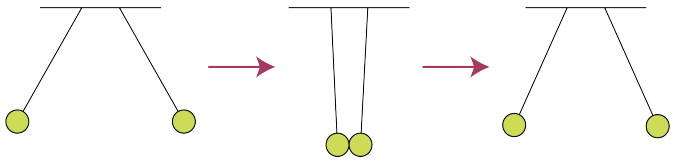
Problem of the Week
Problem B and Solution
On the Rebound

Problem

Two identical rubber balls are hanging from the ends of two ropes that are attached to the ceiling of a gym. Pendul and Eliisa pull the balls away from each other, then release the balls towards each other so they collide in the middle. The balls then rebound and collide again repeatedly.



The students notice that the balls lose energy with each collision, so that they only rebound back to 90% of the distance between them before the collision. For example, if the initial distance between the balls was 6 m, then after the first collision they only rebound back to 90% of 6 m, or $0.9 \times 6 = 5.4$ m between them. Once the rebound distance between the balls is less than 2 m, they will no longer collide and will just swing freely.

Pendul and Eliisa start with the balls 6 m apart. Use the table to determine the total number of collisions between the balls before they stop colliding.

Number of Collisions	Distance Between Balls (m)
0	6
1	$0.9 \times 6 = 5.4$

Extension: Pendul and Eliisa want to try again, but start with the balls 4 m apart. Will there be more or fewer collisions than when they started with the balls 6 m apart? Explain.

Solution

The completed table is shown. All calculations have been rounded to two decimal places.

Number of Collisions	Distance Between Balls (m)
0	6
1	$0.9 \times 6 = 5.4$
2	$0.9 \times 5.4 = 4.86$
3	$0.9 \times 4.86 = 4.37$
4	$0.9 \times 4.37 = 3.93$
5	$0.9 \times 3.93 = 3.54$
6	$0.9 \times 3.54 = 3.19$
7	$0.9 \times 3.19 = 2.87$
8	$0.9 \times 2.87 = 2.58$
9	$0.9 \times 2.58 = 2.32$
10	$0.9 \times 2.32 = 2.09$
11	$0.9 \times 2.09 = 1.88$

After 11 collisions, the balls will rebound back to a distance of 1.88 m apart. Since this is less than 2 m, they will not collide again. Therefore, there will be a total of 11 collisions between the balls.

Solution to Extension:

If they started with the balls 4 m apart, there would be fewer collisions than when they started with the balls 6 m apart. Looking at the table, we see that after the 4^{th} collision the balls are just slightly less than 4 m apart. From there, it takes $11 - 4 = 7$ more collisions before the balls will no longer collide. Thus, if the balls were initially 4 m apart, they would collide approximately 7 times.