Problem of the Week

Problem B and Solution

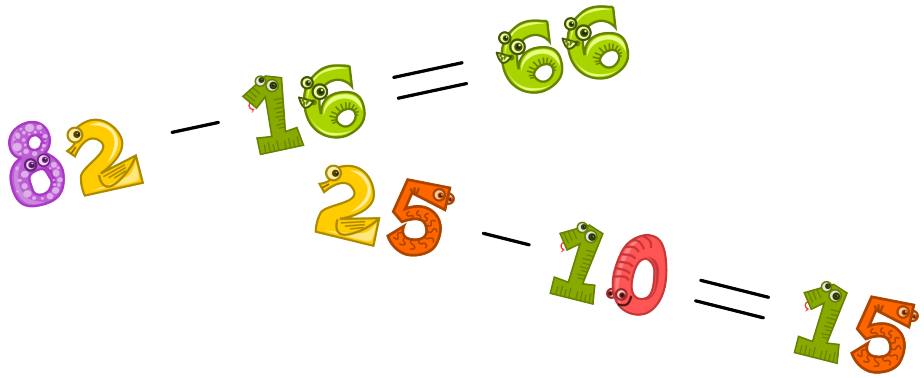
Not a Big Difference

Problem

Yago takes a two-digit whole number and subtracts the product of its digits. He calls the result a *Yago Number*. He repeats this process with other two-digit numbers to find more Yago Numbers.

For example, the product of the digits of 82 is $8 \times 2 = 16$. Then $82 - 16 = 66$, so 66 is a Yago Number. Similarly, the product of the digits of 25 is $2 \times 5 = 10$. Then $25 - 10 = 15$, so 15 is another Yago Number.

What are the largest and smallest Yago Numbers that you can find? Justify your answers.



Solution

We can start by looking for patterns in the Yago Numbers. First look at what happens when we keep the tens digit in our original two-digit number the same but change the ones digit. The results when the tens digit is 2, 5, and 8 are shown in the following tables.

Original Number	20	21	22	23	24	25	26	27	28	29
Yago Number	20	19	18	17	16	15	14	13	12	11

Original Number	50	51	52	53	54	55	56	57	58	59
Yago Number	50	46	42	38	34	30	26	22	18	14

Original Number	80	81	82	83	84	85	86	87	88	89
Yago Number	80	73	66	59	52	45	38	31	24	17

From this, we see that for each tens digit, a ones digit of 0 produces the largest Yago Number and a ones digit of 9 produces the smallest Yago Number.

We can conclude that the largest Yago Number must have a ones digit of 0. Now we consider all two-digit numbers with a ones digit of 0. For each of these the product of the digits will be 0, since 0 multiplied by anything will always equal 0. So for any two-digit number with a ones digit of 0, its Yago Number will equal the original number. Thus, the largest Yago Number is the largest two-digit number with a ones digit of 0, which is 90.

To find the smallest Yago Number, we look at all the two-digit numbers with a ones digit of 9. These are shown in the following table.

Original Number	19	29	39	49	59	69	79	89	99
Yago Number	10	11	12	13	14	15	16	17	18

The smallest Yago Number is therefore 10.

It's worth noting that all two-digit numbers with a tens digit of 1 actually have a Yago Number of 10. We leave it up to the reader to verify this.