



Problem of the Week

Problem B and Solution

Hidden Fractions

Problem

Zoe is trying to find fractions that satisfy some of the following conditions:

- (1) The denominator of the fraction is divisible by 3.
 - (2) All the digits in the fraction are even.
 - (3) The decimal value of the fraction is greater than 1 but less than 1.5.
 - (4) The denominator of the fraction is a multiple of 2.
 - (5) The decimal value of the fraction is between 0.25 and 0.6.
 - (6) The decimal value of the fraction does not include a 9.
 - (7) The numerator of the fraction is even.
 - (8) The decimal value of the fraction is closer to 0.75 than to 0.55.
- (a) Find a fraction that satisfies at least three of the conditions.
 - (b) Can you find a fraction that satisfies all of these conditions? If yes, give such a fraction. If no, explain why no such fraction exists.
 - (c) What is the least number of fractions Zoe needs to find such that together they meet all eight conditions?

Solution

- (a) Answers will vary. Some examples:
 - $\frac{2}{10}$ satisfies conditions (4), (6), and (7).
 - $\frac{4}{9}$ satisfies conditions (1), (5), (6), and (7).
 - $\frac{7}{9}$ satisfies conditions (1), (6), and (8).
- (b) No such fraction exists. This is because conditions (3) and (5) are contradictory, as a number cannot be both between 0.25 and 0.6 **and** be greater than 1. Similarly, conditions (5) and (8) are also contradictory.
- (c) From part (b), we know that we will need at least 2 fractions. It turns out that it is possible to satisfy all eight conditions together with just two fractions. Consider the fractions $\frac{1}{2}$ and $\frac{8}{6}$.

The fraction $\frac{1}{2} = 0.5$ satisfies conditions (4), (5), and (6).

The fraction $\frac{8}{6} = 1.333\dots$ satisfies conditions (1), (2), (3), (4), (6), (7), (8).

Answers will vary. Many other fraction pairs exist.