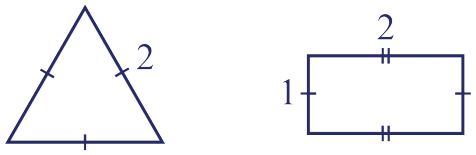
Problem of the Week
Problem B and Solution
The Long and the Short of It

Problem

Geordie drew an equilateral triangle with side length 2 and a rectangle with length 2 and width 1. He then noticed that each shape had a perimeter of 6.



(a) Geordie drew an equilateral triangle with side length 3. Can Geordie draw a rectangle that has the same perimeter as the equilateral triangle, if the side lengths of the rectangle must be whole numbers? Explain.

(b) Geordie drew equilateral triangles with side lengths 4, 5, 6, and 7. For which of these can he draw a rectangle with the same perimeter, if the side lengths of the rectangle must be whole numbers? Explain.

(c) If the side lengths of an equilateral triangle and a rectangle are all whole numbers, then:

(i) What numbers could be the perimeter of the triangle?

(ii) What numbers could be the perimeter of the rectangle?

(iii) What numbers could be the perimeter of both the triangle and the rectangle?

(iv) If the triangle and the rectangle have the same perimeter, what must be true about the side length of the triangle?

Solution

(a) If the side length of the equilateral triangle is 3, then its perimeter is $3 \times 3 = 9$. We will try different lengths and widths for the rectangle to see if we can find ones that give us a perimeter of 9. We will use the formula for the perimeter of a rectangle, which is $P = 2l + 2w$.

- Suppose the rectangle has length 1. Then $9 = 2 \times 1 + 2w$, so $9 = 2 + 2w$. Then $7 = 2w$, so $w = 7 \div 2 = 3.5$. However this is not a whole number, so this is not a possibility.
- Suppose the rectangle has length 2. Then $9 = 2 \times 2 + 2w$, so $9 = 4 + 2w$. Then $5 = 2w$, so $w = 5 \div 2 = 2.5$. However this is not a whole number, so this is not a possibility.
- Suppose the rectangle has length 3. Then $9 = 2 \times 3 + 2w$, so $9 = 6 + 2w$. Then $3 = 2w$, so $w = 3 \div 2 = 1.5$. However this is not a whole number, so this is not a possibility.
- Suppose the rectangle has length 4. Then $9 = 2 \times 4 + 2w$, so $9 = 8 + 2w$. Then $1 = 2w$, so $w = 1 \div 2 = 0.5$. However this is not a whole number, so this is not a possibility.
- Suppose the rectangle has length 5. Then since $5 + 5 = 10$ and $10 > 9$, it is not possible for this rectangle to have a perimeter of 9.

Therefore, Geordie cannot draw a rectangle with a perimeter of 9 if its side lengths must be whole numbers.

In general, if the length and width of a rectangle are whole numbers, then $2l$ and $2w$ will always be even numbers because any whole number multiplied by 2 results in an even number. So $2l + 2w$ will also be an even number, since an even number plus an even number results in an even number. Since $P = 2l + 2w$, it follow that P also must be an even number. This explains why we cannot draw a rectangle with a perimeter of 9, since 9 is an odd number.

(b) Using our results from part (a), we know that if the side lengths of a rectangle are whole numbers, then its perimeter must be even. In other words, if the perimeter is not even, then the side lengths of the rectangle cannot all be whole numbers.

- An equilateral triangle with side length 4 has perimeter $4 \times 3 = 12$, which is even. Geordie can draw a rectangle with perimeter 12; one example is a rectangle with length 5 and width 1.

- An equilateral triangle with side length 5 has perimeter $5 \times 3 = 15$. Since this is odd, Geordie cannot draw a rectangle with perimeter 15.
- An equilateral triangle with side length 6 has perimeter $6 \times 3 = 18$, which is even. Geordie can draw a rectangle with perimeter 18; one example is a rectangle with length 6 and width 3.
- An equilateral triangle with side length 7 has perimeter $7 \times 3 = 21$. Since this is odd, Geordie cannot draw a rectangle with perimeter 21.

(c) Based on our results from parts (a) and (b), we can conclude the following:

(i) The perimeter of the equilateral triangle must be a multiple of 3, since the perimeter is equal to 3 times its side length.

(ii) The perimeter of the rectangle must be an even number.

(iii) If a number is the perimeter of both the equilateral triangle and the rectangle, then it must be both a multiple of 3 and even. This is equivalent to saying it must be a multiple of 6.

(iv) If the equilateral triangle and the rectangle have the same perimeter, then this perimeter must be even. Therefore, the side length of the triangle must be even.