# Problem of the Week
## Problem B and Solution
## Gregorian Calendars

### Problem

The Gregorian Calendar is used in most parts of the world today. In order to keep this calendar in synch with the solar year (the time for the Earth to complete one orbit around the sun), it has leap years with an extra day in February. Leap years generally occur every four years, in years that are divisible by 4. However, years divisible by 100 are excluded, UNLESS they are divisible by 400. For example, the year 2000 was a leap year, but 1900 was not.

The year 2025 has a different calendar than the year 2024 since it started on a different day of the week. January 1, 2024 was on a Monday, while January 1, 2025 was on a Wednesday.

(a) January 1, 2023 was on a Sunday. Explain why January 1, 2024 was one day of the week later than in 2023, while January 1, 2025 was two days of the week later than in 2024.

(b) January 1, 1992 and January 1, 2025 were both on a Wednesday. Did 1992 have the same yearly calendar as 2025?

(c) How many different yearly calendars are there in total? Two yearly calendars are considered the same if each date occurred on the same day of the week.


## Solution

(a) Since the year 2023 had 365 days, and a week has 7 days, there were $365 \div 7 = 52\frac{1}{7}$ weeks in 2023. That is, there were 52 full weeks plus 1 day. Thus, all of the next year's dates fall one day of the week later than the previous year. This is why January 1, 2024 was one day of the week later than January 1, 2023.

However, 2024 was a leap year with 366 days, so it had 52 full weeks plus 2 days. Thus, January 1, 2025 was two days later in the week (a Wednesday) than January 1, 2024 (a Monday).

(b) Despite the fact that January 1 in 1992 and in 2025 were both on a Wednesday, the years won't have the same yearly calendar because 1992 was a leap year. This is since 1992 is divisible by 4, but not 100.

(c) There are 7 possible weekdays on which January 1 can occur. For each of these possibilities, there is a calendar that includes February 29 and a calendar that does not. Thus, there are $7 \times 2 = 14$ different yearly calendars.