute, quelle quantité d'eau boit-il avec succès ?
- (d) Après une heure, tu mesures la quantité d'eau sur le plancher et tu constates que Chase en a renversé 45 mL. Combien d'eau a-t-il réussi à boire ?
- (e) Combien de lapées par jour Chase devra-t-il prendre pour boire la quantité d'eau dont il a besoin quotidiennement ?





Problem of the Week

Problem B and Solution

Doggy Drinking

Problem

When a dog takes a drink, he laps up water by pushing his tongue into the water and then pulling it back with some water sticking to it. Some dogs are better at this than others. Chase is a messy drinker, and usually leaves a large puddle on the floor.

- (a) Chase has a mass of 40 kg. Each day he needs to drink 50 mL of water for each kg of body mass. How many litres of water should Chase drink per day?
- (b) If 70% of Chase's body mass is made up of water, how much of Chase's mass is NOT water?
- (c) For every 3.5 mL of water Chase laps up with his tongue, 1.5 mL ends up on the floor. If Chase takes a lap every 5 seconds for one minute, how much water does he successfully drink?
- (d) After an hour, you measure the water on the floor and find that Chase has spilled 45 mL. How much water did he successfully drink?
- (e) How many laps per day will it take Chase to drink the water he needs in one day?

Solution

- (a) Chase should drink $50 \times 40 = 2000$ mL, or 2 L of water per day.
- (b) The percentage of Chase's body mass that is NOT water is $100\% - 70\% = 30\%$. Thus, the amount of Chase's mass that is NOT water is 30% of 40 kg, or $0.3 \times 40 = 12$ kg.
- (c) Since there are 60 seconds in one minute, Chase takes $60 \div 5 = 12$ laps in total. From each lap, he drinks only $3.5 - 1.5 = 2.0$ mL. Thus, in 12 laps he successfully drinks a total of $12 \times 2.0 = 24$ mL of water.
- (d) The 45 mL of water Chase spilled indicates he took $45 \div 1.5 = 30$ laps. Since he drinks 2 mL with each lap, he successfully drank $2 \times 30 = 60$ mL.
- (e) From part (a), Chase needs to drink 2000 mL of water per day. He drinks 2 mL with each lap, so he will need to take $2000 \div 2 = 1000$ laps each day.



Problème de la semaine

Problème B

L'angle d'Archie

Archie Tekt conçoit des maisons en forme de A depuis de nombreuses années. Toutes ses conceptions ont la forme de triangles équilatéraux avec une base horizontale et deux côtés qui se rejoignent au sommet.



Archie Tekt souhaite créer une conception plus unique et a proposé trois différents modèles de maisons en A.

Trace chaque modèle à l'aide d'une règle et d'un rapporteur, à l'échelle de $1 \text{ cm} = 1 \text{ m}$. Mesure ensuite les angles intérieurs inconnus (au degré près) et les longueurs des côtés (au millimètre près ou $\frac{1}{10} \text{ cm}$ près) et inscris ces mesures sur ton diagramme. Note qu'un *angle à la base* correspond à l'angle intérieur entre le plancher et le mur.

1. **Modèle scalène :** Le plancher mesure 10 m de largeur, l'angle à la base est de 105° et l'un des murs a une longueur de 5 m.
2. **Modèle rectangle :** Le mur gauche est perpendiculaire au plancher, et le plancher ainsi que le mur gauche mesurent chacun 5 m de longueur.
3. **Modèle isocèle :** Le plancher mesure 8 m de longueur, le périmètre du modèle est de 22 m et l'un des angles à la base est égal à l'angle intérieur au sommet du toit.

EXTENSION: La partenaire d'Archie aime l'angle à la base du modèle scalène, mais préférerait que les deux murs aient la même longueur. Crois-tu que cela serait possible ? Explique pourquoi ou pourquoi pas.



Problem of the Week

Problem B and Solution

What's Arkie's Angle?

Problem

Arkie Tekt has been designing A-frame houses for years. All his designs are in the shape of equilateral triangles with a horizontal base and two sides that meet at the top.



Arkie Tekt wants to create a more unique design, and has proposed three different A-frame designs.

Sketch each design using a ruler and protractor, with a scale of $1 \text{ cm} = 1 \text{ m}$. Then measure any unknown interior angles (to the nearest degree) and side lengths (to the nearest millimetre or $\frac{1}{10} \text{ cm}$), and record these measurements on your diagram. Note that a *base angle* means an interior angle between the floor and the wall.

1. **Scalene Design:** The floor is 10 m wide, one base angle is 105° , and one of the walls has length equal to 5 m.
2. **Right-Angled Design:** The left wall is perpendicular to the floor, and both the floor and the left wall are 5 m in length.
3. **Isosceles Design:** The floor is 8 m in length, the perimeter of the design is 22 m, and one of the base angles is the same as the interior angle at the roof peak.

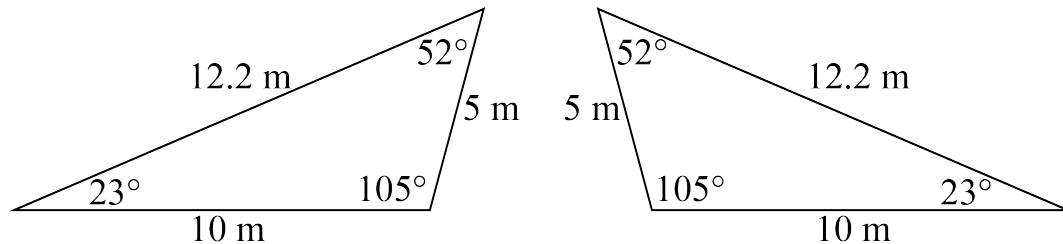
EXTENSION: Arkie's partner likes the base angle given in the scalene design, but would prefer that the side walls be of the same length. Do you think this would work? Explain why or why not.



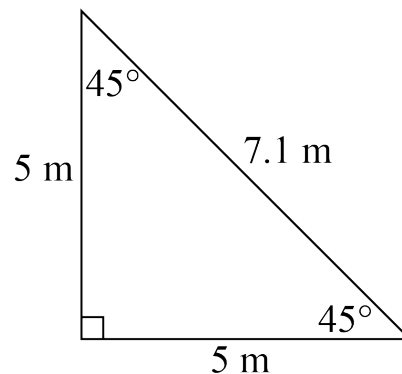
Solution

A sketch of each design is shown, however they are not drawn to scale. Note that there are two possibilities for the scalene and the isosceles designs.

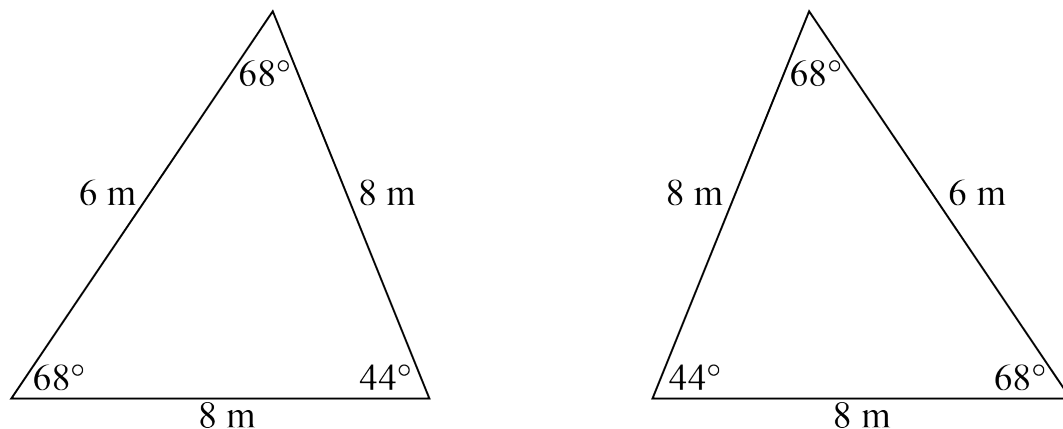
1. Scalene Design:



2. Right-Angled Design:



3. Isosceles Design:



SOLUTION TO EXTENSION:

If the side walls were the same length, then the triangle would be isosceles with two equal base angles. The given base angle is 105° . However $105^\circ + 105^\circ = 210^\circ$, and $210^\circ > 180^\circ$ which is the sum of the angles in triangle. Thus, this design would not work.



Sens du nombre (N)

**Amène-moi à la
couverture**



Problème de la semaine

Problème B

Argent de poche

Macklin a dix pièces de monnaie dans sa poche, d'une valeur totale de 2,30 \$. Une seule des pièces de Macklin est une pièce de cinq cents (0,05 \$) et les neuf autres pièces représentent une combinaison de pièces de dix cents (0,10 \$), de vingt-cinq cents (0,25 \$) et d'un dollar (1,00 \$), avec au moins une pièce de chaque type.

- (a) Combien de pièces de dix cents Macklin possède-t-il ?
- (b) Est-il possible que Macklin ait deux pièces de cinq cents au lieu d'une seule ? Explique ton raisonnement.





Problem of the Week

Problem B and Solution

Loose Change

Problem

Macklin has ten coins in his pocket worth a total of \$2.30. Only one of Macklin's coins is a nickel (worth \$0.05), and the other nine are some combination of dimes (worth \$0.10), quarters (worth \$0.25), and loonies (worth \$1.00), with at least one of each type of coin.

- (a) How many dimes does Macklin have?
- (b) Is it possible that Macklin could have two nickels, not one? Explain your reasoning.

Solution

- (a) We know that Macklin has at least one of each type of coin, so we will start by taking one of each type of coin and recording their total value in the following table.

	Nickels	Dimes	Quarters	Loonies	Total
Number of Coins	1	1	1	1	4
Value (\$)	0.05	0.10	0.25	1.00	1.40

This gives a total value of \$1.40, so we have $\$2.30 - \$1.40 = \$0.90$ remaining. Since \$0.90 is less than the value of a loonie, Macklin cannot have any more loonies. Also, since only one of Macklin's coins is a nickel, Macklin cannot have any more nickels. Thus, we need to find a way to make \$0.90 using only quarters and/or dimes. The only way to do so is by using no quarters and 9 dimes, or by using 2 quarters and 4 dimes, since $(2 \times \$0.25) + (4 \times \$0.10) = \$0.90$.

Using 9 dimes uses a total of $4 + 9 = 13$ coins. Since Macklin has 10 coins, this will not work.

Using 2 quarters and 4 dimes uses a total of $4 + 2 + 4 = 10$ coins, as desired. We will add these coins to our table.

	Nickels	Dimes	Quarters	Loonies	Total
Number of Coins	1	5	3	1	10
Value (\$)	0.05	0.50	0.75	1.00	2.30

Thus, Macklin has 5 dimes in total.



- (b) If two of Macklin's coins were nickels, then we can proceed like in part (a), but include an extra nickel.

	Nickels	Dimes	Quarters	Loonies	Total
Number of Coins	2	1	1	1	5
Value (\$)	0.10	0.10	0.25	1.00	1.45

This gives a total value of \$1.45, so we have $\$2.30 - \$1.45 = \$0.85$ remaining. As before, we can make this amount using only quarters and/or dimes.

Using 3 quarters and 1 dime we can make \$0.85, however this uses a total of $5 + 3 + 1 = 9$ coins, which is not enough.

Using 1 quarter and 6 dimes we can make \$0.85, however this uses a total of $5 + 1 + 6 = 12$ coins, which is too many.

There are no other ways to make \$0.85 using only quarters and/or dimes. Thus, Macklin could not have had two nickels.

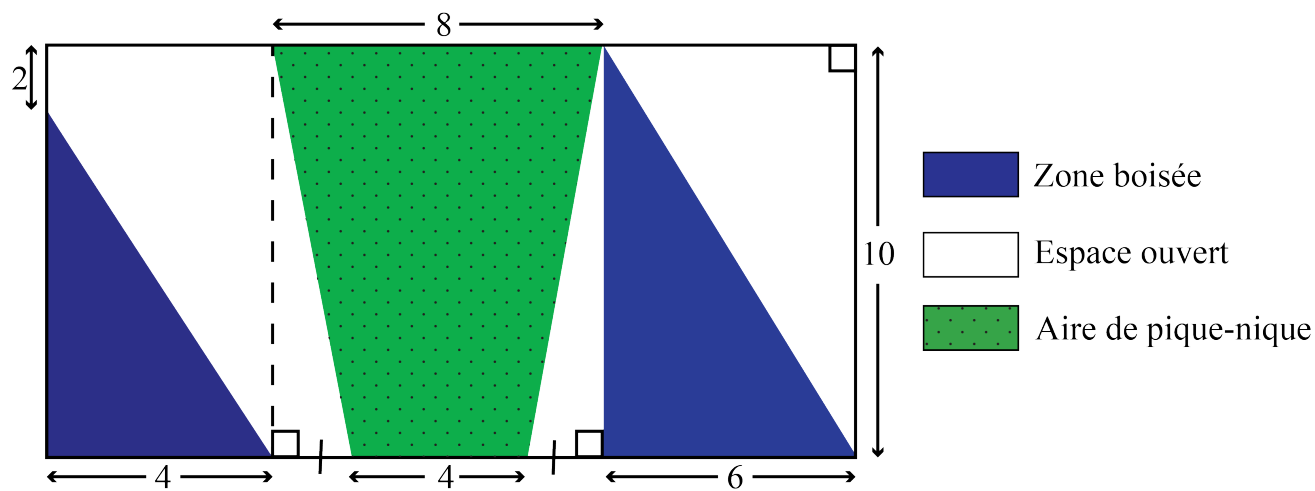


Problème de la semaine

Problème B

Bienvenue au parc de la ville des Formes

La ville des Formes a aménagé un nouveau parc. Le parc est de forme rectangulaire et il est divisé en zones boisées, espaces ouverts et une aire de pique-nique, comme illustré.



Quelle est l'aire totale de chaque type d'espace ?



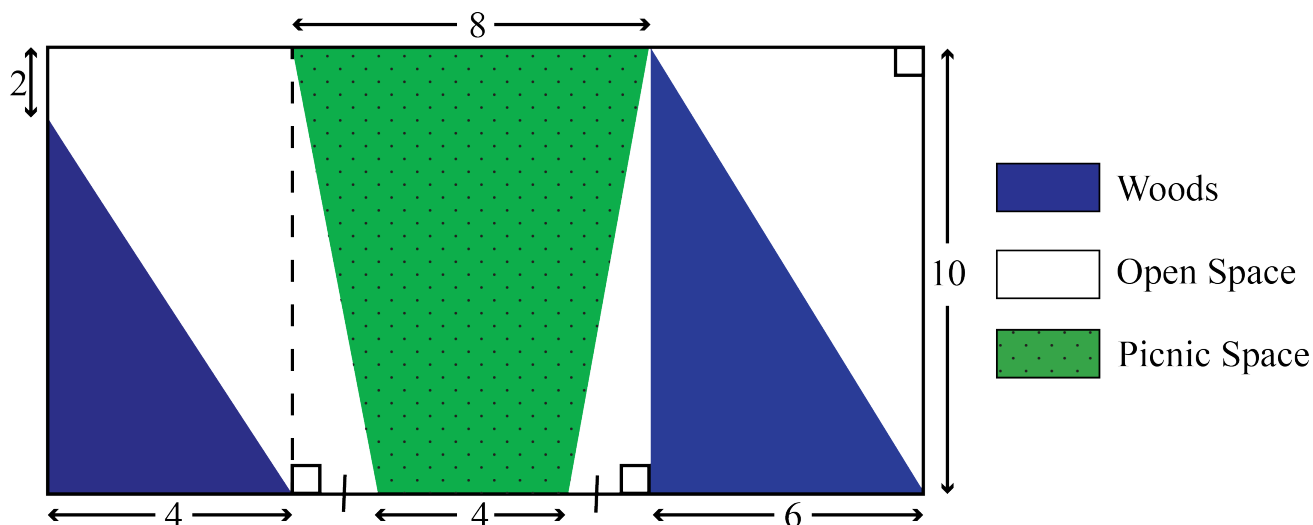
Problem of the Week

Problem B and Solution

Welcome to Shape City Park

Problem

Shape City has built a new park. The park is in the shape of a rectangle, and is divided into woods, open space, and picnic space, as shown.



What is the total area of each type of space?

Solution

Both woods regions are right-angled triangles. One triangle has base 4 and height $10 - 2 = 8$, and thus has area equal to $\frac{1}{2}(4 \times 8) = 16$ square units. The other triangle has base 6 and height 10, and thus has area equal to $\frac{1}{2}(6 \times 10) = 30$ square units. Thus, the area of the woods is $16 + 30 = 46$ square units.

The picnic space is in the shape of a trapezoid with height 10 and parallel sides of length 8 and 4. Thus, the area of the picnic space is $\frac{1}{2}(8 + 4) \times 10 = 60$ square units.

Alternatively, the area of the trapezoidal picnic space can be determined by calculating the area of the rectangular strip containing the trapezoid, and subtracting the two triangular regions of open space. That is, the area of the picnic space is $8 \times 10 - \frac{1}{2}(2 \times 10) - \frac{1}{2}(2 \times 10) = 60$ square units.

The entire park has vertical length 10 and width equal to $4 + 8 + 6 = 18$. Hence the total area of the park is $10 \times 18 = 180$ square units.

The area of the open space is equal to the total area of the park minus the areas of the picnic space and woods, or $180 - 46 - 60 = 74$ square units.



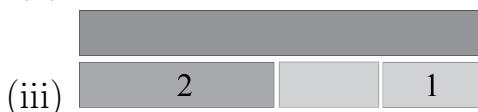
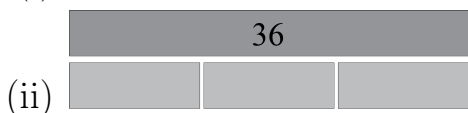
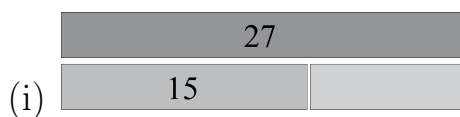
Problème de la semaine

Problème B

Composé ou décomposé

Un modèle à bandes comporte deux rangées de rectangles. La rangée supérieure est constituée d'un rectangle dont la valeur est égale au total, et la rangée inférieure contient des rectangles dont la somme des valeurs est égale au total. Toutes les valeurs sont des nombres entiers.

- (a) Détermine les nombres manquants dans chacun des modèles à bandes ci-dessous. Dans chaque partie, les rectangles de la même nuance de gris ont la même valeur.



- (b) (i) Dans le modèle à bandes illustré, les rectangles de la rangée inférieure ont tous la même valeur. Quelles sont quelques équations que ce modèle à bandes pourrait représenter ?



- (ii) Décris tous les nombres entiers possibles qui pourraient apparaître dans la rangée supérieure de ce modèle à bandes.





Problem of the Week

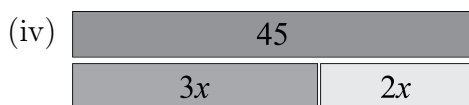
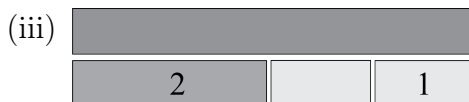
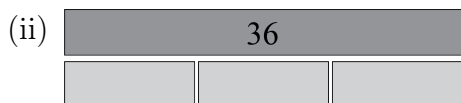
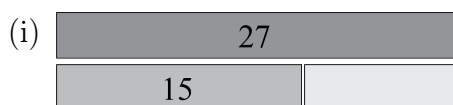
Problem B and Solution

Composed or Decomposed

Problem

A bar model contains two rows of rectangles. The top row is a rectangle with a value equal to the total, and the bottom row contains rectangles whose values sum to the total. All values are whole numbers.

- (a) Determine the numbers missing from each of the bar models below. In each part, rectangles that are the same shade of grey have the same value.



- (b) (i) In the bar model shown, the rectangles in the bottom row all have the same value. What are some equations this bar model could represent?



- (ii) Describe all possible whole numbers that could appear in the top row of this bar model.





Solution

- (a) (i) Since $27 = 15 + 12$, the missing number is 12.
- (ii) Since $36 \div 3 = 12$, the missing number is 12.
- (iii) Since rectangles that are the same shade of grey have the same value, the rectangle in the second row with unknown value has value 1. Thus, the missing number in the top row is $2 + 1 + 1 = 4$.
- (iv) From the bar model, we know $45 = 3x + 2x = 5x$.
Therefore, 45 is divided into 5 equal parts. Since $45 \div 5 = 9$, we know $x = 9$. So the two numbers in the bottom row are $3 \times 9 = 27$ and $2 \times 9 = 18$.
- (b) (i) Answers will vary. Some equations that the bar model could represent include

$$5 \div 5 = \mathbf{1}$$

$$10 \div 5 = \mathbf{2}$$

$$15 \div 5 = \mathbf{3}$$

$$20 \div 5 = \mathbf{4}$$

$$\mathbf{3} \times 5 = 15$$

$$\mathbf{7} \times 5 = 35$$

In each equation, the number in boldface would appear in each of the five rectangles in the bottom row.

- (ii) The whole numbers that could appear in the top row are the positive multiples of 5. That is, 5, 10, 15, 20, 25, and so on.



Problème de la semaine

Problème B

Mettons ces nombres en ordre

- (a) Classe les douze nombres suivants du plus petit au plus grand.

$2,04$; $1,02$; $\frac{85}{100}$; $\frac{175}{100}$; $\frac{7}{4}$; $1\frac{1}{8}$; $0,91$; $\frac{46}{20}$; $0,091$; $\frac{14}{25}$; $\frac{3}{5}$; $1,1$

Représente ces nombres sur une droite numérique.

- (b) Détermine le plus grand écart entre deux nombres consécutifs sur la droite numérique.
- (c) À partir de ta liste de la partie (a), détermine un nombre entre les deux nombres du milieu de la liste, en choisissant un nombre plus proche du plus grand des deux.

$0 < 1,189 < 1,19 < 1,2 < 2$



Problem of the Week

Problem B and Solution

Let's Get These Numbers in Order

Problem

- (a) List the following twelve numbers from least to greatest value.

$$2.04, 1.02, \frac{85}{100}, \frac{175}{100}, \frac{7}{4}, 1\frac{1}{8}, 0.91, \frac{46}{20}, 0.091, \frac{14}{25}, \frac{3}{5}, 1.1$$

Plot these numbers on a number line.

- (b) Determine the largest gap in between two adjacent numbers on the number line.
(c) For your list in part (a), determine a number between the two numbers in the middle of the list, with your number being closer to the larger of the two.

Solution

- (a) The decimals equivalent to the fractional numbers are $\frac{85}{100} = 0.85$, $\frac{175}{100} = 1.75$, $\frac{7}{4} = 1.75$, $1\frac{1}{8} = 1.125$, $\frac{46}{20} = 2.3$, $\frac{14}{25} = 0.56$, and $\frac{3}{5} = 0.6$.

Notice that $\frac{175}{100}$ and $\frac{7}{4}$ have the same value.

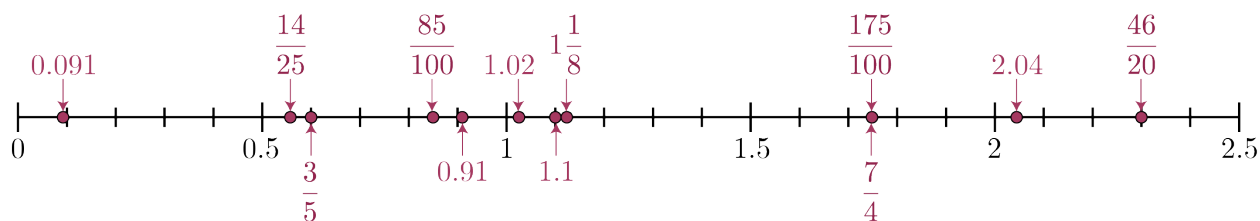
Listed from least to greatest value, the decimals are

$$0.091, 0.56, 0.60, 0.85, 0.91, 1.02, 1.1, 1.125, 1.75 \text{ and } 1.75, 2.04, 2.3$$

In their original format, the numbers, listed from least to greatest value, are

$$0.091, \frac{14}{25}, \frac{3}{5}, \frac{85}{100}, 0.91, 1.02, 1.1, 1\frac{1}{8}, \frac{175}{100} \text{ and } \frac{7}{4} \text{ (in either order), } 2.04, \frac{46}{20}$$

We plot these twelve numbers on a number line from 0 to 2.5.



- (b) The largest gap lies between 1.125 and 1.75, which is 0.625 or $\frac{5}{8}$.
(c) The two middle values are 1.02 and 1.1. A value between these two numbers but closer to 1.1 would also have to be greater than 1.06. One such number is 1.08, but any number with value between 1.06 and 1.1 would satisfy the condition.



Problème de la semaine

Problème B

Jeux et divertissements

L'équipe de jeu-questionnaire de Mariana joue une partie contre d'autres équipes. Le jeu comprend 10 questions et chaque équipe commence avec 5 points. Puis, les équipes gagnent 4 points pour chaque réponse correcte et perdent 3 points pour chaque réponse incorrecte.

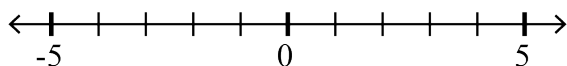


- (a) Le tableau montre la fiche de pointage de l'équipe de Mariana.

(Les questions du jeu étaient difficiles !)

Dans la colonne de droite, inscris leur *pointage cumulatif*, qui représente leur pointage actuel après chaque question.

Note que leur pointage cumulatif peut être positif ou négatif, comme une lecture de température. La droite numérique suivante pourrait t'être utile.



Question	Résultat	Cumulatif
—	—	5
1	X	
2	X	
3	X	
4	✓	
5	X	
6	✓	
7	X	
8	✓	
9	X	
10	✓	

- (b) L'équipe gagnante a obtenu un pointage de 10. À combien de questions ont-ils répondu correctement ?

EXTENSION: Quel est le pointage maximal possible ? Quel est le pointage minimal possible ? Est-ce que tous les nombres entiers entre ces deux valeurs sont des pointages possibles ? Explique ta réponse.



Problem of the Week

Problem B and Solution

Fun and Games

Problem

Mariana's trivia team is playing a game against other teams. The game includes 10 questions, and each team is given 5 points to start. Teams then earn 4 points for each correct answer, and lose 3 points for each incorrect answer.

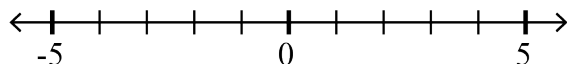


- (a) The table shows the score card for Mariana's team.

(The trivia questions were tough!)

In the right column, fill in their *running score*, which is their current score after each question.

Note that their running score may be positive or negative, like a temperature reading. You may find the following number line helpful.



Question	Result	Running Score
—	—	5
1	X	
2	X	
3	X	
4	✓	
5	X	
6	✓	
7	X	
8	✓	
9	X	
10	✓	

- (b) The winning team had a score of 10. How many questions did they get correct?

EXTENSION: What is the maximum possible score? What is the minimum possible score? Are all the integers in between these values possible scores? Explain.



Solution

(a) The running score is shown in the following table.

Question	Result	Running Score
—	—	5
1	X	$5 - 3 = 2$
2	X	$2 - 3 = -1$
3	X	$-1 - 3 = -4$
4	✓	$-4 + 4 = 0$
5	X	$0 - 3 = -3$
6	✓	$-3 + 4 = 1$
7	X	$1 - 3 = -2$
8	✓	$-2 + 4 = 2$
9	X	$2 - 3 = -1$
10	✓	$-1 + 4 = 3$

(b) The winning team's score went from 5 to 10, so we know they gained 5 points. Notice that the total score for a correct answer followed by an incorrect answer is $4 - 3 = 1$. So 5 pairs of correct and incorrect answers would have a total score of 5. This would also be 10 questions in total, as desired. Therefore, the winning team must have had 5 questions correct.

SOLUTION TO EXTENSION:

If a team got every question correct, they would earn $10 \times 4 = 40$ points, plus the 5 points at the beginning, for a total of $40 + 5 = 45$ points. This is the maximum possible score.

If a team got every question incorrect, they would lose $10 \times 3 = 30$ points, but since they started off with 5 points, their score would be $5 - 30 = -25$ points. This is the minimum possible score.

All the integers in between the minimum and maximum are **not** possible scores. The number of correct answers can be any integer from 0 to 10. Thus, there are 11 possibilities for the number of correct answers. Therefore, there cannot be more than 11 different possible scores. However, there are more than 11 integers between -25 and 45 so not all of them can be possible scores.

Another way to show this is to determine all the 11 possible scores. They are $-25, -18, -11, -4, 3, 10, 17, 24, 31, 38$, and 45 .



Problème de la semaine

Problème B

Des différences différentes

Yago prend un nombre naturel à deux chiffres et soustrait le produit de ses chiffres. Il appelle le résultat un *nombre de Yago*. Il répète ce processus avec d'autres nombres à deux chiffres pour trouver d'autres nombres de Yago.

Par exemple, le produit des chiffres de 82 est $8 \times 2 = 16$. Alors, $82 - 16 = 66$, donc 66 est un nombre de Yago. De même, le produit des chiffres de 25 est $2 \times 5 = 10$. Alors, $25 - 10 = 15$, donc 15 est un autre nombre de Yago.

Quels sont le plus grand nombre de Yago et le plus petit nombre de Yago que tu peux trouver ? Justifie tes réponses.

The illustration shows two examples of the Yago process using cartoon numbers with faces. The first example shows a purple '8' and a yellow '2' followed by a minus sign, a green '1' and a green '6', an equals sign, and two green '6's. The second example shows a yellow '2' and an orange '5' followed by a minus sign, a green '1' and a red '0', an equals sign, and a green '1' and an orange '5'.

$$\begin{array}{l} 82 - 16 = 66 \\ 25 - 10 = 15 \end{array}$$



Problem of the Week

Problem B and Solution

Not a Big Difference

Problem

Yago takes a two-digit whole number and subtracts the product of its digits. He calls the result a *Yago Number*. He repeats this process with other two-digit numbers to find more Yago Numbers.

For example, the product of the digits of 82 is $8 \times 2 = 16$. Then $82 - 16 = 66$, so 66 is a Yago Number. Similarly, the product of the digits of 25 is $2 \times 5 = 10$. Then $25 - 10 = 15$, so 15 is another Yago Number.

What are the largest and smallest Yago Numbers that you can find? Justify your answers.

The illustration shows two mathematical operations using cartoon numbers with faces and limbs. The first operation is $82 - 16 = 66$, where the digits 8 (purple), 2 (yellow), 1 (green), 6 (green), and the result 66 (green) are shown. The second operation is $25 - 10 = 15$, where the digits 2 (yellow), 5 (orange), 1 (green), 0 (red), and the result 15 (green and orange) are shown.



Solution

We can start by looking for patterns in the Yago Numbers. First look at what happens when we keep the tens digit in our original two-digit number the same but change the ones digit. The results when the tens digit is 2, 5, and 8 are shown in the following tables.

Original Number	20	21	22	23	24	25	26	27	28	29
Yago Number	20	19	18	17	16	15	14	13	12	11

Original Number	50	51	52	53	54	55	56	57	58	59
Yago Number	50	46	42	38	34	30	26	22	18	14

Original Number	80	81	82	83	84	85	86	87	88	89
Yago Number	80	73	66	59	52	45	38	31	24	17

From this, we see that for each tens digit, a ones digit of 0 produces the largest Yago Number and a ones digit of 9 produces the smallest Yago Number.

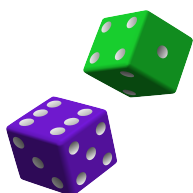
We can conclude that the largest Yago Number must have a ones digit of 0. Now we consider all two-digit numbers with a ones digit of 0. For each of these the product of the digits will be 0, since 0 multiplied by anything will always equal 0. So for any two-digit number with a ones digit of 0, its Yago Number will equal the original number. Thus, the largest Yago Number is the largest two-digit number with a ones digit of 0, which is 90.

To find the smallest Yago Number, we look at all the two-digit numbers with a ones digit of 9. These are shown in the following table.

Original Number	19	29	39	49	59	69	79	89	99
Yago Number	10	11	12	13	14	15	16	17	18

The smallest Yago Number is therefore 10.

It's worth noting that all two-digit numbers with a tens digit of 1 actually have a Yago Number of 10. We leave it up to the reader to verify this.



Problème de la semaine

Problème B

Des dés risqués

Jordyn et Langdon ont deux dés, un violet et un vert. Lorsqu'ils lancent les dés, ils savent qu'il y a un total de $6 \times 6 = 36$ résultats possibles pour les deux nombres qui apparaissent sur le dessus des dés.

Violet	Vert
1 •	• 1
2 •	• 2
3 •	• 3
4 •	• 4
5 •	• 5
6 •	• 6

- (a) Dans le diagramme, relie n'importe quel point à gauche à un point à droite de façon que le produit des deux nombres soit impair. Combien y a-t-il de façons d'obtenir un produit impair ?
- (b) Que t'indique ton résultat de la partie (a) sur le nombre de façons d'obtenir un produit pair ?
- (c) Est-il plus probable d'obtenir un produit de 4 ou un produit de 6 ? Pourquoi ?
- (d) Dans le tableau, inscris les produits impairs possibles dans la première colonne, le nombre de façons d'obtenir chacun de ces produits dans la colonne du milieu et la probabilité théorique de chacun de ces résultats dans la troisième colonne. (Le travail est déjà fait pour le produit impair 1.)

Produit impair	Nombre de façons	Probabilité théorique
1	1	$\frac{1}{36}$

- (e) Lance deux dés 36 fois et note le produit des deux nombres obtenus pour chaque lancer. Compte le nombre de fois que chaque produit impair s'est produit, puis calcule la probabilité expérimentale de lancer ce nombre. Compare la probabilité expérimentale à la probabilité théorique calculée dans la partie (d).



Problem of the Week

Problem B and Solution

Dicey Products

Problem

Jordyn and Langdon have two dice, one purple and one green. When they roll the dice, they know there are a total of $6 \times 6 = 36$ possible outcomes for the top two numbers on the dice.

- (a) In the diagram, join any dot on the left with one on the right such that the product of those two numbers is odd. How many ways are there to obtain an odd product?
- (b) What does your result in part (a) tell you about how many ways there are to obtain an even product?
- (c) Is the product 4 or 6 more likely to occur? Why?
- (d) In the table, enter the possible odd products in the first column, the number of ways that product could occur in the middle column, and the theoretical probability of that outcome in the third column. (This has been done for odd product 1.)



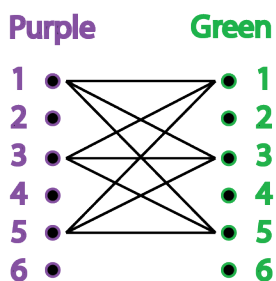
Odd Product	Number of Ways	Theoretical Probability
1	1	$\frac{1}{36}$

- (e) Roll two dice 36 times and record the product of the top two numbers for each roll. Count the number of each odd product that occurred, and then calculate the experimental probability of rolling that number. Compare the experimental probability to the theoretical probability calculated in part (d).



Solution

- (a) Odd products will occur exactly when the top two numbers are odd. We connect each odd number on the left with each odd number on the right to obtain the completed diagram shown. Counting the number of connections, we determine that there are 9 ways to obtain an odd product.



- (b) Since there are 36 possible products in total, and a product must be either even or odd, there must be $36 - 9 = 27$ ways to obtain an even product.
- (c) The product 6 is more likely to occur. This is because $6 = 1 \times 6$, $6 = 6 \times 1$, $6 = 2 \times 3$, and $6 = 3 \times 2$, and so can be formed by 4 different rolls. However, since $4 = 1 \times 4$, $4 = 4 \times 1$, and $4 = 2 \times 2$, it can only be formed by 3 different rolls.
- (d) Noting that there is only one way to get each of 1, 9, and 25, and two ways to get each of 3, 5, and 15, the appropriate data and the theoretical probabilities are calculated.

Odd Product	Number of Ways	Theoretical Probability
1	1	$\frac{1}{36}$
3	2	$\frac{2}{36}$
5	2	$\frac{2}{36}$
9	1	$\frac{1}{36}$
15	2	$\frac{2}{36}$
25	1	$\frac{1}{36}$

- (e) Answers will vary. Note that the result of a single experiment won't necessarily agree very closely with the theoretical probabilities.

SUGGESTION: Average the experimental probabilities obtained by all students in your class for each of the six odd products. Do these average probabilities agree more closely with the theoretical probabilities for each odd product than those of individual students?



Problème de la semaine

Problème B

Des touches louches

Depuis un certain temps, Joanne utilise une calculatrice pour effectuer des multiplications. Elle vient de se rendre compte que la touche 8 de sa calculatrice est brisée.

- (a) Détermine une méthode qui permet à Joanne d'utiliser sa calculatrice défectueuse pour déterminer la valeur de 82×816 .
- (b) Suppose que la touche 8 et la touche 4 soient toutes les deux brisées. Joanne pourrait-elle quand même utiliser sa calculatrice pour déterminer le produit de la partie (a) ?





Problem of the Week

Problem B and Solution

Sticky Keys

Problem

For some time, Joanne has relied on a calculator to multiply. She just realized that the 8 button on her calculator is broken.

- (a) Determine a method for Joanne to use her broken calculator to determine the value of 82×816 .
- (b) Suppose both the 8 button and the 4 button were broken. Could Joanne still use her calculator to determine the product in part (a)?

Solution

- (a) Answers will vary. To solve the problem, Joanne could write each number as a product of whole numbers that do not involve an 8. She can write 82 as 2×41 and 816 as 4×204 . Then $82 \times 816 = 2 \times 41 \times 4 \times 204$, which Joanne can determine using her calculator.

NOTE: If her calculator has brackets, part (a) could also be done using sums. For example, $82 \times 816 = (32 + 50) \times (500 + 316)$.

- (b) If the 4 key were broken as well, Joanne would not be able to solve this problem using the technique in part (a). This is because the only way to obtain a product of 82 using whole numbers is as 1×82 or as 2×41 . This means that she cannot write 82 as a product of whole numbers without one of the numbers involving a 4 or an 8.

However, Joanne could write 82 as a sum of numbers without using 4 or 8. For example, $82 = 30 + 52$. Since $816 = 2 \times 2 \times 2 \times 102$, the product could be found by calculating the sum $30 + 52$, and then multiplying the result by $2 \times 2 \times 2 \times 102$.

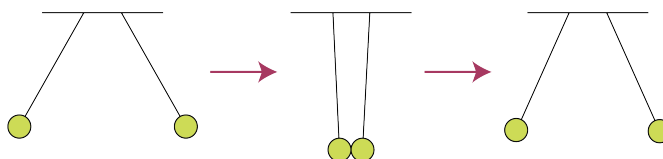


Problème de la semaine

Problème B

Les balles au rebond

Deux balles en caoutchouc identiques sont suspendues à l'extrémité de deux cordes attachées au plafond d'un gymnase. Pendul et Eliisa tirent les balles en les éloignant l'une de l'autre, puis les relâchent pour qu'elles entrent en collision au milieu. Les balles rebondissent, puis entrent de nouveau en collision, à plusieurs reprises.



Les élèves remarquent que les balles perdent de l'énergie à chaque collision, de sorte qu'elles ne rebondissent qu'à 90% de la distance qui les séparait avant la collision. Par exemple, si la distance initiale entre les balles est de 6 m, alors après la première collision, les balles ne rebondissent qu'à 90% de 6 m, soit $0,9 \times 6 = 5,4$ m entre elles. Une fois que la distance de rebond entre les balles est inférieure à 2 m, elles n'entreront plus en collision et se balanceront librement.

Pendul et Eliisa commencent avec une distance initiale de 6 m entre les balles. Utilise le tableau pour déterminer le nombre total de collisions avant que les balles ne cessent de s'entrechoquer.

Nombre de collisions	Distance entre les balles (m)
0	6
1	$0,9 \times 6 = 5,4$

EXTENSION: Pendul et Eliisa veulent faire un nouvel essai, mais en commençant avec une distance de 4 m entre les balles. Y aura-t-il plus ou moins de collisions que lorsqu'ils ont commencé avec les balles à 6 m l'une de l'autre ? Explique ta réponse.

THÈME : SENS DU NOMBRE



Solution

The completed table is shown. All calculations have been rounded to two decimal places.

Number of Collisions	Distance Between Balls (m)
0	6
1	$0.9 \times 6 = 5.4$
2	$0.9 \times 5.4 = 4.86$
3	$0.9 \times 4.86 = 4.37$
4	$0.9 \times 4.37 = 3.93$
5	$0.9 \times 3.93 = 3.54$
6	$0.9 \times 3.54 = 3.19$
7	$0.9 \times 3.19 = 2.87$
8	$0.9 \times 2.87 = 2.58$
9	$0.9 \times 2.58 = 2.32$
10	$0.9 \times 2.32 = 2.09$
11	$0.9 \times 2.09 = 1.88$

After 11 collisions, the balls will rebound back to a distance of 1.88 m apart. Since this is less than 2 m, they will not collide again. Therefore, there will be a total of 11 collisions between the balls.

SOLUTION TO EXTENSION:

If they started with the balls 4 m apart, there would be fewer collisions than when they started with the balls 6 m apart. Looking at the table, we see that after the 4th collision the balls are just slightly less than 4 m apart. From there, it takes $11 - 4 = 7$ more collisions before the balls will no longer collide. Thus, if the balls were initially 4 m apart, they would collide approximately 7 times.



Problème de la semaine

Problème B

Les fractions cachées

Zoé tente de trouver des fractions qui satisfont à certaines des conditions suivantes :

- (1) Le dénominateur de la fraction est divisible par 3.
 - (2) Tous les chiffres de la fraction sont pairs.
 - (3) La valeur décimale de la fraction est supérieure à 1 mais inférieure à 1,5.
 - (4) Le dénominateur de la fraction est un multiple de 2.
 - (5) La valeur décimale de la fraction est comprise entre 0,25 et 0,6.
 - (6) La valeur décimale de la fraction ne contient pas le chiffre 9.
 - (7) Le numérateur de la fraction est pair.
 - (8) La valeur décimale de la fraction est plus proche de 0,75 que de 0,55.
- (a) Trouve une fraction qui satisfait à au moins trois de ces conditions.
 - (b) Peux-tu trouver une fraction qui satisfait à toutes ces conditions ? Si oui, donne une telle fraction. Sinon, explique pourquoi une telle fraction n'existe pas.
 - (c) Quel est le nombre minimal de fractions que Zoé doit trouver de sorte que ces fractions satisfassent ensemble aux huit conditions ?





Problem of the Week

Problem B and Solution

Hidden Fractions

Problem

Zoe is trying to find fractions that satisfy some of the following conditions:

- (1) The denominator of the fraction is divisible by 3.
 - (2) All the digits in the fraction are even.
 - (3) The decimal value of the fraction is greater than 1 but less than 1.5.
 - (4) The denominator of the fraction is a multiple of 2.
 - (5) The decimal value of the fraction is between 0.25 and 0.6.
 - (6) The decimal value of the fraction does not include a 9.
 - (7) The numerator of the fraction is even.
 - (8) The decimal value of the fraction is closer to 0.75 than to 0.55.
- (a) Find a fraction that satisfies at least three of the conditions.
 - (b) Can you find a fraction that satisfies all of these conditions? If yes, give such a fraction. If no, explain why no such fraction exists.
 - (c) What is the least number of fractions Zoe needs to find such that together they meet all eight conditions?

Solution

- (a) Answers will vary. Some examples:
 - $\frac{2}{10}$ satisfies conditions (4), (6), and (7).
 - $\frac{4}{9}$ satisfies conditions (1), (5), (6), and (7).
 - $\frac{7}{9}$ satisfies conditions (1), (6), and (8).
- (b) No such fraction exists. This is because conditions (3) and (5) are contradictory, as a number cannot be both between 0.25 and 0.6 **and** be greater than 1. Similarly, conditions (5) and (8) are also contradictory.
- (c) From part (b), we know that we will need at least 2 fractions. It turns out that it is possible to satisfy all eight conditions together with just two fractions. Consider the fractions $\frac{1}{2}$ and $\frac{8}{6}$.

The fraction $\frac{1}{2} = 0.5$ satisfies conditions (4), (5), and (6).

The fraction $\frac{8}{6} = 1.333\dots$ satisfies conditions (1), (2), (3), (4), (6), (7), (8).

Answers will vary. Many other fraction pairs exist.



Problème de la semaine

Problème B

De long en large

Geordie a tracé un triangle équilatéral de côté 2 et un rectangle de longueur 2 et de largeur 1. Il a alors remarqué que chacune des deux formes avait un périmètre de 6.



- (a) Geordie a tracé un triangle équilatéral de côté 3. Peut-il tracer un rectangle ayant le même périmètre que celui de ce triangle équilatéral, si les dimensions du rectangle doivent être des nombres entiers ? Explique ta réponse.
- (b) Geordie a tracé des triangles équilatéraux de côtés 4, 5, 6 et 7. Pour lesquels de ces triangles peut-il tracer un rectangle ayant le même périmètre, si les dimensions du rectangle doivent être des nombres entiers ? Explique ta réponse.
- (c) Si les mesures des côtés d'un triangle équilatéral et d'un rectangle sont toutes des nombres entiers, alors :
 - (i) Quelles valeurs peut prendre le périmètre du triangle ?
 - (ii) Quelles valeurs peut prendre le périmètre du rectangle ?
 - (iii) Quelles valeurs peuvent représenter à la fois le périmètre du triangle et celui du rectangle ?
 - (iv) Si le triangle et le rectangle ont le même périmètre, quelle condition la mesure du côté du triangle doit-elle satisfaire ?



Problem of the Week

Problem B and Solution

The Long and the Short of It

Problem

Geordie drew an equilateral triangle with side length 2 and a rectangle with length 2 and width 1. He then noticed that each shape had a perimeter of 6.



- (a) Geordie drew an equilateral triangle with side length 3. Can Geordie draw a rectangle that has the same perimeter as the equilateral triangle, if the side lengths of the rectangle must be whole numbers? Explain.
- (b) Geordie drew equilateral triangles with side lengths 4, 5, 6, and 7. For which of these can he draw a rectangle with the same perimeter, if the side lengths of the rectangle must be whole numbers? Explain.
- (c) If the side lengths of an equilateral triangle and a rectangle are all whole numbers, then:
 - (i) What numbers could be the perimeter of the triangle?
 - (ii) What numbers could be the perimeter of the rectangle?
 - (iii) What numbers could be the perimeter of both the triangle and the rectangle?
 - (iv) If the triangle and the rectangle have the same perimeter, what must be true about the side length of the triangle?



Solution

(a) If the side length of the equilateral triangle is 3, then its perimeter is $3 \times 3 = 9$. We will try different lengths and widths for the rectangle to see if we can find ones that give us a perimeter of 9. We will use the formula for the perimeter of a rectangle, which is $P = 2l + 2w$.

- Suppose the rectangle has length 1. Then $9 = 2 \times 1 + 2w$, so $9 = 2 + 2w$. Then $7 = 2w$, so $w = 7 \div 2 = 3.5$. However this is not a whole number, so this is not a possibility.
- Suppose the rectangle has length 2. Then $9 = 2 \times 2 + 2w$, so $9 = 4 + 2w$. Then $5 = 2w$, so $w = 5 \div 2 = 2.5$. However this is not a whole number, so this is not a possibility.
- Suppose the rectangle has length 3. Then $9 = 2 \times 3 + 2w$, so $9 = 6 + 2w$. Then $3 = 2w$, so $w = 3 \div 2 = 1.5$. However this is not a whole number, so this is not a possibility.
- Suppose the rectangle has length 4. Then $9 = 2 \times 4 + 2w$, so $9 = 8 + 2w$. Then $1 = 2w$, so $w = 1 \div 2 = 0.5$. However this is not a whole number, so this is not a possibility.
- Suppose the rectangle has length 5. Then since $5 + 5 = 10$ and $10 > 9$, it is not possible for this rectangle to have a perimeter of 9.

Therefore, Geordie cannot draw a rectangle with a perimeter of 9 if its side lengths must be whole numbers.

In general, if the length and width of a rectangle are whole numbers, then $2l$ and $2w$ will always be even numbers because any whole number multiplied by 2 results in an even number. So $2l + 2w$ will also be an even number, since an even number plus an even number results in an even number. Since $P = 2l + 2w$, it follows that P also must be an even number. This explains why we cannot draw a rectangle with a perimeter of 9, since 9 is an odd number.

(b) Using our results from part (a), we know that if the side lengths of a rectangle are whole numbers, then its perimeter must be even. In other words, if the perimeter is not even, then the side lengths of the rectangle cannot all be whole numbers.

- An equilateral triangle with side length 4 has perimeter $4 \times 3 = 12$, which is even. Geordie can draw a rectangle with perimeter 12; one example is a rectangle with length 5 and width 1.



- An equilateral triangle with side length 5 has perimeter $5 \times 3 = 15$. Since this is odd, Geordie cannot draw a rectangle with perimeter 15.
 - An equilateral triangle with side length 6 has perimeter $6 \times 3 = 18$, which is even. Geordie can draw a rectangle with perimeter 18; one example is a rectangle with length 6 and width 3.
 - An equilateral triangle with side length 7 has perimeter $7 \times 3 = 21$. Since this is odd, Geordie cannot draw a rectangle with perimeter 21.
- (c) Based on our results from parts (a) and (b), we can conclude the following:
- (i) The perimeter of the equilateral triangle must be a multiple of 3, since the perimeter is equal to 3 times its side length.
 - (ii) The perimeter of the rectangle must be an even number.
 - (iii) If a number is the perimeter of both the equilateral triangle and the rectangle, then it must be both a multiple of 3 and even. This is equivalent to saying it must be a multiple of 6.
 - (iv) If the equilateral triangle and the rectangle have the same perimeter, then this perimeter must be even. Therefore, the side length of the triangle must be even.



Problème de la semaine

Problème B

Une soif de chien

Lorsqu'un chien boit, il lape l'eau en plongeant sa langue dans l'eau, puis en la retirant avec un peu d'eau qui y adhère. Certains chiens sont plus habiles que d'autres. Chase est un buveur plutôt maladroit et il laisse généralement une grande flaque d'eau sur le plancher.

- (a) Chase a une masse de 40 kg. Chaque jour, il doit boire 50 mL d'eau par kilogramme de masse corporelle. Combien de litres d'eau Chase doit-il boire par jour ?
- (b) Si 70% de la masse corporelle de Chase est composée d'eau, quelle est sa masse qui n'est PAS de l'eau ?
- (c) Pour chaque 3,5 mL d'eau que Chase lape avec sa langue, 1,5 mL se retrouve sur le plancher. S'il prend une lapée toutes les 5 secondes pendant une minute, quelle quantité d'eau boit-il avec succès ?
- (d) Après une heure, tu mesures la quantité d'eau sur le plancher et tu constates que Chase en a renversé 45 mL. Combien d'eau a-t-il réussi à boire ?
- (e) Combien de lapées par jour Chase devra-t-il prendre pour boire la quantité d'eau dont il a besoin quotidiennement ?





Problem of the Week

Problem B and Solution

Doggy Drinking

Problem

When a dog takes a drink, he laps up water by pushing his tongue into the water and then pulling it back with some water sticking to it. Some dogs are better at this than others. Chase is a messy drinker, and usually leaves a large puddle on the floor.

- (a) Chase has a mass of 40 kg. Each day he needs to drink 50 mL of water for each kg of body mass. How many litres of water should Chase drink per day?
- (b) If 70% of Chase's body mass is made up of water, how much of Chase's mass is NOT water?
- (c) For every 3.5 mL of water Chase laps up with his tongue, 1.5 mL ends up on the floor. If Chase takes a lap every 5 seconds for one minute, how much water does he successfully drink?
- (d) After an hour, you measure the water on the floor and find that Chase has spilled 45 mL. How much water did he successfully drink?
- (e) How many laps per day will it take Chase to drink the water he needs in one day?

Solution

- (a) Chase should drink $50 \times 40 = 2000$ mL, or 2 L of water per day.
- (b) The percentage of Chase's body mass that is NOT water is $100\% - 70\% = 30\%$. Thus, the amount of Chase's mass that is NOT water is 30% of 40 kg, or $0.3 \times 40 = 12$ kg.
- (c) Since there are 60 seconds in one minute, Chase takes $60 \div 5 = 12$ laps in total. From each lap, he drinks only $3.5 - 1.5 = 2.0$ mL. Thus, in 12 laps he successfully drinks a total of $12 \times 2.0 = 24$ mL of water.
- (d) The 45 mL of water Chase spilled indicates he took $45 \div 1.5 = 30$ laps. Since he drinks 2 mL with each lap, he successfully drank $2 \times 30 = 60$ mL.
- (e) From part (a), Chase needs to drink 2000 mL of water per day. He drinks 2 mL with each lap, so he will need to take $2000 \div 2 = 1000$ laps each day.



Problème de la semaine

Problème B

Les formes symboliques

Dans chacune des questions, chaque forme ($\square$, $\bigcirc$, $\triangle$) représente le même nombre entier. Par exemple, $\square + \square = 8$ signifie que $2 \times \square = 8$, donc $\square = 4$. (La valeur de chaque forme peut changer d'une question à l'autre.)

Détermine la valeur de chaque forme dans chaque question.

(a)

$$\square + \square = 8$$

$$\bigcirc + \square = 10$$

(b)

$$\square + \square + \square = 18$$

$$\square + \bigcirc + \bigcirc = 14$$

$$\bigcirc + \square + \triangle = 18$$

(c)

$$\bigcirc + \bigcirc + \bigcirc = 24$$

$$\square \times \bigcirc + \square = 36$$

$$\triangle \times \square + \triangle = 25$$

(d)

$$2 \times \triangle = 28$$

$$\triangle - 10 = \bigcirc$$

$$\bigcirc + \square + \square = 10$$

(e)

$$\square + \triangle + \triangle = 21$$

$$\triangle + \bigcirc - \square = 9$$

$$\square + \square = 18$$



EXTENSION: Crée un problème semblable et mets un ou une camarade de classe au défi de le résoudre.



Problem of the Week

Problem B and Solution

Symbolic Shapes

Problem

In each question, each shape ($\square$, $\bigcirc$, $\triangle$) is equal to the same whole number. For example, $\square + \square = 8$ implies that $2 \times \square = 8$, so $\square = 4$. (The value of each shape can change from question to question.)

Determine the value of each shape in each question.

(a)

$$\square + \square = 8$$

$$\bigcirc + \square = 10$$

(b)

$$\square + \square + \square = 18$$

$$\square + \bigcirc + \bigcirc = 14$$

$$\bigcirc + \square + \triangle = 18$$

(c)

$$\bigcirc + \bigcirc + \bigcirc = 24$$

$$\square \times \bigcirc + \square = 36$$

$$\triangle \times \square + \triangle = 25$$

(d)

$$2 \times \triangle = 28$$

$$\triangle - 10 = \bigcirc$$

$$\bigcirc + \square + \square = 10$$

(e)

$$\square + \triangle + \triangle = 21$$

$$\triangle + \bigcirc - \square = 9$$

$$\square + \square = 18$$



EXTENSION: Create a similar problem and challenge a classmate to solve it.



Solution

- (a) Since $\square + \square = 8$, we know that $2 \times \square = 8$, and so $\square = 8 \div 2 = 4$.
Then, since $\bigcirc + \square = 10$, we know that $\bigcirc + 4 = 10$, and so $\bigcirc = 10 - 4 = 6$.
Thus, $\square = 4$ and $\bigcirc = 6$.
- (b) Since $\square + \square + \square = 18$, we know that $3 \times \square = 18$, and so $\square = 18 \div 3 = 6$.
Then, since $\square + \bigcirc + \bigcirc = 14$, we know that $6 + \bigcirc + \bigcirc = 14$, and so
 $\bigcirc + \bigcirc = 14 - 6 = 8$ and $\bigcirc = 8 \div 2 = 4$.
Then, since $\bigcirc + \square + \triangle = 18$, we have $4 + 6 + \triangle = 18$, and so
 $\triangle = 18 - 10 = 8$.
Thus, $\square = 6$, $\bigcirc = 4$, and $\triangle = 8$.
- (c) Since $\bigcirc + \bigcirc + \bigcirc = 24$, we know that $3 \times \bigcirc = 24$, and so $\bigcirc = 24 \div 3 = 8$.
Then, since $\square \times \bigcirc + \square = 36$, we have $\square \times 8 + \square = 36$, and so $9 \times \square = 36$.
Therefore, $\square = 36 \div 9 = 4$.
Now, since $\triangle \times \square + \triangle = 25$, we have $\triangle \times 4 + \triangle = 25$, and so $5 \times \triangle = 25$
and $\triangle = 25 \div 5 = 5$.
Thus, $\square = 4$, $\bigcirc = 8$, and $\triangle = 5$.
- (d) Since $2 \times \triangle = 28$, we have $\triangle = 28 \div 2 = 14$.
Then, since $\triangle - 10 = \bigcirc$, we have $14 - 10 = \bigcirc$, and so $\bigcirc = 4$.
Then, since $\bigcirc + \square + \square = 10$, we have $4 + \square + \square = 10$, and so
 $\square + \square = 10 - 4 = 6$. Therefore, $\square = 6 \div 2 = 3$.
Thus, $\square = 3$, $\bigcirc = 4$, and $\triangle = 14$.
- (e) For this problem, start with the third equation. Since $\square + \square = 18$, we have
 $2 \times \square = 18$, and so $\square = 18 \div 2 = 9$.
Then, since $\square + \triangle + \triangle = 21$, we have $9 + \triangle + \triangle = 21$, and so
 $\triangle + \triangle = 21 - 9 = 12$. Thus, $2 \times \triangle = 12$, and so $\triangle = 12 \div 2 = 6$.
Now, since $\triangle + \bigcirc - \square = 9$, we have $6 + \bigcirc - 9 = 9$, and so
 $\bigcirc = 9 - 6 + 9 = 12$.
Thus, $\square = 9$, $\bigcirc = 12$, and $\triangle = 6$.



Problème de la semaine

Problème B

Des fleurs pour Sarah

Sarah a aménagé 8 nouvelles plates-bandes dans un endroit ensoleillé et souhaite y planter un jardin fleuri. Les sachets de graines coûtent 2,69 \$ par variété de fleur et elle doit payer des frais d'expédition fixes de 9,00 \$ pour sa commande. Elle aimerait planter plusieurs variétés différentes sans dépenser plus de 53,00 \$.

- (a) Écris une inégalité qui représente le problème de Sarah sous forme mathématique, en utilisant les coûts donnés, le nombre de variétés qu'elle achète et les fonds dont elle dispose.
- (b) Quelles sont toutes les possibilités pour le nombre de sachets de graines que Sarah peut acheter ?
- (c) Si Sarah achète le nombre maximal de sachets de graines qu'elle peut se permettre et qu'elle plante le même nombre de sachets dans chaque plate-bande, combien de sachets plante-t-elle dans chaque plate-bande ?
- (d) En fin de compte, Sarah achète le plus grand nombre possible de sachets de graines, en choisissant quatre variétés au total : des zinnias, des sauges, des marguerites et des cosmos. Elle achète trois fois plus de sachets de zinnias que de sauges et un nombre égal de sachets de marguerites et de cosmos. Si elle achète deux sachets de sauges, combien de sachets de chacune des trois autres variétés a-t-elle achetés ?





Problem of the Week

Problem B and Solution

Flowers for Sarah

Problem

Sarah has constructed 8 new garden beds in a sunny location and is looking to plant a flower garden. Seed packets cost \$2.69 per flower variety, and she must pay a \$9.00 flat rate shipping fee for her order. She would like to plant quite a few different varieties, but not spend more than \$53.00.

- (a) Write an inequality which states Sarah's problem mathematically, using the given costs, the number of varieties she buys, and her available funds.
- (b) What are all possibilities for the number of seed packets Sarah can purchase?
- (c) If Sarah purchases the maximum number of seed packets that she can, and plants the same number of seed packets in each garden bed, then how many packets of seeds does she plant in each garden bed?
- (d) In the end, Sarah buys as many seed packets as she can, purchasing in total four varieties: zinnias, salvia, daisies, and cosmos. She purchases three times as many packets of zinnias as salvia, and equal numbers of packets of daisies and cosmos. If she buys two seed packets of salvia, how many did she buy of each of the other three flower types?

Solution

- (a) Let x represent the number of seed packets Sarah purchases. Then the total cost of her purchase will be $\$2.69 \times x + \9.00 . Since this must be no greater than \$53.00, an appropriate inequality is $\$2.69 \times x + \$9.00 \leq \$53.00$.
- (b) The most Sarah could purchase would occur if $\$2.69 \times x + \$9.00 = \$53.00$. Solving for x gives $2.69 \times x = 53 - 9$, or $x = \frac{44}{2.69} \approx 16.36$. Thus, she can buy at most 16 seed packets. That is, she could purchase any number from 0 to 16 packets.
- (c) If she maximizes her seed purchase, then she will buy 16 packets of seeds. So she will plant $16 \div 8 = 2$ packets of seeds in each of the eight garden beds.
- (d) Since she bought 2 packets of salvia, she bought $3 \times 2 = 6$ packets of zinnias. This leaves $16 - 2 - 6 = 8$ packets to be divided evenly between daisies and cosmos, so she bought $8 \div 2 = 4$ packets of each of those.



Problème de la semaine

Problème B

Égal ou pas égal ?

- (a) Pour chacun des énoncés suivants, remplis le $\bigcirc$ avec $=$, $<$ ou $>$ afin de rendre l'énoncé vrai.

(i) $0,5 + 0,24 \bigcirc \frac{3}{4}$

(v) $25 \times 40 + 321 \bigcirc 153 \times 10$

(ii) $1,10 + \frac{1}{4} \bigcirc \frac{35}{100}$

(vi) $2 \times 24 \div 6 \bigcirc \frac{36}{6}$

(iii) $2 \times 2 \times 5 \bigcirc \frac{100}{5}$

(vii) $35 \times 2 \bigcirc 5 \times 14$

(iv) $100 - 24 + 65 \bigcirc 14 \times 10$

(viii) $2y + 6 \bigcirc y + 3 + y + 3$

- (b) Pour l'énoncé suivant, détermine des valeurs de y qui rendent l'énoncé vrai lorsque le $\bigcirc$ est remplacé par $=$, par $<$ et par $>$.

$$4y + 12 \bigcirc 4 + 2y + 4 + 3y + 4$$



Problem of the Week

Problem B and Solution

Maybe Equals, Maybe Not

Problem

- (a) For each of the following statements, fill in the $\bigcirc$ with $=$, $<$, or $>$ to make the statement true.

(i) $0.5 + 0.24 \bigcirc \frac{3}{4}$

(v) $25 \times 40 + 321 \bigcirc 153 \times 10$

(ii) $1.10 + \frac{1}{4} \bigcirc \frac{35}{100}$

(vi) $2 \times 24 \div 6 \bigcirc \frac{36}{6}$

(iii) $2 \times 2 \times 5 \bigcirc \frac{100}{5}$

(vii) $35 \times 2 \bigcirc 5 \times 14$

(iv) $100 - 24 + 65 \bigcirc 14 \times 10$

(viii) $2y + 6 \bigcirc y + 3 + y + 3$

- (b) In the following statement, determine values of y that make the statement true when the $\bigcirc$ is replaced with $=$, $<$, and $>$.

$$4y + 12 \bigcirc 4 + 2y + 4 + 3y + 4$$



Solution

- (a) To determine which symbol we should put in the $\bigcirc$, we look at the left side and right side of each statement separately.

(i) Left Side	Right Side
$0.5 + 24$ $= 0.74$	$\frac{3}{4}$ $= 0.75$

Since $0.74 < 0.75$, then $0.5 + 0.24 < \frac{3}{4}$.

(ii) Left Side	Right Side
$1.10 + \frac{1}{4}$ $= 1.10 + 0.25 = 1.35$	$\frac{35}{100}$ $= 0.35$

Since $1.35 > 0.35$, then $1.10 + \frac{1}{4} > \frac{35}{100}$.

(iii) Left Side	Right Side
$2 \times 2 \times 5$ $= 4 \times 5 = 20$	$\frac{100}{5}$ $= 20$

Since $20 = 20$, then $2 \times 2 \times 5 = \frac{100}{5}$.

(iv) Left Side	Right Side
$100 - 24 + 65$ $= 76 + 65 = 141$	14×10 $= 140$

Since $141 > 140$, then $100 - 24 + 65 > 14 \times 10$.

(v) Left Side	Right Side
$25 \times 40 + 321$ $= 1000 + 321 = 1321$	153×10 $= 1530$

Since $1321 < 1530$, then $25 \times 40 + 321 < 153 \times 10$.

(vi) Left Side	Right Side
$2 \times 24 \div 6$ $= 48 \div 6 = 8$	$\frac{36}{6}$ $= 6$

Since $8 > 6$, then $2 \times 24 \div 6 > \frac{36}{6}$.

(vii) Left Side	Right Side
35×2 $= 70$	5×14 $= 70$

Since $70 = 70$, then $35 \times 2 = 5 \times 14$.

(viii) Left Side	Right Side
$2y + 6$	$y + 3 + y + 3$ $= y + y + 3 + 3 = 2y + 6$

Since $2y + 6 = 2y + 6$ for any value of y , then $2y + 6 = y + 3 + y + 3$.

- (b) The right side can be rewritten as $4 + 2y + 4 + 3y + 4 = 2y + 3y + 4 + 4 + 4 = 5y + 12$. Then our statement becomes $4y + 12 \bigcirc 5y + 12$.

If y is any value greater than 0, such as 1, then $4y + 12 < 5y + 12$. If $y = 0$, then $4y + 12 = 5y + 12$. If y is any value less than 0, such as -1 , then $4y + 12 > 5y + 12$.



Problème de la semaine

Problème B

Des loutres en liesse !

La boutique de jouets Espace Bonheur a acheté 50 loutres en peluche à 8,00 \$ chacune en vue de les revendre. Elle a également payé des frais de livraison de 30,00 \$.

Les loutres ont été mises en vente à 10,00 \$ chacune pendant tout le mois de décembre. Par la suite, la boutique a vendu le reste des peluches à un prix réduit.

- (a) Si la boutique a vendu 30 loutres en décembre au prix de 10,00 \$ chacune, quel doit être le prix réduit par loutre afin d'atteindre le seuil de rentabilité pour cet article (c'est-à-dire qu'il n'y ait ni profit net ni perte) ?
- (b) Pour chaque loutre, quel pourcentage du prix initial de 10,00 \$ représente l'économie réalisée grâce au prix réduit ?





Problem of the Week

Problem B and Solution

Otters in a Happy Space

Problem

Happy Space toy store bought 50 plush toy otters for \$8.00 each for resale. They also paid a \$30 shipping fee.

They priced the toy otters at \$10.00 each throughout December, after which they sold the remainder at a reduced sale price.

- (a) If they sold 30 toy otters in December for \$10.00 each, what must the reduced sale price per otter be in order that they break even on this item (that is, they have no net profit nor loss)?
- (b) For each otter, what percentage of the \$10.00 price is the savings from the reduced price?



Solution

- (a) To break even, Happy Space must recover their total cost for the otters, which is $(\$8.00 \times 50) + \$30.00 = \$430.00$.

The income from the sale of 30 otters at \$10 each is $30 \times \$10.00 = \300 .

Thus, they need to recover $\$430 - \$300 = \$130$ from the sale of the remaining 20 otters at a reduced price in order to break even.

This requires that the otters be priced at $\$130 \div 20 = \6.50 . So the appropriate reduced sale price is \$6.50 to break even.

- (b) The reduced sale price of \$6.50 is $\$10.00 - \$6.50 = \$3.50$ less than the original price. Thus, as a percentage, this savings is

$$\frac{\$3.50}{\$10.00} \times 100\% = 0.35 \times 100\% = 35\%$$