Problem of the Week

Problems and Solutions 2025-2026

Problem B

Grade 5/6

The CENTRE for EDUCATION
in MATHEMATICS and COMPUTING
cemc.uwaterloo.ca

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Smoothie Choices

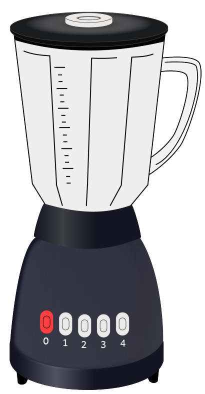
Justin and his siblings make smoothies every morning. Their smoothie recipe uses 3 cups of fruit. Sometimes they use only one type of fruit and sometimes they use more than one type of fruit, but they always use exactly 1, 2, or 3 cups of each fruit.

Smoothies taste different when they change the amounts of each fruit. For example, a smoothie with 2 cups of mango and 1 cup of blueberries is different than a smoothie with 1 cup of mango and 2 cups of blueberries.

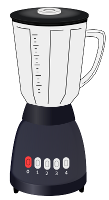
(a) If they have only strawberries and bananas, how many different smoothies can they make?

(b) If they have strawberries, bananas, and raspberries, how many different smoothies can they make?

(c) If they have strawberries, bananas, raspberries, and peaches, how many different smoothies can they make?



Theme: Algebra



Problem of the Week
Problem B and Solution
Smoothie Choices

Problem

Justin and his siblings make smoothies every morning. Their smoothie recipe uses 3 cups of fruit. Sometimes they use only one type of fruit and sometimes they use more than one type of fruit, but they always use exactly 1, 2, or 3 cups of each fruit.

Smoothies taste different when they change the amounts of each fruit. For example, a smoothie with 2 cups of mango and 1 cup of blueberries is different than a smoothie with 1 cup of mango and 2 cups of blueberries.

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(c) If they have strawberries, bananas, raspberries, and peaches, how many different smoothies can they make?

Solution

(a) To determine the number of smoothies they can make using only bananas and/or strawberries, we will write out all the options. Let S represent 1 cup of strawberries and B represent 1 cup of bananas. The smoothies they can make are as follows:

$$SSS,\ SSB,\ SBB,\ BBB$$

Thus, they can make 4 different smoothies if they have only bananas and strawberries.

(b) Let R represent 1 cup of raspberries. The smoothies they can make using strawberries, bananas, and/or raspberries are as follows:

$$SSS,\ BBB,\ RRR,\ SSB,\ SBB,\ SSR,\ SRR,\ BBR,\ BRR,\ SBR$$

Thus, they can make 10 different smoothies if they have strawberries, bananas, and raspberries.

(c) Let P represent 1 cup of peaches. Since there's more to count, we'll organize the smoothies by the number of cups of peaches.

- **Smoothies with** 0 **cups of peaches**: From part (b), we know that they can make 10 different smoothies if they have strawberries, bananas, and raspberries.
- **Smoothies with** 1 **cup of peaches**: The possibilities for the remaining 2 cups of fruit are SS, BB, RR, SB, SR, and BR. Therefore, they can make 6 different smoothies if 1 cup is peaches.
- **Smoothies with** 2 **cups of peaches**: The possibilities for the remaining 1 cup of fruit are S, B, and R. Therefore, they can make 3 different smoothies if 2 cups are peaches.
- **Smoothies with** 3 **cups of peaches**: This can be done in only one way as PPP.

Thus, in total they can make $10 + 6 + 3 + 1 = 20$ different smoothies if they have strawberries, bananas, raspberries, and peaches.

Problem of the Week

Problem B

Grain Guess

Caden's class wants to win a prize for the closest estimate to the number of grains of rice in a 1 kg bag of rice.

They have been given three clues:

Clue (1): The number of grains is a 5-digit even number.

Clue (2): The sum of the five digits in the number is 15.

Clue (3): The number, rounded to the nearest hundred, is 51 200.

(a) Help Caden's class win the prize. Use the given clues to determine all possible estimates of the number of grains of rice in the bag.

(b) Kendala, one of Caden's classmates, found that the mass of a single grain of rice depends on the type, ranging from about 20 to 45 milligrams. When she used this to calculate how many grains were in the 1 kg bag, did her answer suggest the rice was of the lighter or heavier type?

Theme: Algebra

Problem of the Week

Problem B and Solution

Grain Guess

Problem

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Solution

(a) From Clues (1) and (3), we know that the number is an even number between 51 150 and 51 249.

Therefore, the number is of the form $51\,1XY$ or $51\,2XY$, where X and Y are digits.

Suppose the number is of the form $51\,1XY$. We know that Y is even, and $X \geq 5$. Since $5 + 1 + 1 = 7$, then from Clue (2), $X + Y = 8$. Therefore, we could have $X = 6$ and $Y = 2$, or $X = 8$ and $Y = 0$.

Suppose the number is of the form $51\,2XY$. We know that Y is even and $X \leq 4$. Since $5 + 1 + 2 = 8$, then from Clue (2), $X + Y = 7$. Since Y is an even digit, we could have $X = 3$ and $Y = 4$, or $X = 1$ and $Y = 6$.

Therefore, the estimates that satisfy all three clues are 51 162, 51 180, 51 234, and 51 216.

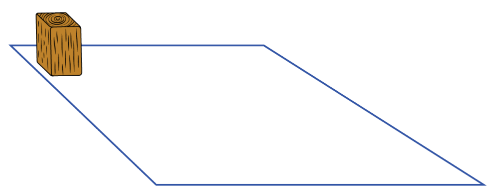
(b) Since 1 kg $=$ 1000 g $=$ 1 000 000 mg, for the lighter rice, Kendala would calculate $1\,000\,000 \div 20 = 50\,000$ grains of rice. For the heavier rice, she would calculate $1\,000\,000 \div 45 \approx 22\,222$ grains of rice. Thus, her answers indicate that the class's rice was of the lighter type, since that answer is closest to their estimate.

Problem of the Week
Problem B
Stamp it Out

Evelina is using a wooden block as a stamp to make decorative wrapping paper. The wooden block measures 3 cm by 3 cm by 4 cm and Evelina is stamping with one of the 3 cm by 3 cm square faces of the block.

(a) Evelina creates a row of 10 stamped squares with a gap of 0.5 cm in between each square. Determine the length of this row.

(b) Evelina has a square piece of wrapping paper with side length 77.5 cm. She leaves a gap of 0.5 cm around each edge and then creates rows of stamped squares with a gap of 0.5 cm in between each square as well as between each row. How many stamped squares in total will be on this piece of wrapping paper?



Themes: Algebra, Geometry & Measurement

Problem of the Week
Problem B and Solution
Stamp it Out

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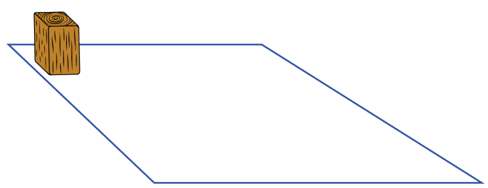
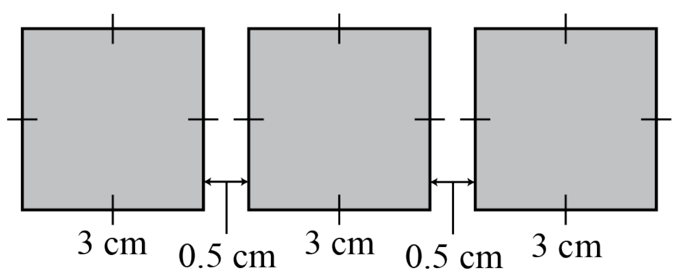


Solution

(a) We start by drawing a diagram of the first three stamped squares in the row.



From here we show two different ways to solve the problem.

Solution 1

In this solution we use a table to help us determine the length of the row. The first stamped square has a length of 3 cm, and each square after that adds $3 + 0.5 = 3.5$ cm to the length.

Number of Stamped Squares	Length of Row (cm)
1	3
2	$3 + 3.5 = 6.5$
3	$6.5 + 3.5 = 10$
4	$10 + 3.5 = 13.5$
5	$13.5 + 3.5 = 17$
6	$17 + 3.5 = 20.5$
7	$20.5 + 3.5 = 24$
8	$24 + 3.5 = 27.5$
9	$27.5 + 3.5 = 31$
10	$31 + 3.5 = 34.5$

Thus, the length of the row is 34.5 cm.

Solution 2

In this solution we use an algebraic expression help us determine the length of the row. This solution is more efficient, especially as the row gets longer. We know the length of each stamped square is 3 cm and the length of each gap between the squares is 0.5 cm. If we let s represent the total number of squares and g represent the total number of gaps between the squares, then the length of the row, in cm, is $3 \times s + 0.5 \times g$.

If there are 10 squares in the row then there are 9 gaps between them. Then we can use our equation to determine the length of the row.

$$\begin{aligned} 3 \times s + 0.5 \times g &= 3 \times 10 + 0.5 \times 9 \\ &= 30 + 4.5 \\ &= 34.5 \end{aligned}$$

Thus, the length of the row is 34.5 cm.

(b) We will start by determining the number of stamped squares in each row. Since the piece of wrapping paper is a square, this will also be the number of rows of stamped squares.

If we subtract the gap at the start of the row, then the length of the row will be $77.5 - 0.5 = 77$ cm. This is helpful because each square is followed by a gap, so the total length of the square and its gap is $3 + 0.5 = 3.5$ cm. Then we can divide to determine the number of squares in the row. Since $77 \div 3.5 = 22$, it follows that there are 22 squares in the row.

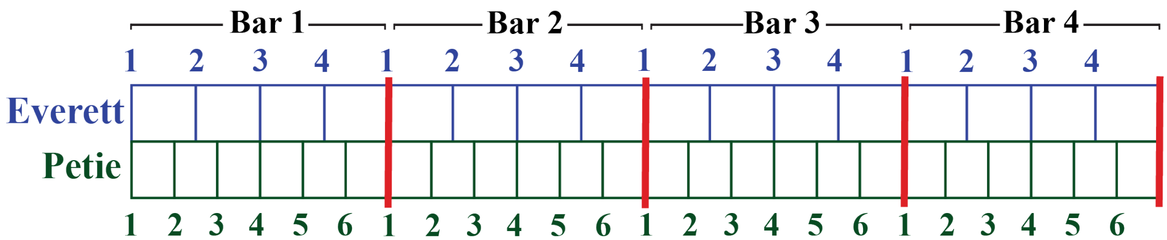
Since there are also 22 rows of squares, the total number of squares is $22 \times 22 = 484$.

Problem of the Week

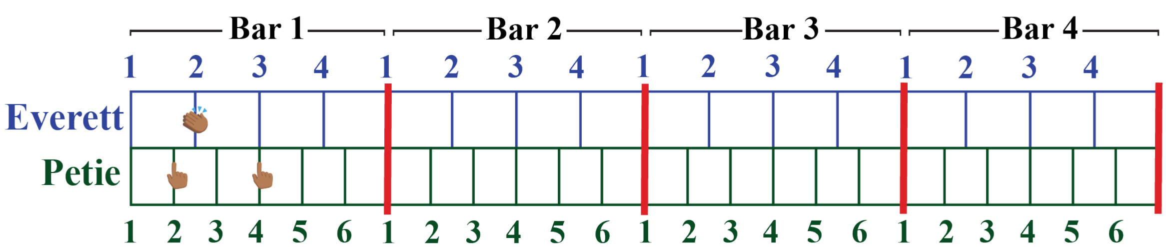
Problem B

Playing to the Beat

A "bar" is a segment of music, broken up into evenly spaced "beats". In music class, Everett claps his hands in a rhythm with four beats per bar. Petie taps on his desk in a rhythm with six beats per bar. The first four bars for Everett and Petie are shown in the diagram. The numbers above the vertical lines indicate the beat number in each bar for Everett and the numbers below the vertical lines indicate the beat number in each bar for Petie.



(a) In Bar 1, Everett claps on the second beat. After that, Everett claps on every third beat (so his next clap is on the first beat in Bar 2). Petie taps on the second and fourth beats in every bar. The diagram below shows Everett's clap and Petie's taps in Bar 1. Complete the diagram to show their taps and claps over the next three bars. In Bar 1, Everett claps between Petie's two taps. When does Everett next clap between Petie's two taps in a bar?



(b) If Petie and Everett continue tapping and clapping this way, does a repeated pattern of sounds occur? If so, how many bars are there in this pattern before it repeats?

(c) How could Petie change his approach so that, in every bar, at least one of Everett's claps happens between Petie's two taps?

Suggestion: Work with a classmate to tap and clap in Everett's and Petie's rhythms.

Theme: Algebra

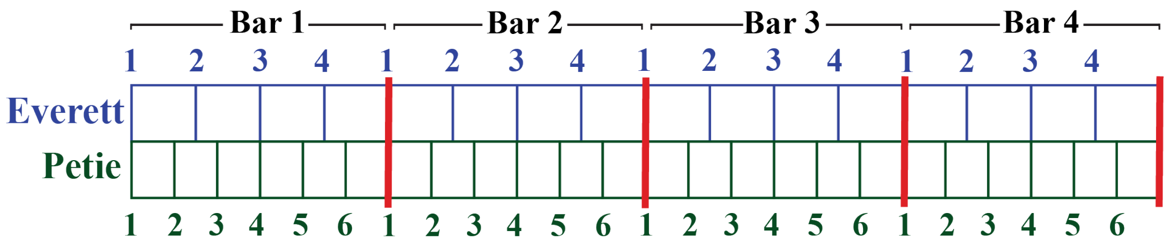
Problem of the Week

Problem B and Solution

Playing to the Beat

Problem

A "bar" is a segment of music, broken up into evenly spaced "beats". In music class, Everett claps his hands in a rhythm with four beats per bar. Petie taps on his desk in a rhythm with six beats per bar. The first four bars for Everett and Petie are shown in the diagram. The numbers above the vertical lines indicate the beat number in each bar for Everett and the numbers below the vertical lines indicate the beat number in each bar for Petie.



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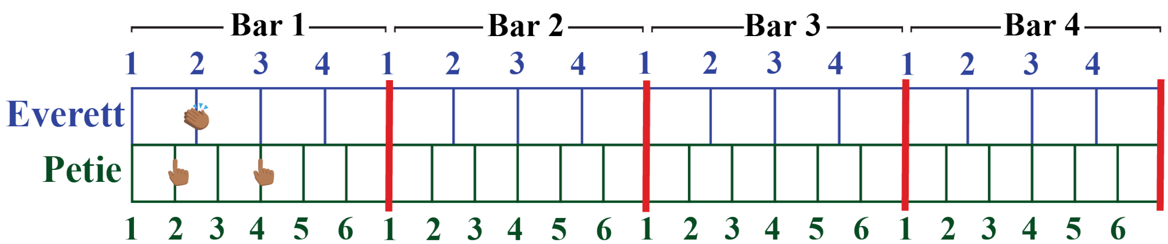

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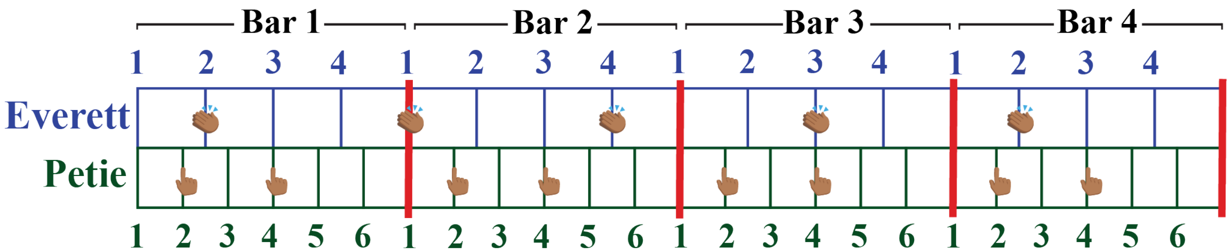
Suggestion: Work with a classmate to tap and clap in Everett's and Petie's rhythms.

Solution

(a) The completed diagram is shown.



Everett next claps between Petie's two taps in Bar 4.

(b) Notice that Bar 4 matches Bar 1. Thus, a repeated pattern of sounds occurs after there are three bars in the pattern. That is, the pattern repeats every three bars.

(c) Answers will vary. Petie could tap on the first and sixth beats in each bar. Alternatively, Petie could tap on the second and sixth beats in each bar.

Problem of the Week
Problem B
Symbolic Shapes

In each question, each shape ($\square$, $\bigcirc$, $\triangle$) is equal to the same whole number. For example, $\square + \square = 8$ implies that $2 \times \square = 8$, so $\square = 4$. (The value of each shape can change from question to question.)

Determine the value of each shape in each question.

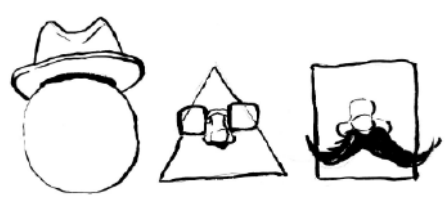
(a)
$$\square + \square = 8$$
$$\bigcirc + \square = 10$$

(b)
$$\square + \square + \square = 18$$
$$\square + \bigcirc + \bigcirc = 14$$
$$\bigcirc + \square + \triangle = 18$$

(c)
$$\bigcirc + \bigcirc + \bigcirc = 24$$
$$\square \times \bigcirc + \square = 36$$
$$\triangle \times \square + \triangle = 25$$

(d)
$$2 \times \triangle = 28$$
$$\triangle - 10 = \bigcirc$$
$$\bigcirc + \square + \square = 10$$

(e)
$$\square + \triangle + \triangle = 21$$
$$\triangle + \bigcirc - \square = 9$$
$$\square + \square = 18$$



Extension: Create a similar problem and challenge a classmate to solve it.

Themes: Algebra, Number Sense

Problem of the Week
Problem B and Solution
Symbolic Shapes

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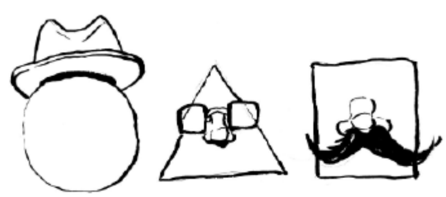

Extension: Create a similar problem and challenge a classmate to solve it.

Solution

(a) Since $\square + \square = 8$, we know that $2 \times \square = 8$, and so $\square = 8 \div 2 = 4$.
Then, since $\bigcirc + \square = 10$, we know that $\bigcirc + 4 = 10$, and so $\bigcirc = 10 - 4 = 6$.
Thus, $\square = 4$ and $\bigcirc = 6$.

(b) Since $\square + \square + \square = 18$, we know that $3 \times \square = 18$, and so $\square = 18 \div 3 = 6$.
Then, since $\square + \bigcirc + \bigcirc = 14$, we know that $6 + \bigcirc + \bigcirc = 14$, and so $\bigcirc + \bigcirc = 14 - 6 = 8$ and $\bigcirc = 8 \div 2 = 4$.
Then, since $\bigcirc + \square + \triangle = 18$, we have $4 + 6 + \triangle = 18$, and so $\triangle = 18 - 10 = 8$.
Thus, $\square = 6$, $\bigcirc = 4$, and $\triangle = 8$.

(c) Since $\bigcirc + \bigcirc + \bigcirc = 24$, we know that $3 \times \bigcirc = 24$, and so $\bigcirc = 24 \div 3 = 8$.
Then, since $\square \times \bigcirc + \square = 36$, we have $\square \times 8 + \square = 36$, and so $9 \times \square = 36$.
Therefore, $\square = 36 \div 9 = 4$.
Now, since $\triangle \times \square + \triangle = 25$, we have $\triangle \times 4 + \triangle = 25$, and so $5 \times \triangle = 25$ and $\triangle = 25 \div 5 = 5$.
Thus, $\square = 4$, $\bigcirc = 8$, and $\triangle = 5$.

(d) Since $2 \times \triangle = 28$, we have $\triangle = 28 \div 2 = 14$.
Then, since $\triangle - 10 = \bigcirc$, we have $14 - 10 = \bigcirc$, and so $\bigcirc = 4$.
Then, since $\bigcirc + \square + \square = 10$, we have $4 + \square + \square = 10$, and so $\square + \square = 10 - 4 = 6$. Therefore, $\square = 6 \div 2 = 3$.
Thus, $\square = 3$, $\bigcirc = 4$, and $\triangle = 14$.

(e) For this problem, start with the third equation. Since $\square + \square = 18$, we have $2 \times \square = 18$, and so $\square = 18 \div 2 = 9$.
Then, since $\square + \triangle + \triangle = 21$, we have $9 + \triangle + \triangle = 21$, and so $\triangle + \triangle = 21 - 9 = 12$. Thus, $2 \times \triangle = 12$, and so $\triangle = 12 \div 2 = 6$.
Now, since $\triangle + \bigcirc - \square = 9$, we have $6 + \bigcirc - 9 = 9$, and so $\bigcirc = 9 - 6 + 9 = 12$.
Thus, $\square = 9$, $\bigcirc = 12$, and $\triangle = 6$.

Problem of the Week
Problem B
Flowers for Sarah

Sarah has constructed 8 new garden beds in a sunny location and is looking to plant a flower garden. Seed packets cost \$2.69 per flower variety, and she must pay a \$9.00 flat rate shipping fee for her order. She would like to plant quite a few different varieties, but not spend more than \$53.00.

(a) Write an inequality which states Sarah's problem mathematically, using the given costs, the number of varieties she buys, and her available funds.

(b) What are all possibilities for the number of seed packets Sarah can purchase?

(c) If Sarah purchases the maximum number of seed packets that she can, and plants the same number of seed packets in each garden bed, then how many packets of seeds does she plant in each garden bed?

(d) In the end, Sarah buys as many seed packets as she can, purchasing in total four varieties: zinnias, salvia, daisies, and cosmos. She purchases three times as many packets of zinnias as salvia, and equal numbers of packets of daisies and cosmos. If she buys two seed packets of salvia, how many did she buy of each of the other three flower types?

Themes: Algebra, Number Sense

Problem of the Week
Problem B and Solution
Flowers for Sarah

Problem

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Solution

(a) Let x represent the number of seed packets Sarah purchases. Then the total cost of her purchase will be $\$2.69 \times x + \$9.00$. Since this must be no greater than \$53.00, an appropriate inequality is $\$2.69 \times x + \$9.00 \leq \$53.00$.

(b) The most Sarah could purchase would occur if $\$2.69 \times x + \$9.00 = \$53.00$. Solving for x gives $2.69 \times x = 53 - 9$, or $x = \frac{44}{2.69} \approx 16.36$. Thus, she can buy at most 16 seed packets. That is, she could purchase any number from 0 to 16 packets.

(c) If she maximizes her seed purchase, then she will buy 16 packets of seeds. So she will plant $16 \div 8 = 2$ packets of seeds in each of the eight garden beds.

(d) Since she bought 2 packets of salvia, she bought $3 \times 2 = 6$ packets of zinnias. This leaves $16 - 2 - 6 = 8$ packets to be divided evenly between daisies and cosmos, so she bought $8 \div 2 = 4$ packets of each of those.

Computational Thinking (C)

Take me to the cover

Problem of the Week

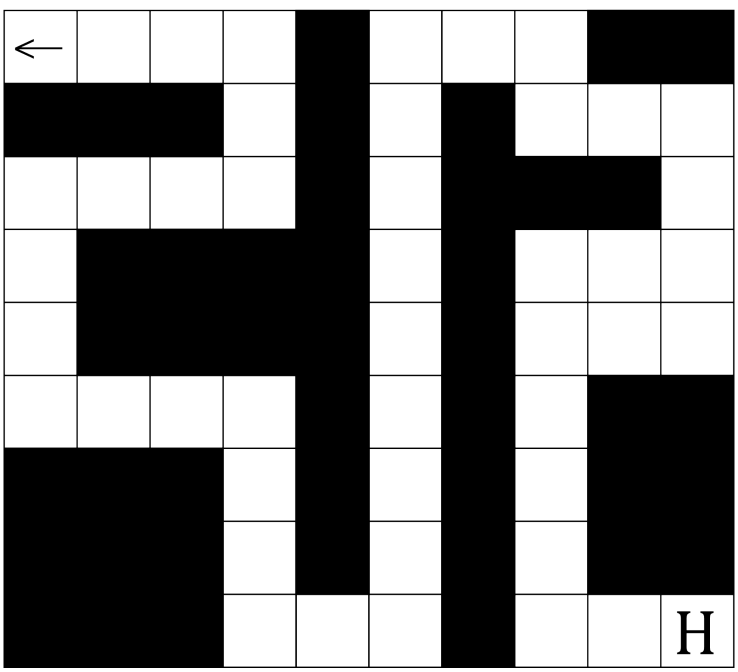
Problem B

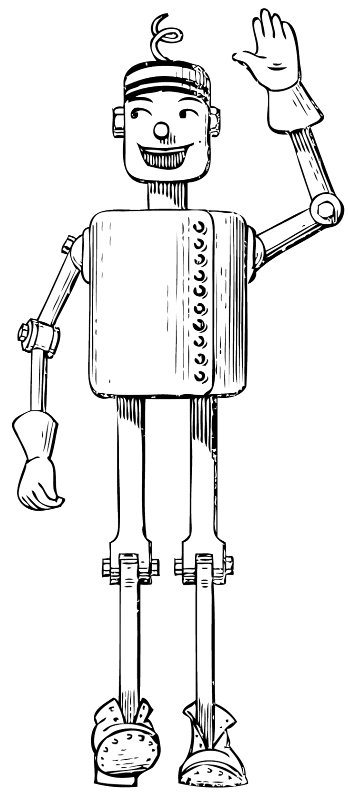
Help Send Freddy Home

Freddy the Robot starts in the top left square of the maze below, facing left (symbolized by $\leftarrow$). Freddy wants to get to his Home Charging Station in the bottom right corner of the maze, marked with an **H**. Freddy can only turn right (clockwise on the diagram), and will only turn the number of degrees which you tell him. As well, he will go exactly the number of squares you tell him and in the direction that the arrow is currently pointing.

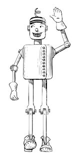
For example, if he starts facing $\leftarrow$, and your command said, “Turn 180 degrees and go 3 spaces”, he would turn 180° right (clockwise) and then move 3 spaces forward (to the right).

Write a program of commands that gets Freddy to his Home Charging Station.





Themes: Computational Thinking, Geometry & Measurement



Problem of the Week

Problem B and Solution

Help Send Freddy Home

Problem

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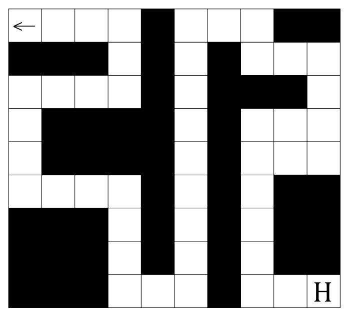


Solution

A program of commands is below. Note that there two options for commands 12 and 14. The second option is in parentheses.

1. Turn 180° and go 3 spaces.
2. Turn 90° and go 2 spaces.
3. Turn 90° and go 3 spaces.
4. Turn 270° and go 3 spaces.
5. Turn 270° and go 3 spaces.
6. Turn 90° and go 3 spaces.
7. Turn 270° and go 2 spaces.
8. Turn 270° and go 8 spaces.
9. Turn 90° and go 2 spaces.
10. Turn 90° and go 1 space.
11. Turn 270° and go 2 spaces.
12. Turn 90° and go 3 (or 2) spaces.
13. Turn 90° and go 2 spaces.
14. Turn 270° and go 4 (or 5) spaces.
15. Turn 270° and go 2 spaces.

Now Freddy is at his Home Charging Station!

Problem of the Week

Problem B

Coding Figures

Ayanna writes programs using the following commands that direct her robot to draw shapes.

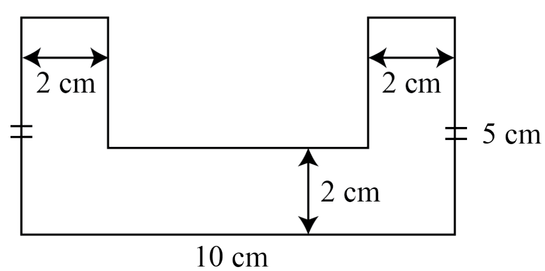
Command	**Explanation**
`Pencil down`	Puts the pencil in position to draw as the robot moves
`Move`	Moves the robot a given distance forward
`Turn CW`	Rotates the robot one quarter turn (90°) clockwise
`Turn CCW`	Rotates the robot one quarter turn (90°) counter clockwise

(a) For each program, follow the code to draw the shape. You may find grid paper helpful.

(i)
```
Pencil down
Repeat 4 times
  Move 5 cm
  Turn CW
End
```

(ii)
```
Pencil down
Move 4 cm
Turn CCW
Move 2 cm
Turn CW
Move 2 cm
Turn CW
Move 6 cm
Turn CW
Move 6 cm
Turn CW
Move 4 cm
End
```

(b) Write a program that would direct Ayanna's robot to draw the given rectangular U-shape.



Themes: Computational Thinking, Geometry & Measurement

Problem of the Week
Problem B and Solution
Coding Figures

Problem

Ayanna writes programs using the following commands that direct her robot to draw shapes.

Command	Explanation
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(a) For each program, follow the code to draw the shape. You may find grid paper helpful.

(i)
```
Pencil down
Repeat 4 times
  Move 5 cm
  Turn CW
End
```

(ii)
```
Pencil down
Move 4 cm
Turn CCW
Move 2 cm
Turn CW
Move 2 cm
Turn CW
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Turn CW
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Turn CW
Move 4 cm
End
```

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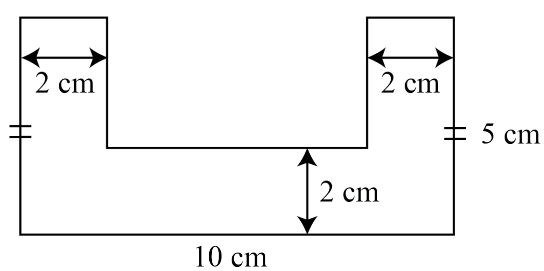
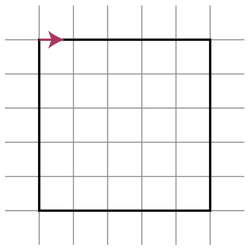


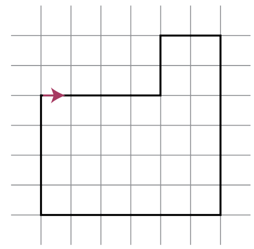
Solution

(a) For each program, the shape is shown on 1 cm grid paper with an arrow indicating the initial direction of the robot. Note that for part (i), the shape will look the same regardless of the initial direction. However for part (ii), the shape will be rotated depending on the initial direction.

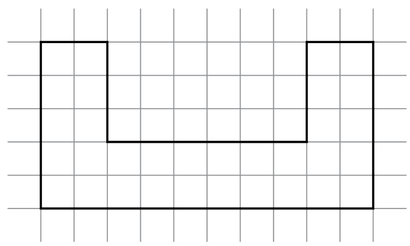
(i)



(ii)



(b) We start by determining the length of each side of the shape. We do this by drawing the shape on grid paper, as shown.



There are several different programs that can be written to direct Ayanna's robot to draw the given shape, depending on the robot's starting position and direction. In the following program, the robot starts in the bottom-left corner, facing to the right.

```
Pencil down
Move 10 cm
Turn CCW
Move 5 cm
Turn CCW
Move 2 cm
Turn CCW
Move 3 cm
Turn CW
Move 6 cm
Turn CW
Move 3 cm
Turn CCW
Move 2 cm
Turn CCW
Move 5 cm
End
```

Problem of the Week

Problem B

Hidden Fractions

Zoe is trying to find fractions that satisfy some of the following conditions:

(1) The denominator of the fraction is divisible by 3.

(2) All the digits in the fraction are even.

(3) The decimal value of the fraction is greater than 1 but less than 1.5.

(4) The denominator of the fraction is a multiple of 2.

(5) The decimal value of the fraction is between 0.25 and 0.6.

(6) The decimal value of the fraction does not include a 9.

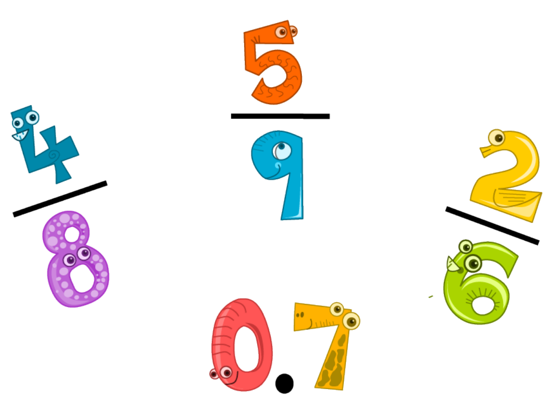
(7) The numerator of the fraction is even.

(8) The decimal value of the fraction is closer to 0.75 than to 0.55.

(a) Find a fraction that satisfies at least three of the conditions.

(b) Can you find a fraction that satisfies all of these conditions? If yes, give such a fraction. If no, explain why no such fraction exists.

(c) What is the least number of fractions Zoe needs to find such that together they meet all eight conditions?



Themes: Computational Thinking, Number Sense

Problem of the Week
Problem B and Solution
Hidden Fractions

Problem

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(b) Can you find a fraction that satisfies all of these conditions? If yes, give such a fraction. If no, explain why no such fraction exists.

(c) What is the least number of fractions Zoe needs to find such that together they meet all eight conditions?

Solution

(a) Answers will vary. Some examples:

- $\frac{2}{10}$ satisfies conditions (4), (6), and (7).
- $\frac{4}{9}$ satisfies conditions (1), (5), (6), and (7).
- $\frac{7}{9}$ satisfies conditions (1), (6), and (8).

(b) No such fraction exists. This is because conditions (3) and (5) are contradictory, as a number cannot be both between 0.25 and 0.6 **and** be greater than 1. Similarly, conditions (5) and (8) are also contradictory.

(c) From part (b), we know that we will need at least 2 fractions. It turns out that it is possible to satisfy all eight conditions together with just two fractions. Consider the fractions $\frac{1}{2}$ and $\frac{8}{6}$.

The fraction $\frac{1}{2} = 0.5$ satisfies conditions (4), (5), and (6).

The fraction $\frac{8}{6} = 1.333\ldots$ satisfies conditions (1), (2), (3), (4), (6), (7), (8).

Answers will vary. Many other fraction pairs exist.

Data Management (D)

Take me to the cover

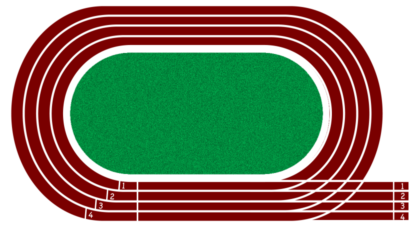
Problem of the Week

Problem B

Who Wins the Best Lane?

Donovan Bailey Elementary School has organized a relay race as part of its track and field day. Four teams, A, B, C, and D, are participating.

To assign each team to a lane on the track, the principal draws the team names randomly. The first team drawn will run in Lane 1, the next in Lane 2, the next in Lane 3, and the remaining team will run in Lane 4.



(a) What is the theoretical probability that Team A will be assigned to Lane 1? Express your answer as a fraction, a decimal, and a percentage.

(b) The principal decides to draw teams several times, and assign Lane 1 to the team that was drawn first most often. The results are in the table.

Team	A	B	C	D
Number of Times First	20	6	12	10

Use these results to calculate the experimental probability of Team A being assigned to Lane 1. How does this compare to the theoretical probability from part (a)?

(c) Use the theoretical probability from part (a) to calculate the number of times you would expect each team to win Lane 1. Do any of the experimental results in part (b) equal this expected result?

Extension:

Draw a spinner with four sections so that the theoretical probability of landing on each section is the same as the experimental probability calculated from the table.

Problem of the Week

Problem B and Solution

Who Wins the Best Lane?

Problem

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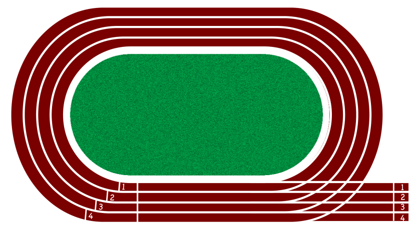

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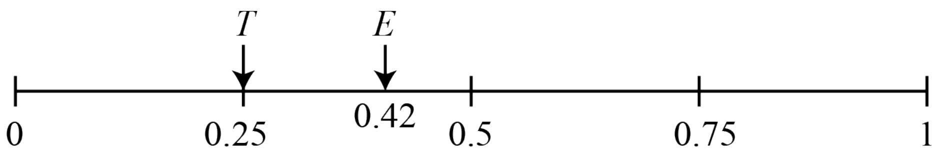
Extension:
Draw a spinner with four sections so that the theoretical probability of landing on each section is the same as the experimental probability calculated from the table.

Solution

(a) Since there are 4 teams, and each team has an equal chance of being assigned to Lane 1, the theoretical probability that Team A will be assigned to Lane 1 is $\frac{1}{4}$, or 0.25, or 25%.

(b) The principal drew teams $20 + 6 + 12 + 10 = 48$ times. Thus, the experimental probability of Team A being assigned to Lane 1 was $\frac{20}{48}$, or $\frac{5}{12}$, which is approximately 0.42, or 42%. This is significantly higher than the theoretical probability from part (a). They are shown on the following probability number line, where E represents the experimental probability and T represents the theoretical probability.



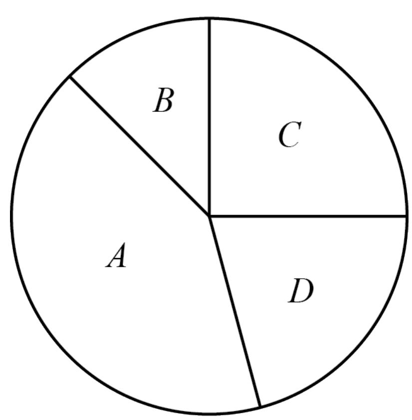
(c) Since the principal drew teams 48 times, and each team had an equal chance of getting drawn each time, we would expect that the number of times each team would win Lane 1 would be $48 \div 4 = 12$. The experimental result for Team C is equal to this expected result.

Solution to Extension:

When creating the spinner, we note that the probability of landing on section C should be $\frac{1}{4}$. Since a circle has $360°$, this means that the centre angle for section C should be $\frac{1}{4}$ of $360°$, which is $90°$.

We notice that the number of times first for Team B is half the number of times first for Team C. Thus, the centre angle for section B should be half of $90°$, which is $45°$. This leaves a section with a centre angle of $360° - 90° - 45° = 225°$ that will be split between sections A and D.

We notice that the number of times first for Team A is twice the number of times first for Team D. Thus, section A should be twice as large as section D. Since they're sharing a section with a centre angle of $225°$, we can determine that $\frac{2}{3}$ of this section will be section A and $\frac{1}{3}$ will be section D. Therefore, the centre angle for section A will be $\frac{2}{3}$ of $225°$, which is $150°$, and the centre angle for section D will be $\frac{1}{3}$ of $225°$, which is $75°$. Now we use our protractor to draw the spinner as shown.

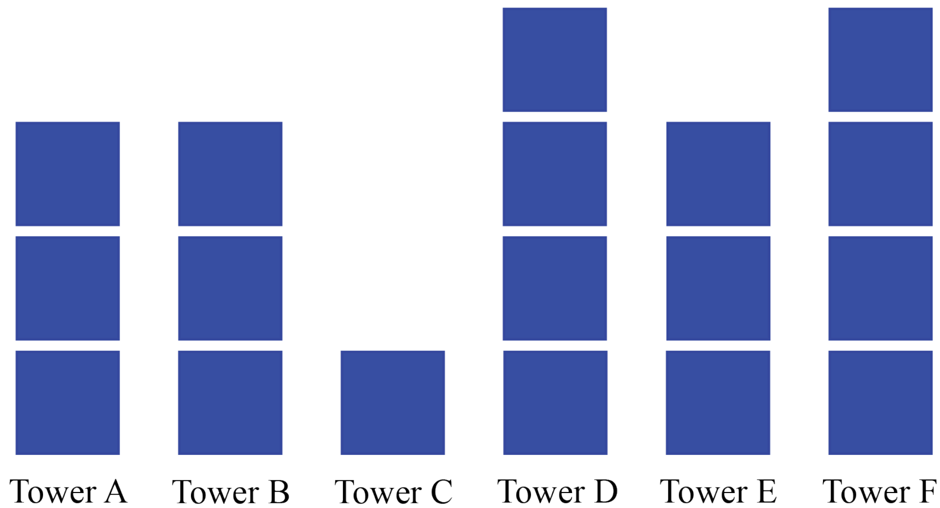


Problem of the Week

Problem B

Means à la Mode

Octavia has a large bin of blocks. Using 18 blue blocks she makes exactly six towers. The height of the tower is the total number of blocks used in that tower.



(a) Determine the mean, the median, and the mode of the heights of the towers made by Octavia.

(b) Create a different set of six towers using 18 blocks where the mean height is greater than the mode height.

(c) Create a different set of six towers using 18 blocks where the mean height is greater than the median height.

(d) Create a different set of six towers using 18 blocks where the median height is greater than the mean height.

Theme: Data Management

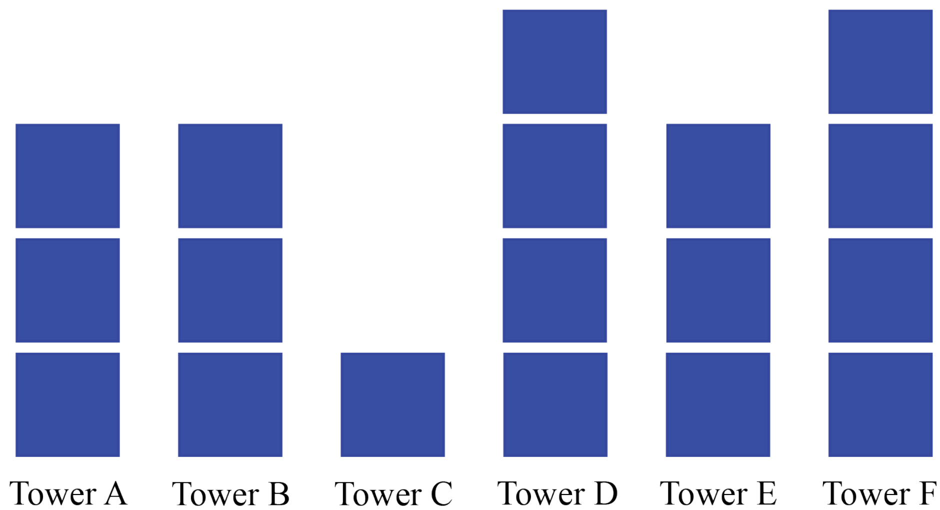
Problem of the Week

Problem B and Solution

Means à la Mode

Problem

Octavia has a large bin of blocks. Using 18 blue blocks she makes exactly six towers. The height of the tower is the total number of blocks used in that tower.



(a) Determine the mean, the median, and the mode of the heights of the towers made by Octavia.

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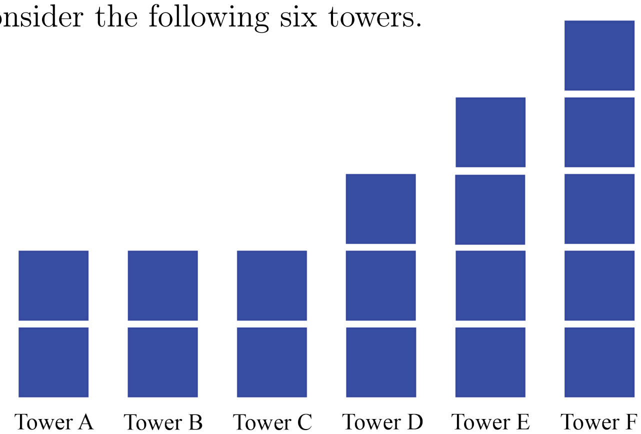
(c) Create a different set of six towers using 18 blocks where the mean height is greater than the median height.

(d) Create a different set of six towers using 18 blocks where the median height is greater than the mean height.

Solution

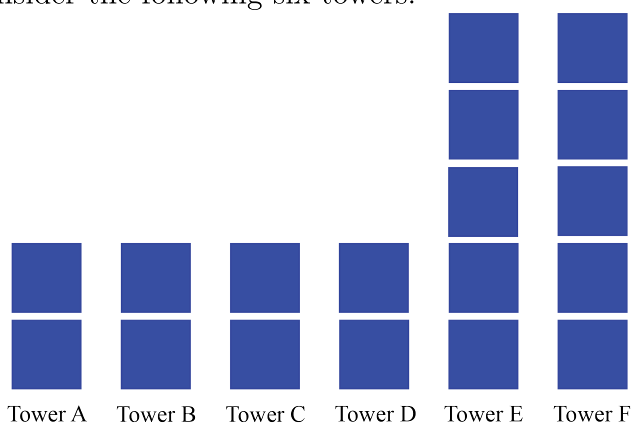
(a) Written in increasing order, the tower heights are 1, 3, 3, 3, 4, 4. Thus the mode is 3, the median is $(3 + 3) \div 2 = 3$, and the mean is $(1 + 3 + 3 + 3 + 4 + 4) \div 6 = 3$.

(b) Answers will vary. Consider the following six towers.



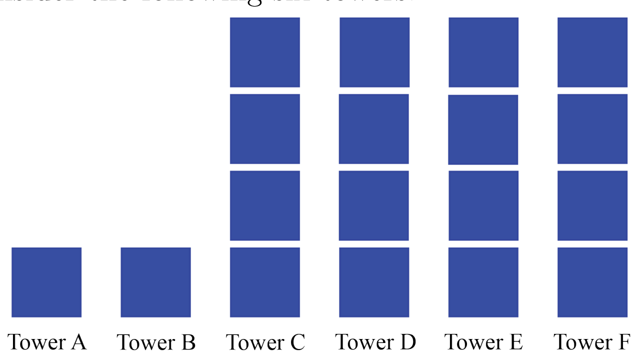
The tower heights are 2, 2, 2, 3, 4, 5. The mode is 2 and the mean is $(2 + 2 + 2 + 3 + 4 + 5) \div 6 = 3$. Thus, the mean is greater than the mode.

(c) Answers will vary. Consider the following six towers.

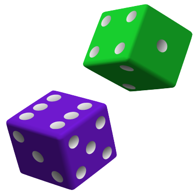


The tower heights are 2, 2, 2, 2, 5, 5. The median is $(2 + 2) \div 2 = 2$ and the mean is $(2 + 2 + 2 + 2 + 5 + 5) \div 6 = 3$. Thus, the mean is greater than the median.

(d) Answers will vary. Consider the following six towers.



The tower heights are 1, 1, 4, 4, 4, 4. The median is $(4 + 4) \div 2 = 4$, and the mean is $(1 + 1 + 4 + 4 + 4 + 4) \div 6 = 3$. Thus, the median is greater than the mean.

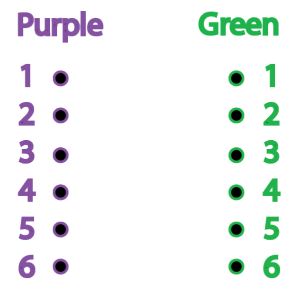


Problem of the Week

Problem B

Dicey Products

Jordyn and Langdon have two dice, one purple and one green. When they roll the dice, they know there are a total of $6 \times 6 = 36$ possible outcomes for the top two numbers on the dice.



(a) In the diagram, join any dot on the left with one on the right such that the product of those two numbers is odd. How many ways are there to obtain an odd product?

(b) What does your result in part (a) tell you about how many ways there are to obtain an even product?

(c) Is the product 4 or 6 more likely to occur? Why?

(d) In the table, enter the possible odd products in the first column, the number of ways that product could occur in the middle column, and the theoretical probability of that outcome in the third column. (This has been done for odd product 1.)

Odd Product	Number of Ways	Theoretical Probability
1	1	$\frac{1}{36}$

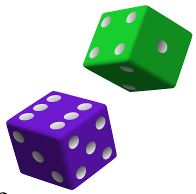
(e) Roll two dice 36 times and record the product of the top two numbers for each roll. Count the number of each odd product that occurred, and then calculate the experimental probability of rolling that number. Compare the experimental probability to the theoretical probability calculated in part (d).

Themes: Data Management, Number Sense

Problem of the Week

Problem B and Solution

Dicey Products



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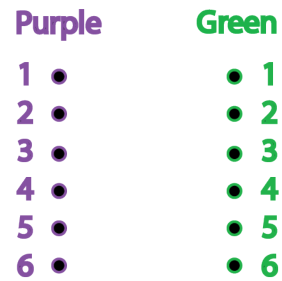

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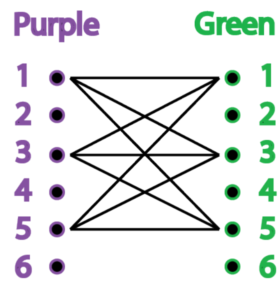
Odd Product	Number of Ways	Theoretical Probability
1	1	$\frac{1}{36}$

(e) Roll two dice 36 times and record the product of the top two numbers for each roll. Count the number of each odd product that occurred, and then calculate the experimental probability of rolling that number. Compare the experimental probability to the theoretical probability calculated in part (d).

Solution

(a) Odd products will occur exactly when the top two numbers are odd. We connect each odd number on the left with each odd number on the right to obtain the completed diagram shown. Counting the number of connections, we determine that there are 9 ways to obtain an odd product.



(b) Since there are 36 possible products in total, and a product must be either even or odd, there must be $36 - 9 = 27$ ways to obtain an even product.

(c) The product 6 is more likely to occur. This is because $6 = 1 \times 6$, $6 = 6 \times 1$, $6 = 2 \times 3$, and $6 = 3 \times 2$, and so can be formed by 4 different rolls. However, since $4 = 1 \times 4$, $4 = 4 \times 1$, and $4 = 2 \times 2$, it can only be formed by 3 different rolls.

(d) Noting that there is only one way to get each of 1, 9, and 25, and two ways to get each of 3, 5, and 15, the appropriate data and the theoretical probabilities are calculated.

Odd Product	Number of Ways	Theoretical Probability
1	1	$\frac{1}{36}$
3	2	$\frac{2}{36}$
5	2	$\frac{2}{36}$
9	1	$\frac{1}{36}$
15	2	$\frac{2}{36}$
25	1	$\frac{1}{36}$

(e) Answers will vary. Note that the result of a single experiment won't necessarily agree very closely with the theoretical probabilities.

SUGGESTION: Average the experimental probabilities obtained by all students in your class for each of the six odd products. Do these average probabilities agree more closely with the theoretical probabilities for each odd product than those of individual students?

Problem of the Week

Problem B

Three Coins in a Toss

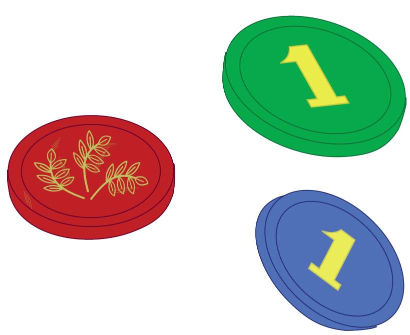
Jack and Jill have made up a game where they take turns tossing a red coin, a blue coin, and a green coin. Each coin has heads (H) on one side and tails (T) on the other side. They earn points according to the following rules.

1. If the toss results in all three coins being heads (HHH), or all three being tails (TTT), then Jack gets one point.
2. If the toss results in any other combination of heads and tails (e.g., TTH), then Jill gets one point.
3. The winner of the game is the player with the most points after 10 tosses of all three coins.

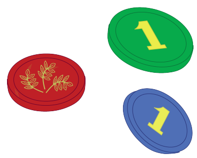
(a) Play this game with a partner. Assign one player to be Jack and the other to be Jill.

(b) A game is considered a *fair game* if each player is equally likely to win the game. Explain why this game is **not** a fair game.

(c) Explain how you could change the rules to make this a fair game.



Theme: Data Management



Problem of the Week
Problem B and Solution
Three Coins in a Toss

Problem

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(a) Play this game with a partner. Assign one player to be Jack and the other to be Jill.

(b) A game is considered a *fair game* if each player is equally likely to win the game. Explain why this game is **not** a fair game.

(c) Explain how you could change the rules to make this a fair game.

Solution

(a) After playing the game, check how many pairs had Jack win and how many had Jill win.

(b) First we determine the possible outcomes of each toss of the three coins. In total there are eight possible outcomes:

TTT, TTH, THT, HTT, THH, HTH, HHT, HHH

From here we see that Jack gets a point for only 2 out of the 8 outcomes, while Jill gets a point for 6 out of the 8 outcomes. Thus, the theoretical probability of Jack getting a point is $\frac{2}{8} = \frac{1}{4}$, while the theoretical probability of Jill getting a point is $\frac{6}{8} = \frac{3}{4}$. Since Jill is more likely to get a point on any toss, she is more likely to win the game. Thus, it is not a fair game.

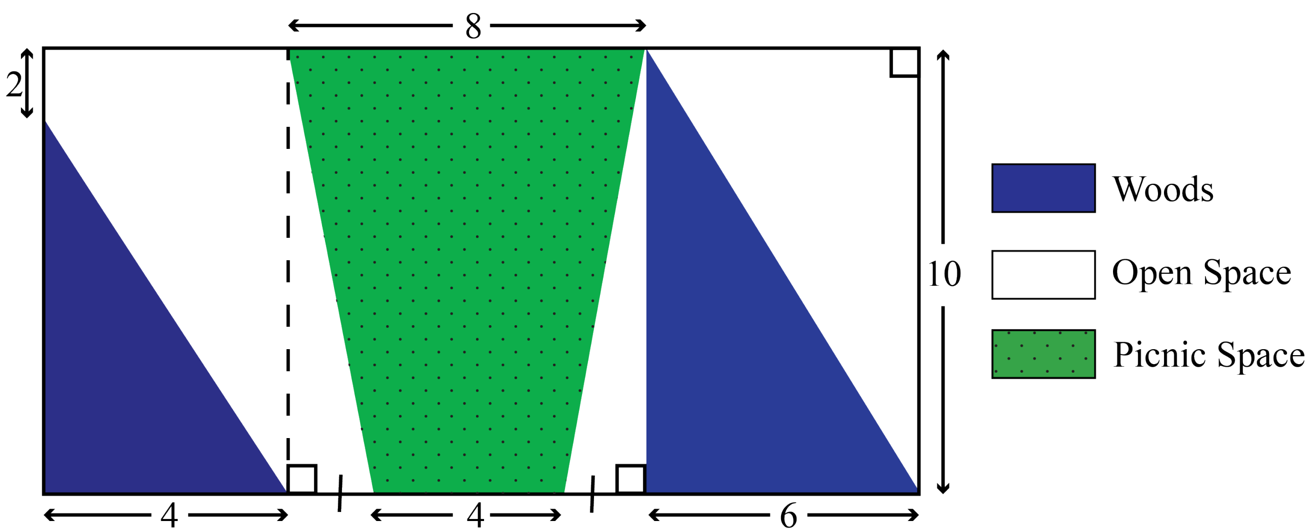
(c) There are many different ways to change the rules to make it a fair game. One way is to give Jack a point if there are at least 2 heads, and give Jill a point if there are at least 2 tails. Then Jack would get a point for four outcomes (HHH, THH, HTH, and HHT), and Jill would get a point for the remaining four outcomes (TTT, TTH, THT, and HTT). This would be a fair game because each player is equally likely to earn a point on any toss. Thus, each player is equally likely to win the game.

Geometry & Measurement (G)

Take me to the cover

Problem of the Week
Problem B
Welcome to Shape City Park

Shape City has built a new park. The park is in the shape of a rectangle, and is divided into woods, open space, and picnic space, as shown.



What is the total area of each type of space?

Themes: Geometry & Measurement, Number Sense

Problem of the Week
Problem B and Solution
Welcome to Shape City Park

Problem

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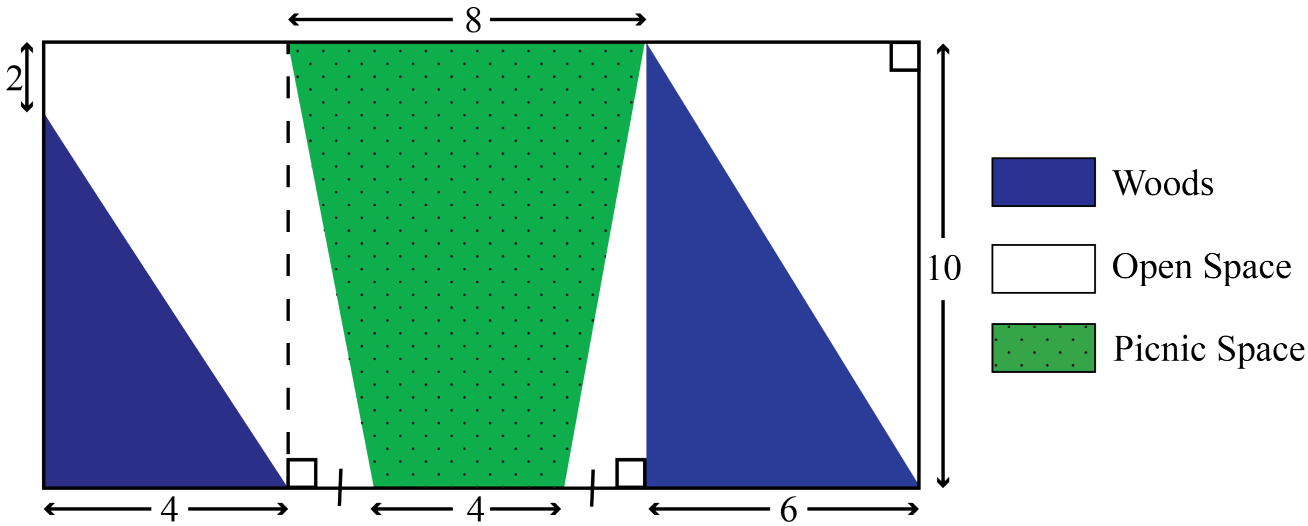

What is the total area of each type of space?

Solution

Both woods regions are right-angled triangles. One triangle has base 4 and height $10 - 2 = 8$, and thus has area equal to $\frac{1}{2}(4 \times 8) = 16$ square units. The other triangle has base 6 and height 10, and thus has area equal to $\frac{1}{2}(6 \times 10) = 30$ square units. Thus, the area of the woods is $16 + 30 = 46$ square units.

The picnic space is in the shape of a trapezoid with height 10 and parallel sides of length 8 and 4. Thus, the area of the picnic space is $\frac{1}{2}(8 + 4) \times 10 = 60$ square units.

Alternatively, the area of the trapezoidal picnic space can be determined by calculating the area of the rectangular strip containing the trapezoid, and subtracting the two triangular regions of open space. That is, the area of the picnic space is $8 \times 10 - \frac{1}{2}(2 \times 10) - \frac{1}{2}(2 \times 10) = 60$ square units.

The entire park has vertical length 10 and width equal to $4 + 8 + 6 = 18$. Hence the total area of the park is $10 \times 18 = 180$ square units.

The area of the open space is equal to the total area of the park minus the areas of the picnic space and woods, or $180 - 46 - 60 = 74$ square units.

Problem of the Week

Problem B

Frank Mixes Things Up

Frank wants to make a fruit punch drink for a family party. The recipe for his fruit punch drink is:

500 mL cranberry juice
1.8 L orange juice
1450 mL pineapple juice
350 mL ginger ale
2.4 L water

(a) Frank has three different-sized containers: a 6 L jug, a 5700 mL jug, and a 6.8 L jug. Which jug(s) could Frank use to hold his fruit punch?

(b) If the costs of his four flavoured drinks average to \$4.56 per litre, approximately how much will it cost Frank to make his fruit punch? Assume Frank does not need to pay for water.

(c) There will be 31 family members at the party and Frank wants to make enough fruit punch so that each person can drink one cup (250 mL). Currently his recipe will not make enough fruit punch for everyone. Adjust the amounts of each ingredient in the recipe so that Frank has enough of his fruit punch drink for everyone, but not too much extra.

Problem of the Week
Problem B and Solution
Frank Mixes Things Up

Problem

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Solution

(a) Since there are 1000 mL in one litre, we can convert all volumes in the recipe to litres. This gives 0.5 L of cranberry juice, 1.45 L of pineapple juice, and 0.35 L of ginger ale. Now we can find the total volume of the fruit punch by adding up the volumes of each of the five ingredients. This gives $0.5 + 1.8 + 1.45 + 0.35 + 2.4 = 6.5$ L.

In litres, the capacity of the jugs are 6 L, 5.7 L, and 6.8 L. Therefore, the only jug large enough to hold 6.5 L of fruit punch is the 6.8 L jug.

(b) First we find the total volume of the four flavoured drinks. We could add up the volumes of the four flavoured drinks, or simply subtract the volume of water from the total we found in part (a). This gives $6.5 - 2.4 = 4.1$ L. Since the costs of the four flavoured drinks average to \$4.56 per litre, and their total volume is 4.1 L, then it will cost Frank approximately $\$4.56 \times 4.1 \approx \$18.70$ to make the fruit punch.

(c) One recipe makes a total of 6.5 L or 6500 mL of fruit punch. In order to make enough for everybody, Frank should make $31 \times 250 = 7750$ mL. So his recipe is short by $7750 - 6500 = 1250$ mL.

We could just double the recipe, but since the difference is only 1250 mL, that would make much more fruit punch than Frank actually wants. Notice that $7750 \div 6500 \approx 1.1923$. So if we multiply each of the volumes in the recipe by 1.2, then there will be enough fruit punch, without having too much extra. This gives the following volumes, in mL:

- Cranberry juice: $500 \text{ mL} \times 1.2 = 600$ mL
- Orange juice: $1800 \text{ mL} \times 1.2 = 2160$ mL
- Pineapple juice: $1450 \text{ mL} \times 1.2 = 1740$ mL
- Ginger ale: $350 \text{ mL} \times 1.2 = 420$ mL
- Water: $2400 \text{ mL} \times 1.2 = 2880$ mL

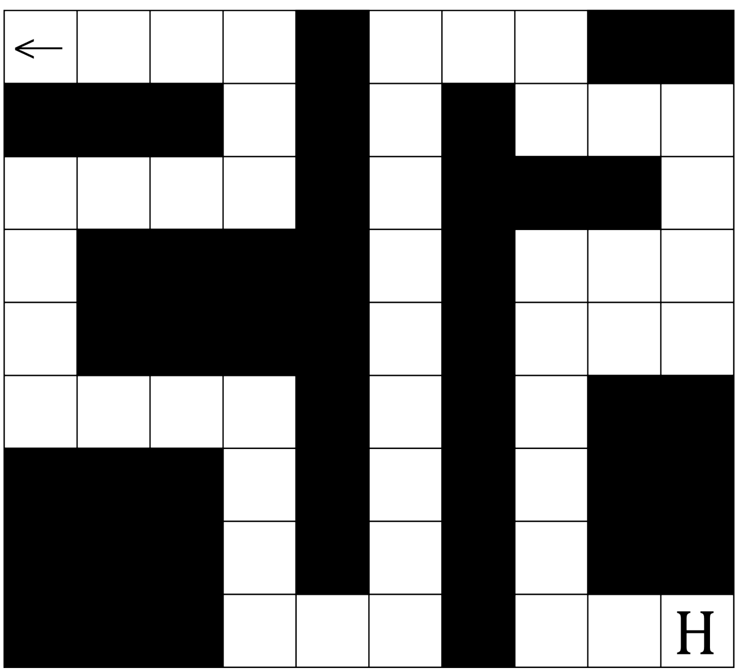
The total volume of the fruit punch will then be $600 + 2160 + 1740 + 420 + 2880 = 7800$ mL, which is enough for everyone, with only $7800 - 7750 = 50$ mL extra.

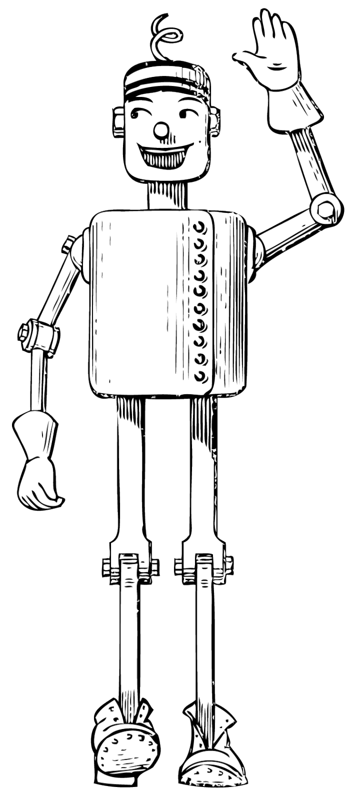
Problem of the Week
Problem B
Help Send Freddy Home

Freddy the Robot starts in the top left square of the maze below, facing left (symbolized by ←). Freddy wants to get to his Home Charging Station in the bottom right corner of the maze, marked with an **H**. Freddy can only turn right (clockwise on the diagram), and will only turn the number of degrees which you tell him. As well, he will go exactly the number of squares you tell him and in the direction that the arrow is currently pointing.

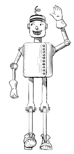
For example, if he starts facing ←, and your command said, "Turn 180 degrees and go 3 spaces", he would turn 180° right (clockwise) and then move 3 spaces forward (to the right).

Write a program of commands that gets Freddy to his Home Charging Station.





Themes: Computational Thinking, Geometry & Measurement



Problem of the Week

Problem B and Solution

Help Send Freddy Home

Problem

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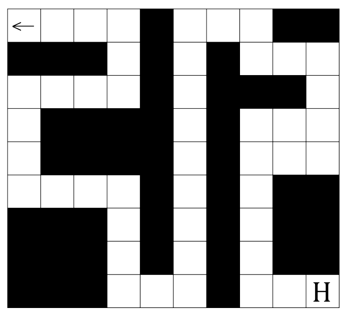


Solution

A program of commands is below. Note that there two options for commands 12 and 14. The second option is in parentheses.

1. Turn 180° and go 3 spaces.
2. Turn 90° and go 2 spaces.
3. Turn 90° and go 3 spaces.
4. Turn 270° and go 3 spaces.
5. Turn 270° and go 3 spaces.
6. Turn 90° and go 3 spaces.
7. Turn 270° and go 2 spaces.
8. Turn 270° and go 8 spaces.
9. Turn 90° and go 2 spaces.
10. Turn 90° and go 1 space.
11. Turn 270° and go 2 spaces.
12. Turn 90° and go 3 (or 2) spaces.
13. Turn 90° and go 2 spaces.
14. Turn 270° and go 4 (or 5) spaces.
15. Turn 270° and go 2 spaces.

Now Freddy is at his Home Charging Station!

Problem of the Week

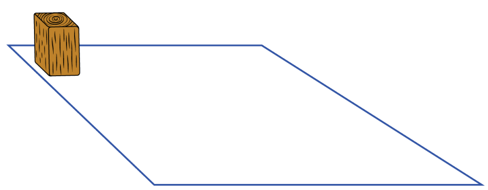
Problem B

Stamp it Out

Evelina is using a wooden block as a stamp to make decorative wrapping paper. The wooden block measures 3 cm by 3 cm by 4 cm and Evelina is stamping with one of the 3 cm by 3 cm square faces of the block.

(a) Evelina creates a row of 10 stamped squares with a gap of 0.5 cm in between each square. Determine the length of this row.

(b) Evelina has a square piece of wrapping paper with side length 77.5 cm. She leaves a gap of 0.5 cm around each edge and then creates rows of stamped squares with a gap of 0.5 cm in between each square as well as between each row. How many stamped squares in total will be on this piece of wrapping paper?



Themes: Algebra, Geometry & Measurement

Problem of the Week
Problem B and Solution
Stamp it Out

Problem

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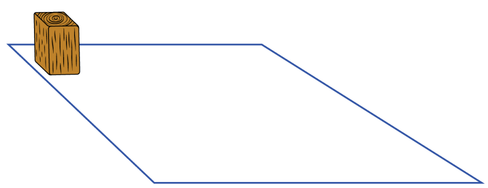
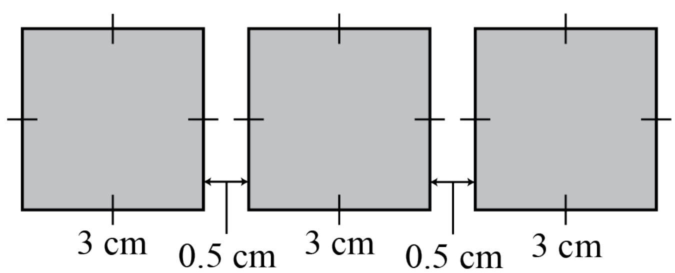


Solution

(a) We start by drawing a diagram of the first three stamped squares in the row.



From here we show two different ways to solve the problem.

Solution 1

In this solution we use a table to help us determine the length of the row. The first stamped square has a length of 3 cm, and each square after that adds $3 + 0.5 = 3.5$ cm to the length.

Number of Stamped Squares	Length of Row (cm)
1	3
2	$3 + 3.5 = 6.5$
3	$6.5 + 3.5 = 10$
4	$10 + 3.5 = 13.5$
5	$13.5 + 3.5 = 17$
6	$17 + 3.5 = 20.5$
7	$20.5 + 3.5 = 24$
8	$24 + 3.5 = 27.5$
9	$27.5 + 3.5 = 31$
10	$31 + 3.5 = 34.5$

Thus, the length of the row is 34.5 cm.

Solution 2

In this solution we use an algebraic expression help us determine the length of the row. This solution is more efficient, especially as the row gets longer. We know the length of each stamped square is 3 cm and the length of each gap between the squares is 0.5 cm. If we let s represent the total number of squares and g represent the total number of gaps between the squares, then the length of the row, in cm, is $3 \times s + 0.5 \times g$.

If there are 10 squares in the row then there are 9 gaps between them. Then we can use our equation to determine the length of the row.

$$\begin{aligned} 3 \times s + 0.5 \times g &= 3 \times 10 + 0.5 \times 9 \\ &= 30 + 4.5 \\ &= 34.5 \end{aligned}$$

Thus, the length of the row is 34.5 cm.

(b) We will start by determining the number of stamped squares in each row. Since the piece of wrapping paper is a square, this will also be the number of rows of stamped squares.

If we subtract the gap at the start of the row, then the length of the row will be $77.5 - 0.5 = 77$ cm. This is helpful because each square is followed by a gap, so the total length of the square and its gap is $3 + 0.5 = 3.5$ cm. Then we can divide to determine the number of squares in the row. Since $77 \div 3.5 = 22$, it follows that there are 22 squares in the row.

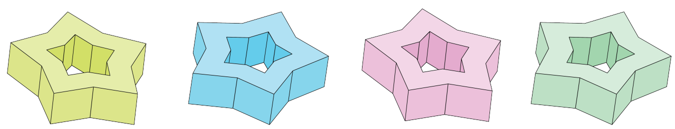
Since there are also 22 rows of squares, the total number of squares is $22 \times 22 = 484$.

Problem of the Week

Problem B

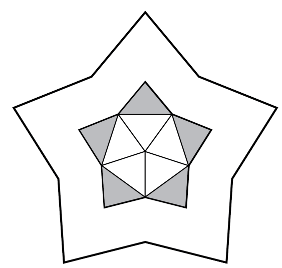
Icy Star Rings

Michel makes coloured ice in the shape of a star ring by pouring coloured water into a special mold and freezing it. Some of his ice stars are shown.

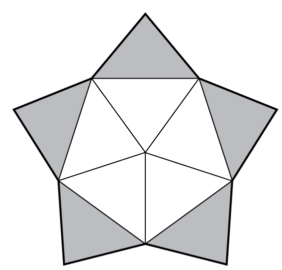


Michel's mold consists of an outer star and an inner star, and water is poured between the inner and outer star walls.

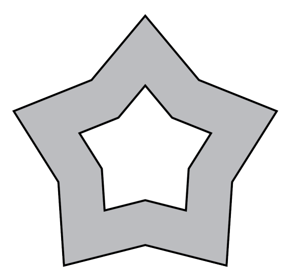
(a) The diagram shows the inner and outer stars from Michel's mold. The inner star is divided into five outer triangles, shown in grey, and five inner triangles, shown in white. The outer triangles each have a base of 3 cm and a height of 1.7 cm. The inner triangles each have a base of 3 cm and a height of 2 cm. Determine the area of the inner star.



(b) If we ignore the inner star, then we can divide the outer star into five outer triangles and five inner triangles, like we did with the inner star in part (a). The base and height of each of the triangles in the outer star are twice the base and height of their corresponding triangle in the inner star. Determine the area of the outer star.



(c) The shaded region represents the area between the inner and outer star walls. Use your calculations from parts (a) and (b) to determine the area of this region.



Theme: Geometry & Measurement

Problem of the Week
Problem B and Solution
Icy Star Rings

Problem

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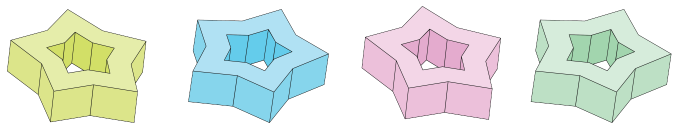

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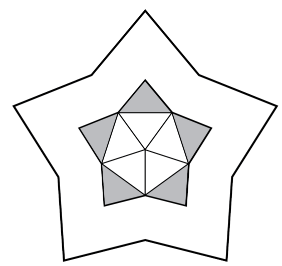

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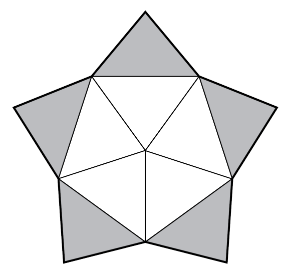

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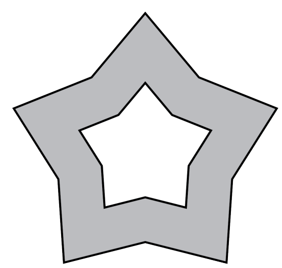

Solution

(a) The area of each outer triangle is $3 \times 1.7 \div 2 = 5.1 \div 2 = 2.55$ cm^2.
Since there are 5 outer triangles, their total area is $5 \times 2.55 = 12.75$ cm^2.
The area of each inner triangle is $3 \times 2 \div 2 = 3$ cm^2.
Since there are 5 inner triangles, their total area is $5 \times 3 = 15$ cm^2.
Therefore, the area of the inner star is $12.75 + 15 = 27.75$ cm^2.

(b) The base of each outer triangle is $2 \times 3 = 6$ cm and the height is $2 \times 1.7 = 3.4$ cm.
The area of each outer triangle is then $6 \times 3.4 \div 2 = 20.4 \div 2 = 10.2$ cm^2.
Since there are 5 outer triangles, their total area is $5 \times 10.2 = 51$ cm^2.
The base of each inner triangle is $2 \times 3 = 6$ cm and the height is $2 \times 2 = 4$ cm.
The area of each inner triangle is then $6 \times 4 \div 2 = 24 \div 2 = 12$ cm^2.
Since there are 5 inner triangles, their total area is $5 \times 12 = 60$ cm^2.
Therefore, the area of the outer star is $51 + 60 = 111$ cm^2.

Notice that this is 4 times the area of the inner star. Does it make sense why the area was multiplied by 4 when we doubled the dimensions?

(c) The area of the shaded region is the area of the outer star minus the area of the inner star. This is $111 - 27.75 = 83.25$ cm^2.

Problem of the Week

Problem B

Gregorian Calendars

The Gregorian Calendar is used in most parts of the world today. In order to keep this calendar in synch with the solar year (the time for the Earth to complete one orbit around the sun), it has leap years with an extra day in February. Leap years generally occur every four years, in years that are divisible by 4. However, years divisible by 100 are excluded, UNLESS they are divisible by 400. For example, the year 2000 was a leap year, but 1900 was not.

The year 2025 has a different calendar than the year 2024 since it started on a different day of the week. January 1, 2024 was on a Monday, while January 1, 2025 was on a Wednesday.

(a) January 1, 2023 was on a Sunday. Explain why January 1, 2024 was one day of the week later than in 2023, while January 1, 2025 was two days of the week later than in 2024.

(b) January 1, 1992 and January 1, 2025 were both on a Wednesday. Did 1992 have the same yearly calendar as 2025?

(c) How many different yearly calendars are there in total? Two yearly calendars are considered the same if each date occurred on the same day of the week.

Theme: Geometry & Measurement

Problem of the Week
Problem B and Solution
Gregorian Calendars

Problem

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(c) How many different yearly calendars are there in total? Two yearly calendars are considered the same if each date occurred on the same day of the week.

Solution

(a) Since the year 2023 had 365 days, and a week has 7 days, there were $365 \div 7 = 52\frac{1}{7}$ weeks in 2023. That is, there were 52 full weeks plus 1 day. Thus, all of the next year's dates fall one day of the week later than the previous year. This is why January 1, 2024 was one day of the week later than January 1, 2023.

However, 2024 was a leap year with 366 days, so it had 52 full weeks plus 2 days. Thus, January 1, 2025 was two days later in the week (a Wednesday) than January 1, 2024 (a Monday).

(b) Despite the fact that January 1 in 1992 and in 2025 were both on a Wednesday, the years won't have the same yearly calendar because 1992 was a leap year. This is since 1992 is divisible by 4, but not 100.

(c) There are 7 possible weekdays on which January 1 can occur. For each of these possibilities, there is a calendar that includes February 29 and a calendar that does not. Thus, there are $7 \times 2 = 14$ different yearly calendars.

Problem of the Week

Problem B

Coding Figures

Ayanna writes programs using the following commands that direct her robot to draw shapes.

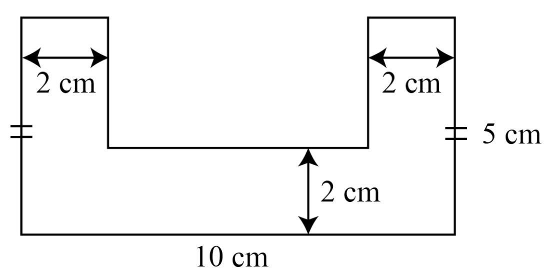
Command	Explanation
`Pencil down`	Puts the pencil in position to draw as the robot moves
`Move`	Moves the robot a given distance forward
`Turn CW`	Rotates the robot one quarter turn (90°) clockwise
`Turn CCW`	Rotates the robot one quarter turn (90°) counter clockwise

(a) For each program, follow the code to draw the shape. You may find grid paper helpful.

(i)
```
Pencil down
Repeat 4 times
  Move 5 cm
  Turn CW
End
```

(ii)
```
Pencil down
Move 4 cm
Turn CCW
Move 2 cm
Turn CW
Move 2 cm
Turn CW
Move 6 cm
Turn CW
Move 6 cm
Turn CW
Move 4 cm
End
```

(b) Write a program that would direct Ayanna's robot to draw the given rectangular U-shape.



Themes: Computational Thinking, Geometry & Measurement

Problem of the Week
Problem B and Solution
Coding Figures

Problem

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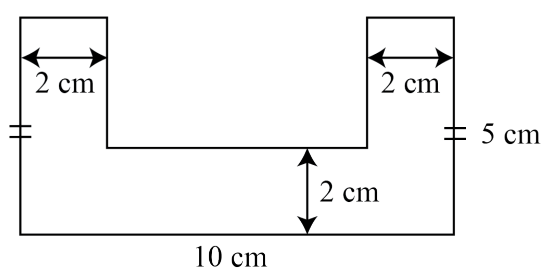
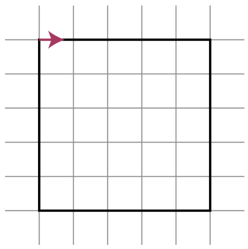


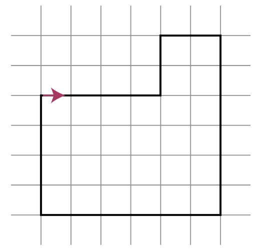
Solution

(a) For each program, the shape is shown on 1 cm grid paper with an arrow indicating the initial direction of the robot. Note that for part (i), the shape will look the same regardless of the initial direction. However for part (ii), the shape will be rotated depending on the initial direction.

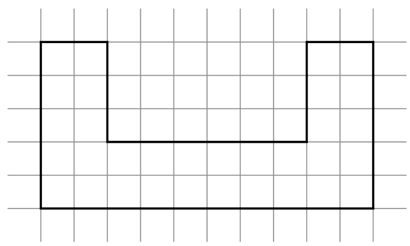
(i)



(ii)



(b) We start by determining the length of each side of the shape. We do this by drawing the shape on grid paper, as shown.



There are several different programs that can be written to direct Ayanna's robot to draw the given shape, depending on the robot's starting position and direction. In the following program, the robot starts in the bottom-left corner, facing to the right.

```
Pencil down
Move 10 cm
Turn CCW
Move 5 cm
Turn CCW
Move 2 cm
Turn CCW
Move 3 cm
Turn CW
Move 6 cm
Turn CW
Move 3 cm
Turn CCW
Move 2 cm
Turn CCW
Move 5 cm
End
```

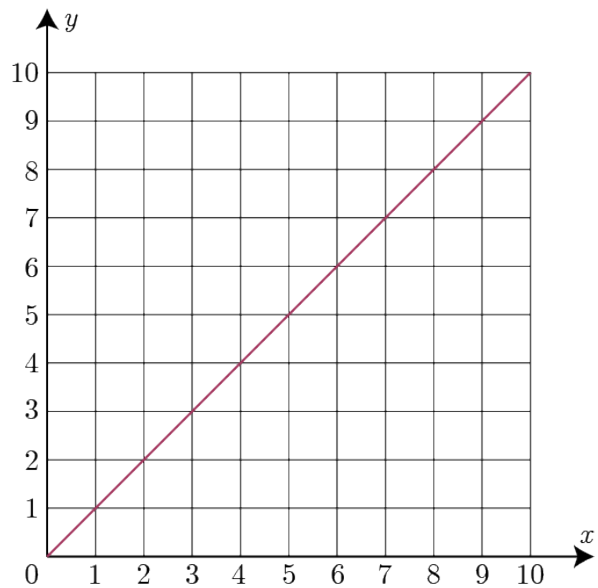
Problem of the Week

Problem B

A Capital Transformation

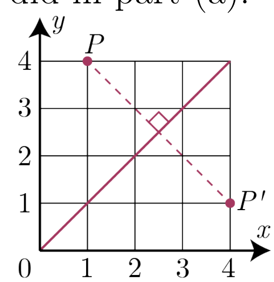
(a) Plot the given points on the grid, then connect the points in the order they appear in the table. Finish by connecting the last point with the first.

x	y
1	10
6	10
6	9
4	9
4	5
3	5
3	9
1	9



(b) Reflect each of the points you plotted in part (a) across the diagonal line in the grid above, then connect the reflected points like you did in part (a).

Tip: When you reflect a point across a line, the reflected point lies on the opposite side of the line, the same perpendicular distance away as the original point. This is shown in the diagram where P is the original point and P' is its reflection across the diagonal line.



(c) How do the coordinates of each reflected point from part (b) compare with the coordinates of their original point?

(d) Starting with the original points, can you use 2 or 3 different transformations in a row to get the same final image you drew in part (b)? If so, explain the transformations you used.

Theme: Geometry & Measurement

Problem of the Week

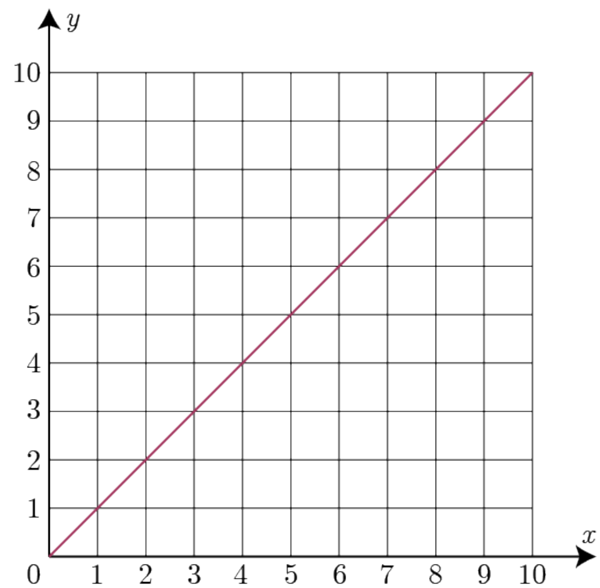
Problem B and Solution

A Capital Transformation

Problem

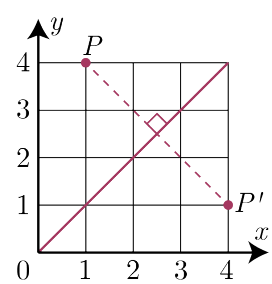
(a) Plot the given points on the grid, then connect the points in the order they appear in the table. Finish by connecting the last point with the first.

x	y
1	10
6	10
6	9
4	9
4	5
3	5
3	9
1	9



(b) Reflect each of the points you plotted in part (a) across the diagonal line in the grid above, then connect the reflected points like you did in part (a).

Tip: When you reflect a point across a line, the reflected point lies on the opposite side of the line, the same perpendicular distance away as the original point. This is shown in the diagram where P is the original point and P' is its reflection across the diagonal line.

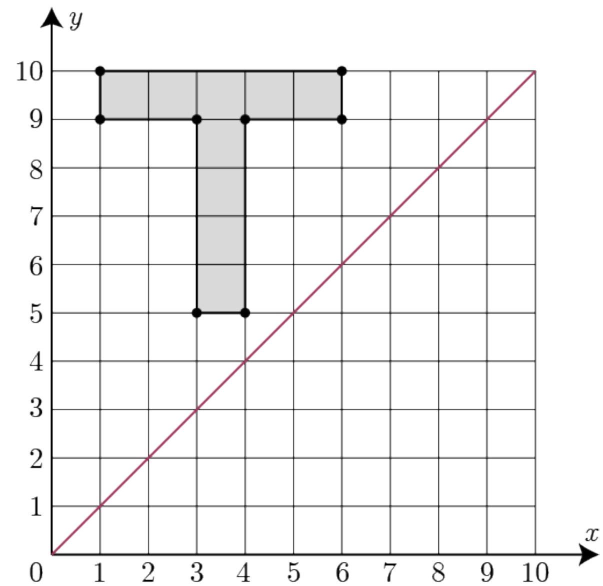


(c) How do the coordinates of each reflected point from part (b) compare with the coordinates of their original point?

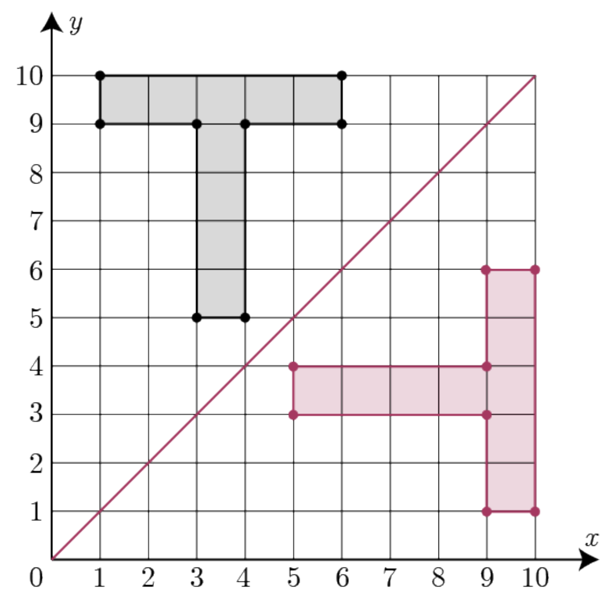
(d) Starting with the original points, can you use 2 or 3 different transformations in a row to get the same final image you drew in part (b)? If so, explain the transformations you used.

Solution

(a) The plotted points are shown on the grid. When joined in order, they form the letter T.



(b) The reflected points are shown on the grid. When joined in order, they form a sideways letter T.



(c) The coordinates for each point are reversed in its reflected point. For example, the reflection of the point $(1, 10)$ is $(10, 1)$.

(d) There are several different ways to get the same final image using multiple transformations. One way is as follows:

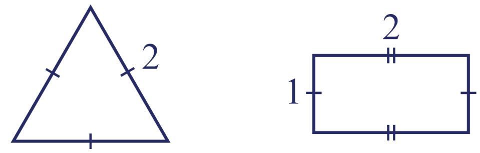
Step 1: Rotate each point 90° clockwise about the point $(3, 5)$, which is the bottom-left corner of the T.

Step 2: Translate each point 2 units to the right.

Step 3: Translate each point 1 unit down.

Problem of the Week
Problem B
The Long and the Short of It

Geordie drew an equilateral triangle with side length 2 and a rectangle with length 2 and width 1. He then noticed that each shape had a perimeter of 6.



(a) Geordie drew an equilateral triangle with side length 3. Can Geordie draw a rectangle that has the same perimeter as the equilateral triangle, if the side lengths of the rectangle must be whole numbers? Explain.

(b) Geordie drew equilateral triangles with side lengths 4, 5, 6, and 7. For which of these can he draw a rectangle with the same perimeter, if the side lengths of the rectangle must be whole numbers? Explain.

(c) If the side lengths of an equilateral triangle and a rectangle are all whole numbers, then:

(i) What numbers could be the perimeter of the triangle?

(ii) What numbers could be the perimeter of the rectangle?

(iii) What numbers could be the perimeter of both the triangle and the rectangle?

(iv) If the triangle and the rectangle have the same perimeter, what must be true about the side length of the triangle?

Themes: Geometry & Measurement, Number Sense

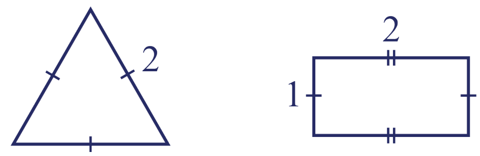
Problem of the Week

Problem B and Solution

The Long and the Short of It

Problem

Geordie drew an equilateral triangle with side length 2 and a rectangle with length 2 and width 1. He then noticed that each shape had a perimeter of 6.



(a) Geordie drew an equilateral triangle with side length 3. Can Geordie draw a rectangle that has the same perimeter as the equilateral triangle, if the side lengths of the rectangle must be whole numbers? Explain.

(b) Geordie drew equilateral triangles with side lengths 4, 5, 6, and 7. For which of these can he draw a rectangle with the same perimeter, if the side lengths of the rectangle must be whole numbers? Explain.

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(i) What numbers could be the perimeter of the triangle?

(ii) What numbers could be the perimeter of the rectangle?

(iii) What numbers could be the perimeter of both the triangle and the rectangle?

(iv) If the triangle and the rectangle have the same perimeter, what must be true about the side length of the triangle?

Solution

(a) If the side length of the equilateral triangle is 3, then its perimeter is $3 \times 3 = 9$. We will try different lengths and widths for the rectangle to see if we can find ones that give us a perimeter of 9. We will use the formula for the perimeter of a rectangle, which is $P = 2l + 2w$.

- Suppose the rectangle has length 1. Then $9 = 2 \times 1 + 2w$, so $9 = 2 + 2w$. Then $7 = 2w$, so $w = 7 \div 2 = 3.5$. However this is not a whole number, so this is not a possibility.
- Suppose the rectangle has length 2. Then $9 = 2 \times 2 + 2w$, so $9 = 4 + 2w$. Then $5 = 2w$, so $w = 5 \div 2 = 2.5$. However this is not a whole number, so this is not a possibility.
- Suppose the rectangle has length 3. Then $9 = 2 \times 3 + 2w$, so $9 = 6 + 2w$. Then $3 = 2w$, so $w = 3 \div 2 = 1.5$. However this is not a whole number, so this is not a possibility.
- Suppose the rectangle has length 4. Then $9 = 2 \times 4 + 2w$, so $9 = 8 + 2w$. Then $1 = 2w$, so $w = 1 \div 2 = 0.5$. However this is not a whole number, so this is not a possibility.
- Suppose the rectangle has length 5. Then since $5 + 5 = 10$ and $10 > 9$, it is not possible for this rectangle to have a perimeter of 9.

Therefore, Geordie cannot draw a rectangle with a perimeter of 9 if its side lengths must be whole numbers.

In general, if the length and width of a rectangle are whole numbers, then $2l$ and $2w$ will always be even numbers because any whole number multiplied by 2 results in an even number. So $2l + 2w$ will also be an even number, since an even number plus an even number results in an even number. Since $P = 2l + 2w$, it follow that P also must be an even number. This explains why we cannot draw a rectangle with a perimeter of 9, since 9 is an odd number.

(b) Using our results from part (a), we know that if the side lengths of a rectangle are whole numbers, then its perimeter must be even. In other words, if the perimeter is not even, then the side lengths of the rectangle cannot all be whole numbers.

- An equilateral triangle with side length 4 has perimeter $4 \times 3 = 12$, which is even. Geordie can draw a rectangle with perimeter 12; one example is a rectangle with length 5 and width 1.

- An equilateral triangle with side length 5 has perimeter $5 \times 3 = 15$. Since this is odd, Geordie cannot draw a rectangle with perimeter 15.
- An equilateral triangle with side length 6 has perimeter $6 \times 3 = 18$, which is even. Geordie can draw a rectangle with perimeter 18; one example is a rectangle with length 6 and width 3.
- An equilateral triangle with side length 7 has perimeter $7 \times 3 = 21$. Since this is odd, Geordie cannot draw a rectangle with perimeter 21.

(c) Based on our results from parts (a) and (b), we can conclude the following:

(i) The perimeter of the equilateral triangle must be a multiple of 3, since the perimeter is equal to 3 times its side length.

(ii) The perimeter of the rectangle must be an even number.

(iii) If a number is the perimeter of both the equilateral triangle and the rectangle, then it must be both a multiple of 3 and even. This is equivalent to saying it must be a multiple of 6.

(iv) If the equilateral triangle and the rectangle have the same perimeter, then this perimeter must be even. Therefore, the side length of the triangle must be even.

Problem of the Week

Problem B

Doggy Drinking

When a dog takes a drink, he laps up water by pushing his tongue into the water and then pulling it back with some water sticking to it. Some dogs are better at this than others. Chase is a messy drinker, and usually leaves a large puddle on the floor.

(a) Chase has a mass of 40 kg. Each day he needs to drink 50 mL of water for each kg of body mass. How many litres of water should Chase drink per day?

(b) If 70% of Chase's body mass is made up of water, how much of Chase's mass is NOT water?

(c) For every 3.5 mL of water Chase laps up with his tongue, 1.5 mL ends up on the floor. If Chase takes a lap every 5 seconds for one minute, how much water does he successfully drink?

(d) After an hour, you measure the water on the floor and find that Chase has spilled 45 mL. How much water did he successfully drink?

(e) How many laps per day will it take Chase to drink the water he needs in one day?

Themes: Geometry & Measurement, Number Sense

Problem of the Week
Problem B and Solution
Doggy Drinking

Problem

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(e) How many laps per day will it take Chase to drink the water he needs in one day?

Solution

(a) Chase should drink $50 \times 40 = 2000$ mL, or 2 L of water per day.

(b) The percentage of Chase's body mass that is NOT water is $100\% - 70\% = 30\%$. Thus, the amount of Chase's mass that is NOT water is 30% of 40 kg, or $0.3 \times 40 = 12$ kg.

(c) Since there are 60 seconds in one minute, Chase takes $60 \div 5 = 12$ laps in total. From each lap, he drinks only $3.5 - 1.5 = 2.0$ mL. Thus, in 12 laps he successfully drinks a total of $12 \times 2.0 = 24$ mL of water.

(d) The 45 mL of water Chase spilled indicates he took $45 \div 1.5 = 30$ laps. Since he drinks 2 mL with each lap, he successfully drank $2 \times 30 = 60$ mL.

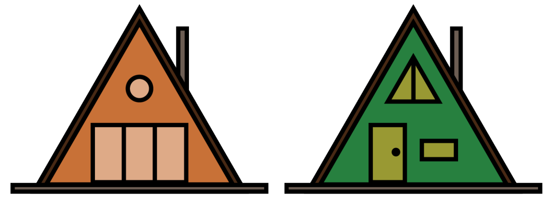
(e) From part (a), Chase needs to drink 2000 mL of water per day. He drinks 2 mL with each lap, so he will need to take $2000 \div 2 = 1000$ laps each day.

Problem of the Week

Problem B

What's Arkie's Angle?

Arkie Tekt has been designing A-frame houses for years. All his designs are in the shape of equilateral triangles with a horizontal base and two sides that meet at the top.



Arkie Tekt wants to create a more unique design, and has proposed three different A-frame designs.

Sketch each design using a ruler and protractor, with a scale of 1 cm = 1 m. Then measure any unknown interior angles (to the nearest degree) and side lengths (to the nearest millimetre or $\frac{1}{10}$ cm), and record these measurements on your diagram. Note that a *base angle* means an interior angle between the floor and the wall.

1. **Scalene Design:** The floor is 10 m wide, one base angle is 105°, and one of the walls has length equal to 5 m.
2. **Right-Angled Design:** The left wall is perpendicular to the floor, and both the floor and the left wall are 5 m in length.
3. **Isosceles Design:** The floor is 8 m in length, the perimeter of the design is 22 m, and one of the base angles is the same as the interior angle at the roof peak.

Extension: Arkie's partner likes the base angle given in the scalene design, but would prefer that the side walls be of the same length. Do you think this would work? Explain why or why not.

Theme: Geometry & Measurement

Problem of the Week

Problem B and Solution

What's Arkie's Angle?

Problem

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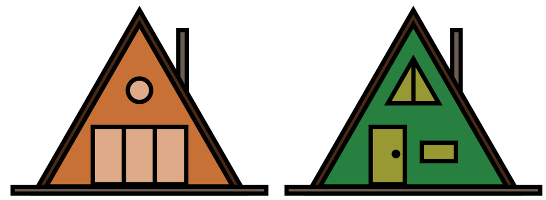

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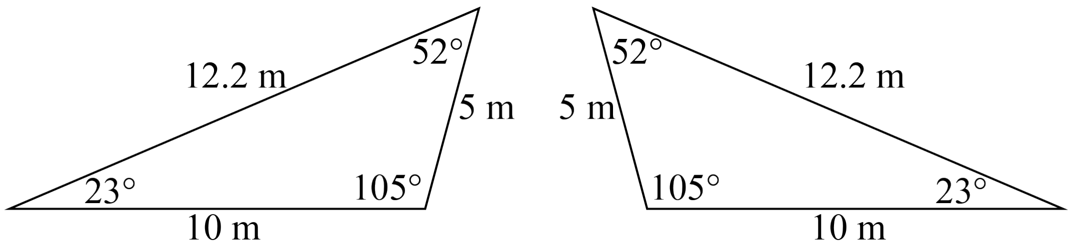
3. **Isosceles Design:** The floor is 8 m in length, the perimeter of the design is 22 m, and one of the base angles is the same as the interior angle at the roof peak.

Extension: Arkie's partner likes the base angle given in the scalene design, but would prefer that the side walls be of the same length. Do you think this would work? Explain why or why not.

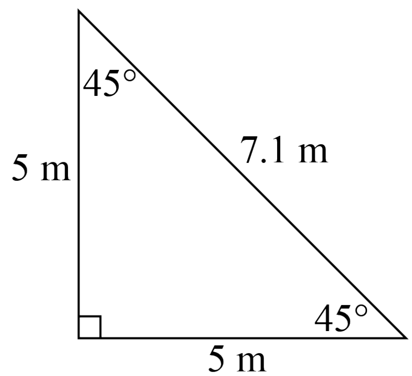
Solution

A sketch of each design is shown, however they are not drawn to scale. Note that there are two possibilities for the scalene and the isosceles designs.

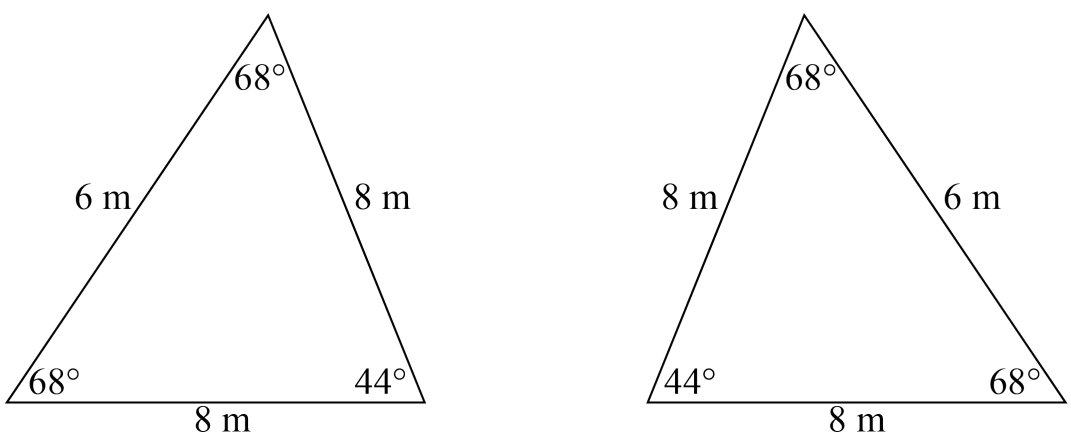
1. **Scalene Design:**



2. **Right-Angled Design:**



3. **Isosceles Design:**



SOLUTION TO EXTENSION:

If the side walls were the same length, then the triangle would be isosceles with two equal base angles. The given base angle is 105°. However $105^\circ + 105^\circ = 210^\circ$, and $210^\circ > 180^\circ$ which is the sum of the angles in triangle. Thus, this design would not work.

Number Sense (N)

Take me to the cover

Problem of the Week

Problem B

Loose Change

Macklin has ten coins in his pocket worth a total of \$2.30. Only one of Macklin's coins is a nickel (worth \$0.05), and the other nine are some combination of dimes (worth \$0.10), quarters (worth \$0.25), and loonies (worth \$1.00), with at least one of each type of coin.

(a) How many dimes does Macklin have?

(b) Is it possible that Macklin could have two nickels, not one? Explain your reasoning.

Problem of the Week

Problem B and Solution

Loose Change

Problem

Macklin has ten coins in his pocket worth a total of \$2.30. Only one of Macklin's coins is a nickel (worth \$0.05), and the other nine are some combination of dimes (worth \$0.10), quarters (worth \$0.25), and loonies (worth \$1.00), with at least one of each type of coin.

(a) How many dimes does Macklin have?

(b) Is it possible that Macklin could have two nickels, not one? Explain your reasoning.

Solution

(a) We know that Macklin has at least one of each type of coin, so we will start by taking one of each type of coin and recording their total value in the following table.

	Nickels	Dimes	Quarters	Loonies	Total
Number of Coins	1	1	1	1	4
Value (\$)	0.05	0.10	0.25	1.00	1.40

This gives a total value of \$1.40, so we have $\$2.30 - \$1.40 = \$0.90$ remaining. Since \$0.90 is less than the value of a loonie, Macklin cannot have any more loonies. Also, since only one of Macklin's coins is a nickel, Macklin cannot have any more nickels. Thus, we need to find a way to make \$0.90 using only quarters and/or dimes. The only way to do so is by using no quarters and 9 dimes, or by using 2 quarters and 4 dimes, since
$(2 \times \$0.25) + (4 \times \$0.10) = \$0.90$.

Using 9 dimes uses a total of $4 + 9 = 13$ coins. Since Macklin has 10 coins, this will not work.

Using 2 quarters and 4 dimes uses a total of $4 + 2 + 4 = 10$ coins, as desired. We will add these coins to our table.

	Nickels	Dimes	Quarters	Loonies	Total
Number of Coins	1	5	3	1	10
Value (\$)	0.05	0.50	0.75	1.00	2.30

Thus, Macklin has 5 dimes in total.

(b) If two of Macklin's coins were nickels, then we can proceed like in part (a), but include an extra nickel.

	Nickels	**Dimes**	**Quarters**	**Loonies**	**Total**
Number of Coins	2	1	1	1	5
Value ($)	0.10	0.10	0.25	1.00	1.45

This gives a total value of \$1.45, so we have \$2.30 − \$1.45 = \$0.85 remaining. As before, we can make this amount using only quarters and/or dimes.

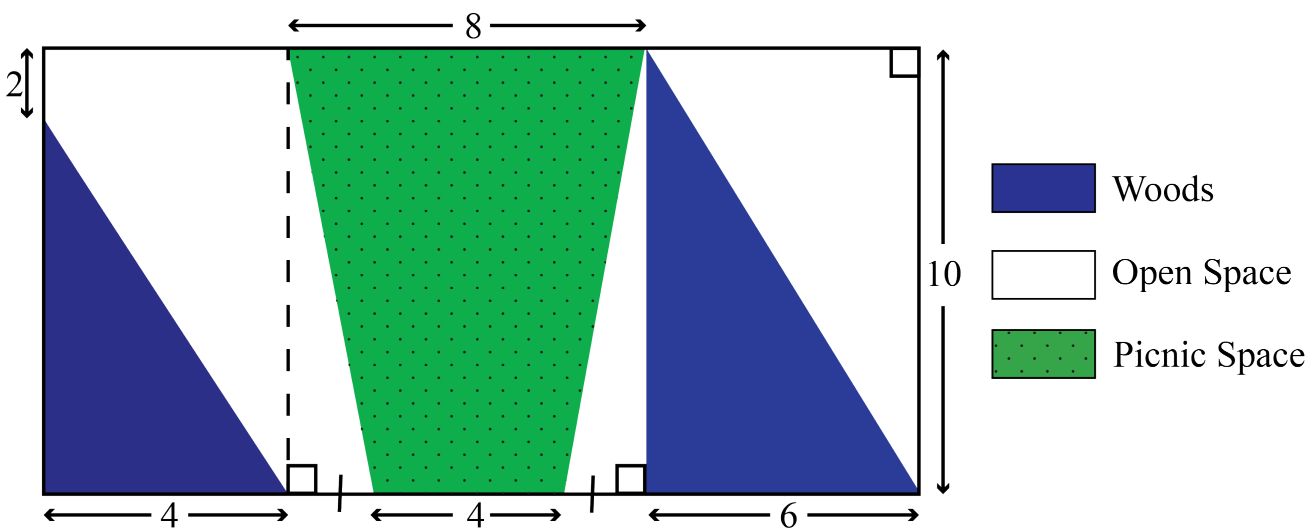
Using 3 quarters and 1 dime we can make \$0.85, however this uses a total of $5 + 3 + 1 = 9$ coins, which is not enough.

Using 1 quarter and 6 dimes we can make \$0.85, however this uses a total of $5 + 1 + 6 = 12$ coins, which is too many.

There are no other ways to make \$0.85 using only quarters and/or dimes. Thus, Macklin could not have had two nickels.

Problem of the Week
Problem B
Welcome to Shape City Park

Shape City has built a new park. The park is in the shape of a rectangle, and is divided into woods, open space, and picnic space, as shown.



What is the total area of each type of space?

Themes: Geometry & Measurement, Number Sense

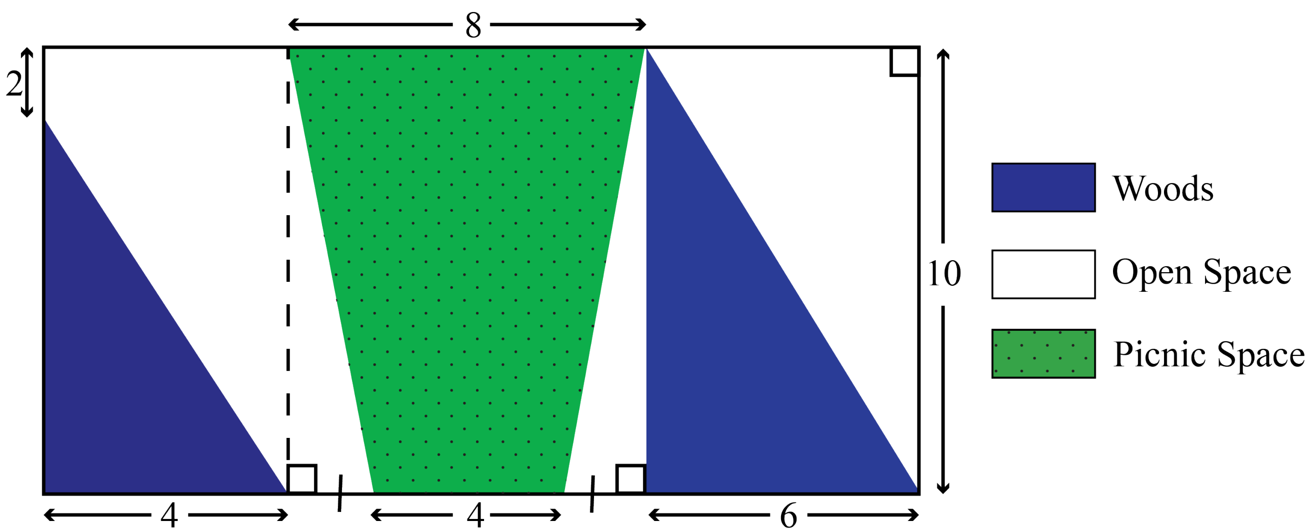
Problem of the Week

Problem B and Solution

Welcome to Shape City Park

Problem

Shape City has built a new park. The park is in the shape of a rectangle, and is divided into woods, open space, and picnic space, as shown.



What is the total area of each type of space?

Solution

Both woods regions are right-angled triangles. One triangle has base 4 and height $10 - 2 = 8$, and thus has area equal to $\frac{1}{2}(4 \times 8) = 16$ square units. The other triangle has base 6 and height 10, and thus has area equal to $\frac{1}{2}(6 \times 10) = 30$ square units. Thus, the area of the woods is $16 + 30 = 46$ square units.

The picnic space is in the shape of a trapezoid with height 10 and parallel sides of length 8 and 4. Thus, the area of the picnic space is $\frac{1}{2}(8 + 4) \times 10 = 60$ square units.

Alternatively, the area of the trapezoidal picnic space can be determined by calculating the area of the rectangular strip containing the trapezoid, and subtracting the two triangular regions of open space. That is, the area of the picnic space is $8 \times 10 - \frac{1}{2}(2 \times 10) - \frac{1}{2}(2 \times 10) = 60$ square units.

The entire park has vertical length 10 and width equal to $4 + 8 + 6 = 18$. Hence the total area of the park is $10 \times 18 = 180$ square units.

The area of the open space is equal to the total area of the park minus the areas of the picnic space and woods, or $180 - 46 - 60 = 74$ square units.

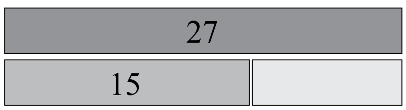
Problem of the Week

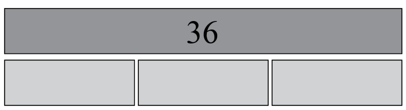
Problem B

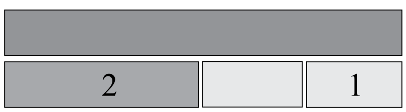
Composed or Decomposed

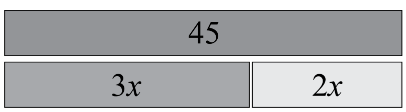
A bar model contains two rows of rectangles. The top row is a rectangle with a value equal to the total, and the bottom row contains rectangles whose values sum to the total. All values are whole numbers.

(a) Determine the numbers missing from each of the bar models below. In each part, rectangles that are the same shade of grey have the same value.

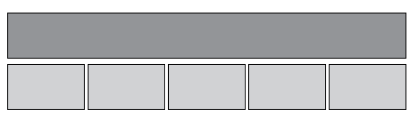
(i)


(ii)


(iii)


(iv)


(b) (i) In the bar model shown, the rectangles in the bottom row all have the same value. What are some equations this bar model could represent?



(ii) Describe all possible whole numbers that could appear in the top row of this bar model.

Theme: Number Sense

Problem of the Week
Problem B and Solution
Composed or Decomposed

Problem

A bar model contains two rows of rectangles. The top row is a rectangle with a value equal to the total, and the bottom row contains rectangles whose values sum to the total. All values are whole numbers.

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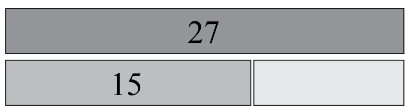
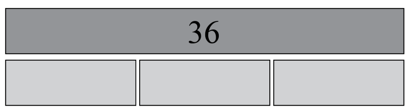
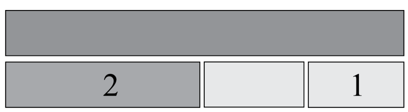
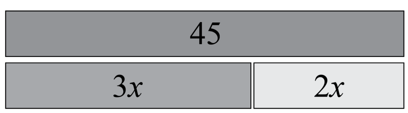


(ii)


(iii)


(iv)


(b) (i) In the bar model shown, the rectangles in the bottom row all have the same value. What are some equations this bar model could represent?

(ii) Describe all possible whole numbers that could appear in the top row of this bar model.

Solution

(a) (i) Since $27 = 15 + 12$, the missing number is 12.

(ii) Since $36 \div 3 = 12$, the missing number is 12.

(iii) Since rectangles that are the same shade of grey have the same value, the rectangle in the second row with unknown value has value 1. Thus, the missing number in the top row is $2 + 1 + 1 = 4$.

(iv) From the bar model, we know $45 = 3x + 2x = 5x$.

Therefore, 45 is divided into 5 equal parts. Since $45 \div 5 = 9$, we know $x = 9$. So the two numbers in the bottom row are $3 \times 9 = 27$ and $2 \times 9 = 18$.

(b) (i) Answers will vary. Some equations that the bar model could represent include

$$\begin{aligned} 5 \div 5 &= \mathbf{1} \\ 10 \div 5 &= \mathbf{2} \\ 15 \div 5 &= \mathbf{3} \\ 20 \div 5 &= \mathbf{4} \\ \mathbf{3} \times 5 &= 15 \\ \mathbf{7} \times 5 &= 35 \end{aligned}$$

In each equation, the number in boldface would appear in each of the five rectangles in the bottom row.

(ii) The whole numbers that could appear in the top row are the positive multiples of 5. That is, 5, 10, 15, 20, 25, and so on.

Problem of the Week

Problem B

Let's Get These Numbers in Order

(a) List the following twelve numbers from least to greatest value.

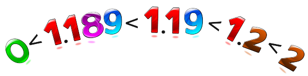
$$2.04,\ \ 1.02,\ \ \tfrac{85}{100},\ \ \tfrac{175}{100},\ \ \tfrac{7}{4},\ \ 1\tfrac{1}{8},\ \ 0.91,\ \ \tfrac{46}{20},\ \ 0.091,\ \ \tfrac{14}{25},\ \ \tfrac{3}{5},\ \ 1.1$$

Plot these numbers on a number line.

(b) Determine the largest gap in between two adjacent numbers on the number line.

(c) For your list in part (a), determine a number between the two numbers in the middle of the list, with your number being closer to the larger of the two.

Theme: Number Sense



Problem of the Week
Problem B and Solution
Let's Get These Numbers in Order

Problem

(a) List the following twelve numbers from least to greatest value.

$$2.04,\ \ 1.02,\ \ \tfrac{85}{100},\ \ \tfrac{175}{100},\ \ \tfrac{7}{4},\ \ 1\tfrac{1}{8},\ \ 0.91,\ \ \tfrac{46}{20},\ \ 0.091,\ \ \tfrac{14}{25},\ \ \tfrac{3}{5},\ \ 1.1$$

Plot these numbers on a number line.

(b) Determine the largest gap in between two adjacent numbers on the number line.

(c) For your list in part (a), determine a number between the two numbers in the middle of the list, with your number being closer to the larger of the two.

Solution

(a) The decimals equivalent to the fractional numbers are $\frac{85}{100} = 0.85$, $\frac{175}{100} = 1.75$, $\frac{7}{4} = 1.75$, $1\frac{1}{8} = 1.125$, $\frac{46}{20} = 2.3$, $\frac{14}{25} = 0.56$, and $\frac{3}{5} = 0.6$.

Notice that $\frac{175}{100}$ and $\frac{7}{4}$ have the same value.

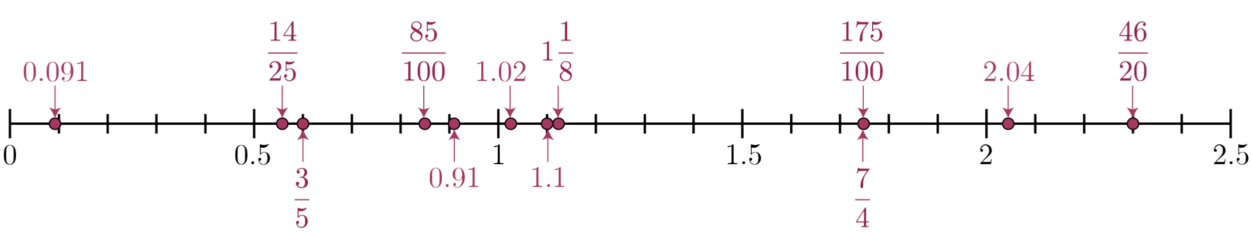
Listed from least to greatest value, the decimals are

$$0.091,\ 0.56,\ 0.60,\ 0.85,\ 0.91,\ 1.02,\ 1.1,\ 1.125,\ 1.75 \text{ and } 1.75,\ 2.04,\ 2.3$$

In their original format, the numbers, listed from least to greatest value, are

$$0.091,\ \tfrac{14}{25},\ \tfrac{3}{5},\ \tfrac{85}{100},\ 0.91,\ 1.02,\ 1.1,\ 1\tfrac{1}{8},\ \tfrac{175}{100} \text{ and } \tfrac{7}{4} \text{ (in either order)},\ 2.04,\ \tfrac{46}{20}$$

We plot these twelve numbers on a number line from 0 to 2.5.



(b) The largest gap lies between 1.125 and 1.75, which is 0.625 or $\frac{5}{8}$.

(c) The two middle values are 1.02 and 1.1. A value between these two numbers but closer to 1.1 would also have to be greater than 1.06. One such number is 1.08, but any number with value between 1.06 and 1.1 would satisfy the condition.

Problem of the Week

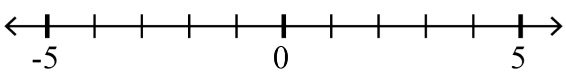
Problem B

Fun and Games

Mariana's trivia team is playing a game against other teams. The game includes 10 questions, and each team is given 5 points to start. Teams then earn 4 points for each correct answer, and lose 3 points for each incorrect answer.

(a) The table shows the score card for Mariana's team. (The trivia questions were tough!) In the right column, fill in their *running score*, which is their current score after each question.

Note that their running score may be positive or negative, like a temperature reading. You may find the following number line helpful.



Question	Result	Running Score
–	–	5
1	X	
2	X	
3	X	
4	✓	
5	X	
6	✓	
7	X	
8	✓	
9	X	
10	✓	

(b) The winning team had a score of 10. How many questions did they get correct?

Extension: What is the maximum possible score? What is the minimum possible score? Are all the integers in between these values possible scores? Explain.

Theme: Number Sense

Problem of the Week

Problem B and Solution

Fun and Games

Problem

Mariana's trivia team is playing a game against other teams. The game includes 10 questions, and each team is given 5 points to start. Teams then earn 4 points for each correct answer, and lose 3 points for each incorrect answer.

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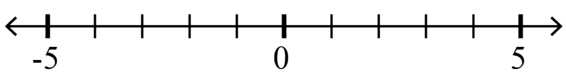

Question	Result	Running Score
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7	X	
8	✓	
9	X	
10	✓	

(b) The winning team had a score of 10. How many questions did they get correct?

Extension: What is the maximum possible score? What is the minimum possible score? Are all the integers in between these values possible scores? Explain.

Solution

(a) The running score is shown in the following table.

Question	Result	Running Score
–	–	5
1	X	$5 - 3 = 2$
2	X	$2 - 3 = -1$
3	X	$-1 - 3 = -4$
4	✓	$-4 + 4 = 0$
5	X	$0 - 3 = -3$
6	✓	$-3 + 4 = 1$
7	X	$1 - 3 = -2$
8	✓	$-2 + 4 = 2$
9	X	$2 - 3 = -1$
10	✓	$-1 + 4 = 3$

(b) The winning team's score went from 5 to 10, so we know they gained 5 points. Notice that the total score for a correct answer followed by an incorrect answer is $4 - 3 = 1$. So 5 pairs of correct and incorrect answers would have a total score of 5. This would also be 10 questions in total, as desired. Therefore, the winning team must have had 5 questions correct.

Solution to Extension:

If a team got every question correct, they would earn $10 \times 4 = 40$ points, plus the 5 points at the beginning, for a total of $40 + 5 = 45$ points. This is the maximum possible score.

If a team got every question incorrect, they would lose $10 \times 3 = 30$ points, but since they started off with 5 points, their score would be $5 - 30 = -25$ points. This is the minimum possible score.

All the integers in between the minimum and maximum are **not** possible scores. The number of correct answers can be any integer from 0 to 10. Thus, there are 11 possibilities for the number of correct answers. Therefore, there cannot be more than 11 different possible scores. However, there are more than 11 integers between -25 and 45 so not all of them can be possible scores.

Another way to show this is to determine all the 11 possible scores. They are -25, -18, -11, -4, 3, 10, 17, 24, 31, 38, and 45.

Problem of the Week

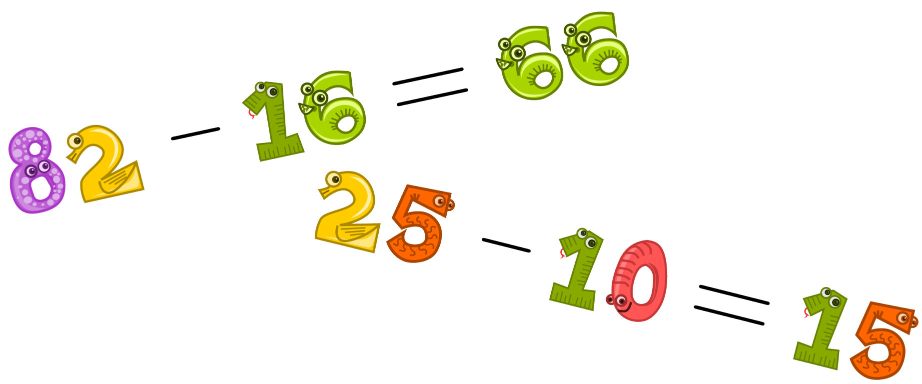
Problem B

Not a Big Difference

Yago takes a two-digit whole number and subtracts the product of its digits. He calls the result a *Yago Number*. He repeats this process with other two-digit numbers to find more Yago Numbers.

For example, the product of the digits of 82 is $8 \times 2 = 16$. Then $82 - 16 = 66$, so 66 is a Yago Number. Similarly, the product of the digits of 25 is $2 \times 5 = 10$. Then $25 - 10 = 15$, so 15 is another Yago Number.

What are the largest and smallest Yago Numbers that you can find? Justify your answers.



Problem of the Week

Problem B and Solution

Not a Big Difference

Problem

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For example, the product of the digits of 82 is $8 \times 2 = 16$. Then $82 - 16 = 66$, so 66 is a Yago Number. Similarly, the product of the digits of 25 is $2 \times 5 = 10$. Then $25 - 10 = 15$, so 15 is another Yago Number.

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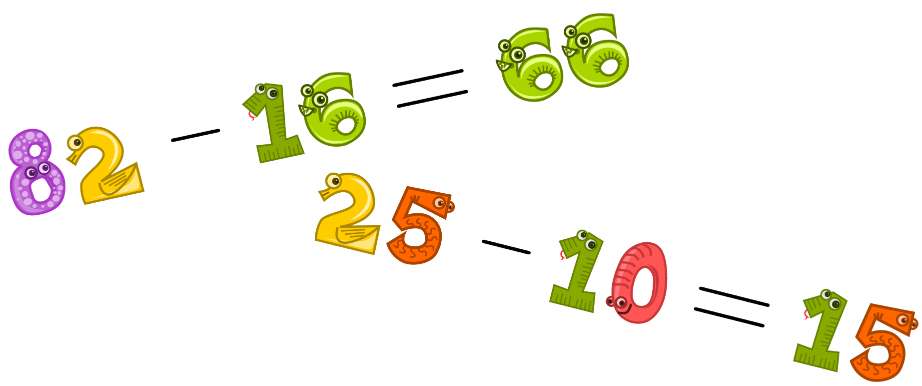


Solution

We can start by looking for patterns in the Yago Numbers. First look at what happens when we keep the tens digit in our original two-digit number the same but change the ones digit. The results when the tens digit is 2, 5, and 8 are shown in the following tables.

Original Number	20	21	22	23	24	25	26	27	28	29
Yago Number	20	19	18	17	16	15	14	13	12	11

Original Number	50	51	52	53	54	55	56	57	58	59
Yago Number	50	46	42	38	34	30	26	22	18	14

Original Number	80	81	82	83	84	85	86	87	88	89
Yago Number	80	73	66	59	52	45	38	31	24	17

From this, we see that for each tens digit, a ones digit of 0 produces the largest Yago Number and a ones digit of 9 produces the smallest Yago Number.

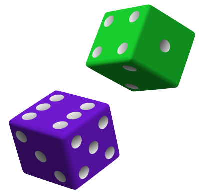
We can conclude that the largest Yago Number must have a ones digit of 0. Now we consider all two-digit numbers with a ones digit of 0. For each of these the product of the digits will be 0, since 0 multiplied by anything will always equal 0. So for any two-digit number with a ones digit of 0, its Yago Number will equal the original number. Thus, the largest Yago Number is the largest two-digit number with a ones digit of 0, which is 90.

To find the smallest Yago Number, we look at all the two-digit numbers with a ones digit of 9. These are shown in the following table.

Original Number	19	29	39	49	59	69	79	89	99
Yago Number	10	11	12	13	14	15	16	17	18

The smallest Yago Number is therefore 10.

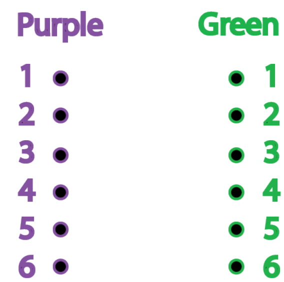
It's worth noting that all two-digit numbers with a tens digit of 1 actually have a Yago Number of 10. We leave it up to the reader to verify this.



Problem of the Week
Problem B
Dicey Products

Jordyn and Langdon have two dice, one purple and one green. When they roll the dice, they know there are a total of $6 \times 6 = 36$ possible outcomes for the top two numbers on the dice.

(a) In the diagram, join any dot on the left with one on the right such that the product of those two numbers is odd. How many ways are there to obtain an odd product?



(b) What does your result in part (a) tell you about how many ways there are to obtain an even product?

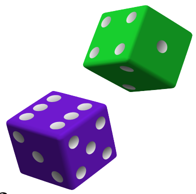
(c) Is the product 4 or 6 more likely to occur? Why?

(d) In the table, enter the possible odd products in the first column, the number of ways that product could occur in the middle column, and the theoretical probability of that outcome in the third column. (This has been done for odd product 1.)

Odd Product	Number of Ways	Theoretical Probability
1	1	$\frac{1}{36}$

(e) Roll two dice 36 times and record the product of the top two numbers for each roll. Count the number of each odd product that occurred, and then calculate the experimental probability of rolling that number. Compare the experimental probability to the theoretical probability calculated in part (d).

Themes: Data Management, Number Sense



Problem of the Week

Problem B and Solution

Dicey Products

Problem

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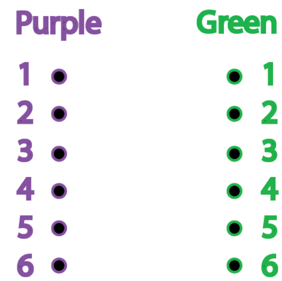

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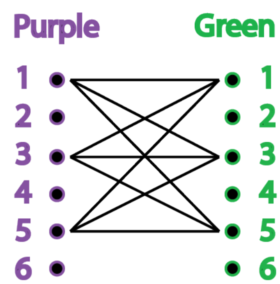
Odd Product	Number of Ways	Theoretical Probability
1	1	$\frac{1}{36}$

(e) Roll two dice 36 times and record the product of the top two numbers for each roll. Count the number of each odd product that occurred, and then calculate the experimental probability of rolling that number. Compare the experimental probability to the theoretical probability calculated in part (d).

Solution

(a) Odd products will occur exactly when the top two numbers are odd. We connect each odd number on the left with each odd number on the right to obtain the completed diagram shown. Counting the number of connections, we determine that there are 9 ways to obtain an odd product.



(b) Since there are 36 possible products in total, and a product must be either even or odd, there must be $36 - 9 = 27$ ways to obtain an even product.

(c) The product 6 is more likely to occur. This is because $6 = 1 \times 6$, $6 = 6 \times 1$, $6 = 2 \times 3$, and $6 = 3 \times 2$, and so can be formed by 4 different rolls. However, since $4 = 1 \times 4$, $4 = 4 \times 1$, and $4 = 2 \times 2$, it can only be formed by 3 different rolls.

(d) Noting that there is only one way to get each of 1, 9, and 25, and two ways to get each of 3, 5, and 15, the appropriate data and the theoretical probabilities are calculated.

Odd Product	Number of Ways	Theoretical Probability
1	1	$\frac{1}{36}$
3	2	$\frac{2}{36}$
5	2	$\frac{2}{36}$
9	1	$\frac{1}{36}$
15	2	$\frac{2}{36}$
25	1	$\frac{1}{36}$

(e) Answers will vary. Note that the result of a single experiment won't necessarily agree very closely with the theoretical probabilities.

SUGGESTION: Average the experimental probabilities obtained by all students in your class for each of the six odd products. Do these average probabilities agree more closely with the theoretical probabilities for each odd product than those of individual students?

Problem of the Week

Problem B

Sticky Keys

For some time, Joanne has relied on a calculator to multiply. She just realized that the 8 button on her calculator is broken.

(a) Determine a method for Joanne to use her broken calculator to determine the value of 82×816.

(b) Suppose both the 8 button and the 4 button were broken. Could Joanne still use her calculator to determine the product in part (a)?

Theme: Number Sense

Problem of the Week
Problem B and Solution
Sticky Keys

Problem

For some time, Joanne has relied on a calculator to multiply. She just realized that the 8 button on her calculator is broken.

(a) Determine a method for Joanne to use her broken calculator to determine the value of 82×816.

(b) Suppose both the 8 button and the 4 button were broken. Could Joanne still use her calculator to determine the product in part (a)?

Solution

(a) Answers will vary. To solve the problem, Joanne could write each number as a product of whole numbers that do not involve an 8. She can write 82 as 2×41 and 816 as 4×204. Then $82 \times 816 = 2 \times 41 \times 4 \times 204$, which Joanne can determine using her calculator.

NOTE: If her calculator has brackets, part (a) could also be done using sums. For example, $82 \times 816 = (32 + 50) \times (500 + 316)$.

(b) If the 4 key were broken as well, Joanne would not be able to solve this problem using the technique in part (a). This is because the only way to obtain a product of 82 using whole numbers is as 1×82 or as 2×41. This means that she cannot write 82 as a product of whole numbers without one of the numbers involving a 4 or an 8.

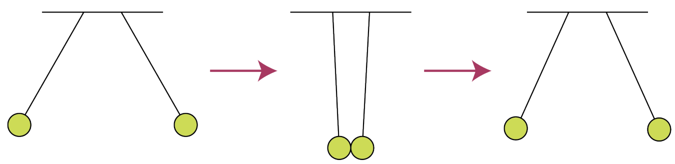
However, Joanne could write 82 as a sum of numbers without using 4 or 8. For example, $82 = 30 + 52$. Since $816 = 2 \times 2 \times 2 \times 102$, the product could be found by calculating the sum $30 + 52$, and then multiplying the result by $2 \times 2 \times 2 \times 102$.

Problem of the Week

Problem B

On the Rebound

Two identical rubber balls are hanging from the ends of two ropes that are attached to the ceiling of a gym. Pendul and Eliisa pull the balls away from each other, then release the balls towards each other so they collide in the middle. The balls then rebound and collide again repeatedly.



The students notice that the balls lose energy with each collision, so that they only rebound back to 90% of the distance between them before the collision. For example, if the initial distance between the balls was 6 m, then after the first collision they only rebound back to 90% of 6 m, or $0.9 \times 6 = 5.4$ m between them. Once the rebound distance between the balls is less than 2 m, they will no longer collide and will just swing freely.

Pendul and Eliisa start with the balls 6 m apart. Use the table to determine the total number of collisions between the balls before they stop colliding.

Number of Collisions	**Distance Between Balls (m)**
0	6
1	$0.9 \times 6 = 5.4$

Extension: Pendul and Eliisa want to try again, but start with the balls 4 m apart. Will there be more or fewer collisions than when they started with the balls 6 m apart? Explain.

Theme: Number Sense

Problem of the Week
Problem B and Solution
On the Rebound

Problem

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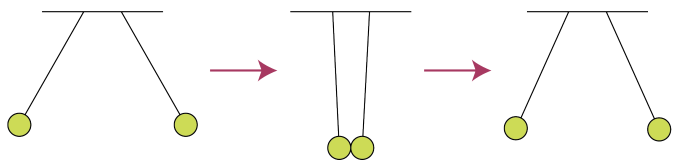

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Extension: Pendul and Eliisa want to try again, but start with the balls 4 m apart. Will there be more or fewer collisions than when they started with the balls 6 m apart? Explain.

Solution

The completed table is shown. All calculations have been rounded to two decimal places.

Number of Collisions	Distance Between Balls (m)
0	6
1	$0.9 \times 6 = 5.4$
2	$0.9 \times 5.4 = 4.86$
3	$0.9 \times 4.86 = 4.37$
4	$0.9 \times 4.37 = 3.93$
5	$0.9 \times 3.93 = 3.54$
6	$0.9 \times 3.54 = 3.19$
7	$0.9 \times 3.19 = 2.87$
8	$0.9 \times 2.87 = 2.58$
9	$0.9 \times 2.58 = 2.32$
10	$0.9 \times 2.32 = 2.09$
11	$0.9 \times 2.09 = 1.88$

After 11 collisions, the balls will rebound back to a distance of 1.88 m apart. Since this is less than 2 m, they will not collide again. Therefore, there will be a total of 11 collisions between the balls.

Solution to Extension:

If they started with the balls 4 m apart, there would be fewer collisions than when they started with the balls 6 m apart. Looking at the table, we see that after the 4^{th} collision the balls are just slightly less than 4 m apart. From there, it takes $11 - 4 = 7$ more collisions before the balls will no longer collide. Thus, if the balls were initially 4 m apart, they would collide approximately 7 times.

Problem of the Week

Problem B

Hidden Fractions

Zoe is trying to find fractions that satisfy some of the following conditions:

(1) The denominator of the fraction is divisible by 3.

(2) All the digits in the fraction are even.

(3) The decimal value of the fraction is greater than 1 but less than 1.5.

(4) The denominator of the fraction is a multiple of 2.

(5) The decimal value of the fraction is between 0.25 and 0.6.

(6) The decimal value of the fraction does not include a 9.

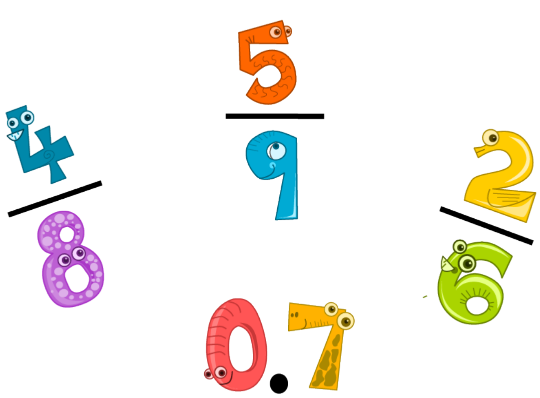
(7) The numerator of the fraction is even.

(8) The decimal value of the fraction is closer to 0.75 than to 0.55.

(a) Find a fraction that satisfies at least three of the conditions.

(b) Can you find a fraction that satisfies all of these conditions? If yes, give such a fraction. If no, explain why no such fraction exists.

(c) What is the least number of fractions Zoe needs to find such that together they meet all eight conditions?



Themes: Computational Thinking, Number Sense

Problem of the Week
Problem B and Solution
Hidden Fractions

Problem

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(b) Can you find a fraction that satisfies all of these conditions? If yes, give such a fraction. If no, explain why no such fraction exists.

(c) What is the least number of fractions Zoe needs to find such that together they meet all eight conditions?

Solution

(a) Answers will vary. Some examples:

- $\frac{2}{10}$ satisfies conditions (4), (6), and (7).
- $\frac{4}{9}$ satisfies conditions (1), (5), (6), and (7).
- $\frac{7}{9}$ satisfies conditions (1), (6), and (8).

(b) No such fraction exists. This is because conditions (3) and (5) are contradictory, as a number cannot be both between 0.25 and 0.6 **and** be greater than 1. Similarly, conditions (5) and (8) are also contradictory.

(c) From part (b), we know that we will need at least 2 fractions. It turns out that it is possible to satisfy all eight conditions together with just two fractions. Consider the fractions $\frac{1}{2}$ and $\frac{8}{6}$.

The fraction $\frac{1}{2} = 0.5$ satisfies conditions (4), (5), and (6).

The fraction $\frac{8}{6} = 1.333\ldots$ satisfies conditions (1), (2), (3), (4), (6), (7), (8).

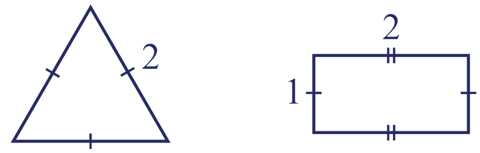
Answers will vary. Many other fraction pairs exist.

Problem of the Week

Problem B

The Long and the Short of It

Geordie drew an equilateral triangle with side length 2 and a rectangle with length 2 and width 1. He then noticed that each shape had a perimeter of 6.



(a) Geordie drew an equilateral triangle with side length 3. Can Geordie draw a rectangle that has the same perimeter as the equilateral triangle, if the side lengths of the rectangle must be whole numbers? Explain.

(b) Geordie drew equilateral triangles with side lengths 4, 5, 6, and 7. For which of these can he draw a rectangle with the same perimeter, if the side lengths of the rectangle must be whole numbers? Explain.

(c) If the side lengths of an equilateral triangle and a rectangle are all whole numbers, then:

(i) What numbers could be the perimeter of the triangle?

(ii) What numbers could be the perimeter of the rectangle?

(iii) What numbers could be the perimeter of both the triangle and the rectangle?

(iv) If the triangle and the rectangle have the same perimeter, what must be true about the side length of the triangle?

Themes: Geometry & Measurement, Number Sense

Problem of the Week
Problem B and Solution
The Long and the Short of It

Problem

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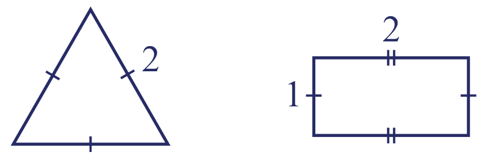

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(iii) What numbers could be the perimeter of both the triangle and the rectangle?

(iv) If the triangle and the rectangle have the same perimeter, what must be true about the side length of the triangle?

Solution

(a) If the side length of the equilateral triangle is 3, then its perimeter is $3 \times 3 = 9$. We will try different lengths and widths for the rectangle to see if we can find ones that give us a perimeter of 9. We will use the formula for the perimeter of a rectangle, which is $P = 2l + 2w$.

- Suppose the rectangle has length 1. Then $9 = 2 \times 1 + 2w$, so $9 = 2 + 2w$. Then $7 = 2w$, so $w = 7 \div 2 = 3.5$. However this is not a whole number, so this is not a possibility.
- Suppose the rectangle has length 2. Then $9 = 2 \times 2 + 2w$, so $9 = 4 + 2w$. Then $5 = 2w$, so $w = 5 \div 2 = 2.5$. However this is not a whole number, so this is not a possibility.
- Suppose the rectangle has length 3. Then $9 = 2 \times 3 + 2w$, so $9 = 6 + 2w$. Then $3 = 2w$, so $w = 3 \div 2 = 1.5$. However this is not a whole number, so this is not a possibility.
- Suppose the rectangle has length 4. Then $9 = 2 \times 4 + 2w$, so $9 = 8 + 2w$. Then $1 = 2w$, so $w = 1 \div 2 = 0.5$. However this is not a whole number, so this is not a possibility.
- Suppose the rectangle has length 5. Then since $5 + 5 = 10$ and $10 > 9$, it is not possible for this rectangle to have a perimeter of 9.

Therefore, Geordie cannot draw a rectangle with a perimeter of 9 if its side lengths must be whole numbers.

In general, if the length and width of a rectangle are whole numbers, then $2l$ and $2w$ will always be even numbers because any whole number multiplied by 2 results in an even number. So $2l + 2w$ will also be an even number, since an even number plus an even number results in an even number. Since $P = 2l + 2w$, it follow that P also must be an even number. This explains why we cannot draw a rectangle with a perimeter of 9, since 9 is an odd number.

(b) Using our results from part (a), we know that if the side lengths of a rectangle are whole numbers, then its perimeter must be even. In other words, if the perimeter is not even, then the side lengths of the rectangle cannot all be whole numbers.

- An equilateral triangle with side length 4 has perimeter $4 \times 3 = 12$, which is even. Geordie can draw a rectangle with perimeter 12; one example is a rectangle with length 5 and width 1.

- An equilateral triangle with side length 5 has perimeter $5 \times 3 = 15$. Since this is odd, Geordie cannot draw a rectangle with perimeter 15.
- An equilateral triangle with side length 6 has perimeter $6 \times 3 = 18$, which is even. Geordie can draw a rectangle with perimeter 18; one example is a rectangle with length 6 and width 3.
- An equilateral triangle with side length 7 has perimeter $7 \times 3 = 21$. Since this is odd, Geordie cannot draw a rectangle with perimeter 21.

(c) Based on our results from parts (a) and (b), we can conclude the following:

(i) The perimeter of the equilateral triangle must be a multiple of 3, since the perimeter is equal to 3 times its side length.

(ii) The perimeter of the rectangle must be an even number.

(iii) If a number is the perimeter of both the equilateral triangle and the rectangle, then it must be both a multiple of 3 and even. This is equivalent to saying it must be a multiple of 6.

(iv) If the equilateral triangle and the rectangle have the same perimeter, then this perimeter must be even. Therefore, the side length of the triangle must be even.

Problem of the Week
Problem B
Doggy Drinking

When a dog takes a drink, he laps up water by pushing his tongue into the water and then pulling it back with some water sticking to it. Some dogs are better at this than others. Chase is a messy drinker, and usually leaves a large puddle on the floor.

(a) Chase has a mass of 40 kg. Each day he needs to drink 50 mL of water for each kg of body mass. How many litres of water should Chase drink per day?

(b) If 70% of Chase's body mass is made up of water, how much of Chase's mass is NOT water?

(c) For every 3.5 mL of water Chase laps up with his tongue, 1.5 mL ends up on the floor. If Chase takes a lap every 5 seconds for one minute, how much water does he successfully drink?

(d) After an hour, you measure the water on the floor and find that Chase has spilled 45 mL. How much water did he successfully drink?

(e) How many laps per day will it take Chase to drink the water he needs in one day?

Themes: Geometry & Measurement, Number Sense

Problem of the Week

Problem B and Solution

Doggy Drinking

Problem

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(d) After an hour, you measure the water on the floor and find that Chase has spilled 45 mL. How much water did he successfully drink?

(e) How many laps per day will it take Chase to drink the water he needs in one day?

Solution

(a) Chase should drink $50 \times 40 = 2000$ mL, or 2 L of water per day.

(b) The percentage of Chase's body mass that is NOT water is $100\% - 70\% = 30\%$. Thus, the amount of Chase's mass that is NOT water is 30% of 40 kg, or $0.3 \times 40 = 12$ kg.

(c) Since there are 60 seconds in one minute, Chase takes $60 \div 5 = 12$ laps in total. From each lap, he drinks only $3.5 - 1.5 = 2.0$ mL. Thus, in 12 laps he successfully drinks a total of $12 \times 2.0 = 24$ mL of water.

(d) The 45 mL of water Chase spilled indicates he took $45 \div 1.5 = 30$ laps. Since he drinks 2 mL with each lap, he successfully drank $2 \times 30 = 60$ mL.

(e) From part (a), Chase needs to drink 2000 mL of water per day. He drinks 2 mL with each lap, so he will need to take $2000 \div 2 = 1000$ laps each day.

Problem of the Week

Problem B

Symbolic Shapes

In each question, each shape ($\square$, $\bigcirc$, $\triangle$) is equal to the same whole number. For example, $\square + \square = 8$ implies that $2 \times \square = 8$, so $\square = 4$. (The value of each shape can change from question to question.)

Determine the value of each shape in each question.

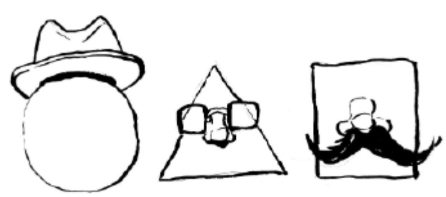
(a)
$$\square + \square = 8$$
$$\bigcirc + \square = 10$$

(b)
$$\square + \square + \square = 18$$
$$\square + \bigcirc + \bigcirc = 14$$
$$\bigcirc + \square + \triangle = 18$$

(c)
$$\bigcirc + \bigcirc + \bigcirc = 24$$
$$\square \times \bigcirc + \square = 36$$
$$\triangle \times \square + \triangle = 25$$

(d)
$$2 \times \triangle = 28$$
$$\triangle - 10 = \bigcirc$$
$$\bigcirc + \square + \square = 10$$

(e)
$$\square + \triangle + \triangle = 21$$
$$\triangle + \bigcirc - \square = 9$$
$$\square + \square = 18$$



EXTENSION: Create a similar problem and challenge a classmate to solve it.

THEMES: ALGEBRA, NUMBER SENSE

Problem of the Week
Problem B and Solution
Symbolic Shapes

Problem

In each question, each shape ($\square$, $\bigcirc$, $\triangle$) is equal to the same whole number. For example, $\square + \square = 8$ implies that $2 \times \square = 8$, so $\square = 4$. (The value of each shape can change from question to question.)

Determine the value of each shape in each question.

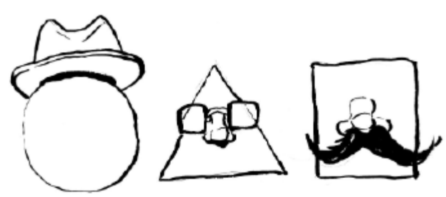
(a)
$$\square + \square = 8$$
$$\bigcirc + \square = 10$$

(b)
$$\square + \square + \square = 18$$
$$\square + \bigcirc + \bigcirc = 14$$
$$\bigcirc + \square + \triangle = 18$$

(c)
$$\bigcirc + \bigcirc + \bigcirc = 24$$
$$\square \times \bigcirc + \square = 36$$
$$\triangle \times \square + \triangle = 25$$

(d)
$$2 \times \triangle = 28$$
$$\triangle - 10 = \bigcirc$$
$$\bigcirc + \square + \square = 10$$

(e)
$$\square + \triangle + \triangle = 21$$
$$\triangle + \bigcirc - \square = 9$$
$$\square + \square = 18$$



EXTENSION: Create a similar problem and challenge a classmate to solve it.

Solution

(a) Since $\square + \square = 8$, we know that $2 \times \square = 8$, and so $\square = 8 \div 2 = 4$.
Then, since $\bigcirc + \square = 10$, we know that $\bigcirc + 4 = 10$, and so $\bigcirc = 10 - 4 = 6$.
Thus, $\square = 4$ and $\bigcirc = 6$.

(b) Since $\square + \square + \square = 18$, we know that $3 \times \square = 18$, and so $\square = 18 \div 3 = 6$.
Then, since $\square + \bigcirc + \bigcirc = 14$, we know that $6 + \bigcirc + \bigcirc = 14$, and so $\bigcirc + \bigcirc = 14 - 6 = 8$ and $\bigcirc = 8 \div 2 = 4$.
Then, since $\bigcirc + \square + \triangle = 18$, we have $4 + 6 + \triangle = 18$, and so $\triangle = 18 - 10 = 8$.
Thus, $\square = 6$, $\bigcirc = 4$, and $\triangle = 8$.

(c) Since $\bigcirc + \bigcirc + \bigcirc = 24$, we know that $3 \times \bigcirc = 24$, and so $\bigcirc = 24 \div 3 = 8$.
Then, since $\square \times \bigcirc + \square = 36$, we have $\square \times 8 + \square = 36$, and so $9 \times \square = 36$.
Therefore, $\square = 36 \div 9 = 4$.
Now, since $\triangle \times \square + \triangle = 25$, we have $\triangle \times 4 + \triangle = 25$, and so $5 \times \triangle = 25$ and $\triangle = 25 \div 5 = 5$.
Thus, $\square = 4$, $\bigcirc = 8$, and $\triangle = 5$.

(d) Since $2 \times \triangle = 28$, we have $\triangle = 28 \div 2 = 14$.
Then, since $\triangle - 10 = \bigcirc$, we have $14 - 10 = \bigcirc$, and so $\bigcirc = 4$.
Then, since $\bigcirc + \square + \square = 10$, we have $4 + \square + \square = 10$, and so $\square + \square = 10 - 4 = 6$. Therefore, $\square = 6 \div 2 = 3$.
Thus, $\square = 3$, $\bigcirc = 4$, and $\triangle = 14$.

(e) For this problem, start with the third equation. Since $\square + \square = 18$, we have $2 \times \square = 18$, and so $\square = 18 \div 2 = 9$.
Then, since $\square + \triangle + \triangle = 21$, we have $9 + \triangle + \triangle = 21$, and so $\triangle + \triangle = 21 - 9 = 12$. Thus, $2 \times \triangle = 12$, and so $\triangle = 12 \div 2 = 6$.
Now, since $\triangle + \bigcirc - \square = 9$, we have $6 + \bigcirc - 9 = 9$, and so $\bigcirc = 9 - 6 + 9 = 12$.
Thus, $\square = 9$, $\bigcirc = 12$, and $\triangle = 6$.

Problem of the Week

Problem B

Flowers for Sarah

Sarah has constructed 8 new garden beds in a sunny location and is looking to plant a flower garden. Seed packets cost \$2.69 per flower variety, and she must pay a \$9.00 flat rate shipping fee for her order. She would like to plant quite a few different varieties, but not spend more than \$53.00.

(a) Write an inequality which states Sarah's problem mathematically, using the given costs, the number of varieties she buys, and her available funds.

(b) What are all possibilities for the number of seed packets Sarah can purchase?

(c) If Sarah purchases the maximum number of seed packets that she can, and plants the same number of seed packets in each garden bed, then how many packets of seeds does she plant in each garden bed?

(d) In the end, Sarah buys as many seed packets as she can, purchasing in total four varieties: zinnias, salvia, daisies, and cosmos. She purchases three times as many packets of zinnias as salvia, and equal numbers of packets of daisies and cosmos. If she buys two seed packets of salvia, how many did she buy of each of the other three flower types?

Themes: Algebra, Number Sense

Problem of the Week
Problem B and Solution
Flowers for Sarah

Problem

Sarah has constructed 8 new garden beds in a sunny location and is looking to plant a flower garden. Seed packets cost \$2.69 per flower variety, and she must pay a \$9.00 flat rate shipping fee for her order. She would like to plant quite a few different varieties, but not spend more than \$53.00.

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Solution

(a) Let x represent the number of seed packets Sarah purchases. Then the total cost of her purchase will be $\$2.69 \times x + \$9.00$. Since this must be no greater than \$53.00, an appropriate inequality is $\$2.69 \times x + \$9.00 \leq \$53.00$.

(b) The most Sarah could purchase would occur if $\$2.69 \times x + \$9.00 = \$53.00$. Solving for x gives $2.69 \times x = 53 - 9$, or $x = \frac{44}{2.69} \approx 16.36$. Thus, she can buy at most 16 seed packets. That is, she could purchase any number from 0 to 16 packets.

(c) If she maximizes her seed purchase, then she will buy 16 packets of seeds. So she will plant $16 \div 8 = 2$ packets of seeds in each of the eight garden beds.

(d) Since she bought 2 packets of salvia, she bought $3 \times 2 = 6$ packets of zinnias. This leaves $16 - 2 - 6 = 8$ packets to be divided evenly between daisies and cosmos, so she bought $8 \div 2 = 4$ packets of each of those.

Problem of the Week

Problem B

Maybe Equals, Maybe Not

(a) For each of the following statements, fill in the ◯ with =, <, or > to make the statement true.

(i) $0.5 + 0.24 \bigcirc \frac{3}{4}$

(ii) $1.10 + \frac{1}{4} \bigcirc \frac{35}{100}$

(iii) $2 \times 2 \times 5 \bigcirc \frac{100}{5}$

(iv) $100 - 24 + 65 \bigcirc 14 \times 10$

(v) $25 \times 40 + 321 \bigcirc 153 \times 10$

(vi) $2 \times 24 \div 6 \bigcirc \frac{36}{6}$

(vii) $35 \times 2 \bigcirc 5 \times 14$

(viii) $2y + 6 \bigcirc y + 3 + y + 3$

(b) In the following statement, determine values of y that make the statement true when the ◯ is replaced with =, <, and >.

$$4y + 12 \bigcirc 4 + 2y + 4 + 3y + 4$$

Theme: Number Sense

Problem of the Week

Problem B and Solution

Maybe Equals, Maybe Not

Problem

(a) For each of the following statements, fill in the ◯ with =, <, or > to make the statement true.

(i) $0.5 + 0.24 \bigcirc \frac{3}{4}$

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(b) In the following statement, determine values of y that make the statement true when the ◯ is replaced with =, <, and >.

$$4y + 12 \bigcirc 4 + 2y + 4 + 3y + 4$$

Solution

(a) To determine which symbol we should put in the ◯, we look at the left side and right side of each statement separately.

(i)

Left Side	**Right Side**
$0.5 + 24$	$\frac{3}{4}$
$= 0.74$	$= 0.75$

Since $0.74 < 0.75$, then $0.5 + 0.24 < \frac{3}{4}$.

(ii)

Left Side	**Right Side**
$1.10 + \frac{1}{4}$	$\frac{35}{100}$
$= 1.10 + 0.25 = 1.35$	$= 0.35$

Since $1.35 > 0.35$, then $1.10 + \frac{1}{4} > \frac{35}{100}$.

(iii)

Left Side	**Right Side**
$2 \times 2 \times 5$	$\frac{100}{5}$
$= 4 \times 5 = 20$	$= 20$

Since $20 = 20$, then $2 \times 2 \times 5 = \frac{100}{5}$.

(iv)

Left Side	**Right Side**
$100 - 24 + 65$	14×10
$= 76 + 65 = 141$	$= 140$

Since $141 > 140$, then $100 - 24 + 65 > 14 \times 10$.

(v)

Left Side	**Right Side**
$25 \times 40 + 321$	153×10
$= 1000 + 321 = 1321$	$= 1530$

Since $1321 < 1530$, then $25 \times 40 + 321 < 153 \times 10$.

(vi)

Left Side	**Right Side**
$2 \times 24 \div 6$	$\frac{36}{6}$
$= 48 \div 6 = 8$	$= 6$

Since $8 > 6$, then $2 \times 24 \div 6 > \frac{36}{6}$.

(vii)

Left Side	**Right Side**
35×2	5×14
$= 70$	$= 70$

Since $70 = 70$, then $35 \times 2 = 5 \times 14$.

(viii)

Left Side	**Right Side**
$2y + 6$	$y + 3 + y + 3$
	$= y+y+3+3 = 2y+6$

Since $2y + 6 = 2y + 6$ for any value of y, then $2y + 6 = y + 3 + y + 3$.

(b) The right side can be rewritten as $4 + 2y + 4 + 3y + 4 = 2y + 3y + 4 + 4 + 4 = 5y + 12$. Then our statement becomes $4y + 12$ ◯ $5y + 12$.

If y is any value greater than 0, such as 1, then $4y + 12 < 5y + 12$. If $y = 0$, then $4y + 12 = 5y + 12$. If y is any value less than 0, such as -1, then $4y + 12 > 5y + 12$.

Problem of the Week

Problem B

Otters in a Happy Space

Happy Space toy store bought 50 plush toy otters for \$8.00 each for resale. They also paid a \$30 shipping fee.

They priced the toy otters at \$10.00 each throughout December, after which they sold the remainder at a reduced sale price.

(a) If they sold 30 toy otters in December for \$10.00 each, what must the reduced sale price per otter be in order that they break even on this item (that is, they have no net profit nor loss)?

(b) For each otter, what percentage of the \$10.00 price is the savings from the reduced price?

Theme: Number Sense

Problem of the Week

Problem B and Solution

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(b) For each otter, what percentage of the \$10.00 price is the savings from the reduced price?

Solution

(a) To break even, Happy Space must recover their total cost for the otters, which is $(\$8.00 \times 50) + \$30.00 = \$430.00$.

The income from the sale of 30 otters at \$10 each is $30 \times \$10.00 = \$300$.

Thus, they need to recover $\$430 - \$300 = \$130$ from the sale of the remaining 20 otters at a reduced price in order to break even.

This requires that the otters be priced at $\$130 \div 20 = \$6.50$. So the appropriate reduced sale price is \$6.50 to break even.

(b) The reduced sale price of \$6.50 is $\$10.00 - \$6.50 = \$3.50$ less than the original price. Thus, as a percentage, this savings is

$$\frac{\$3.50}{\$10.00} \times 100\% = 0.35 \times 100\% = 35\%$$