



## Problem of the Week

### Problem B and Solution

#### Three Coins in a Toss

#### Problem

Jack and Jill have made up a game where they take turns tossing a red coin, a blue coin, and a green coin. Each coin has heads (H) on one side and tails (T) on the other side. They earn points according to the following rules.

1. If the toss results in all three coins being heads (HHH), or all three being tails (TTT), then Jack gets one point.
  2. If the toss results in any other combination of heads and tails (e.g., TTH), then Jill gets one point.
  3. The winner of the game is the player with the most points after 10 tosses of all three coins.
- (a) Play this game with a partner. Assign one player to be Jack and the other to be Jill.
- (b) A game is considered a *fair game* if each player is equally likely to win the game. Explain why this game is **not** a fair game.
- (c) Explain how you could change the rules to make this a fair game.

#### Solution

- (a) After playing the game, check how many pairs had Jack win and how many had Jill win.
- (b) First we determine the possible outcomes of each toss of the three coins. In total there are eight possible outcomes:

$$TTT, TTH, THT, HTT, THH, HTH, HHT, HHH$$

From here we see that Jack gets a point for only 2 out of the 8 outcomes, while Jill gets a point for 6 out of the 8 outcomes. Thus, the theoretical probability of Jack getting a point is  $\frac{2}{8} = \frac{1}{4}$ , while the theoretical probability of Jill getting a point is  $\frac{6}{8} = \frac{3}{4}$ . Since Jill is more likely to get a point on any toss, she is more likely to win the game. Thus, it is not a fair game.

- (c) There are many different ways to change the rules to make it a fair game. One way is to give Jack a point if there are at least 2 heads, and give Jill a point if there are at least 2 tails. Then Jack would get a point for four outcomes (HHH, THH, HTH, and HHT), and Jill would get a point for the remaining four outcomes (TTT, TTH, THT, and HTT). This would be a fair game because each player is equally likely to earn a point on any toss. Thus, each player is equally likely to win the game.