



Problem of the Week

Problem B and Solution

Symbolic Shapes

Problem

In each question, each shape (\square , \circ , \triangle) is equal to the same whole number. For example, $\square + \square = 8$ implies that $2 \times \square = 8$, so $\square = 4$. (The value of each shape can change from question to question.)

Determine the value of each shape in each question.

(a)

$$\begin{aligned}\square + \square &= 8 \\ \circ + \square &= 10\end{aligned}$$

(b)

$$\begin{aligned}\square + \square + \square &= 18 \\ \square + \circ + \circ &= 14 \\ \circ + \square + \triangle &= 18\end{aligned}$$

(c)

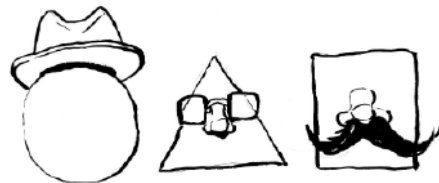
$$\begin{aligned}\circ + \circ + \circ &= 24 \\ \square \times \circ + \square &= 36 \\ \triangle \times \square + \triangle &= 25\end{aligned}$$

(d)

$$\begin{aligned}2 \times \triangle &= 28 \\ \triangle - 10 &= \circ \\ \circ + \square + \square &= 10\end{aligned}$$

(e)

$$\begin{aligned}\square + \triangle + \triangle &= 21 \\ \triangle + \circ - \square &= 9 \\ \square + \square &= 18\end{aligned}$$



EXTENSION: Create a similar problem and challenge a classmate to solve it.



Solution

- (a) Since $\square + \square = 8$, we know that $2 \times \square = 8$, and so $\square = 8 \div 2 = 4$.
Then, since $\circ + \square = 10$, we know that $\circ + 4 = 10$, and so $\circ = 10 - 4 = 6$.
Thus, $\square = 4$ and $\circ = 6$.
- (b) Since $\square + \square + \square = 18$, we know that $3 \times \square = 18$, and so $\square = 18 \div 3 = 6$.
Then, since $\square + \circ + \circ = 14$, we know that $6 + \circ + \circ = 14$, and so
 $\circ + \circ = 14 - 6 = 8$ and $\circ = 8 \div 2 = 4$.
Then, since $\circ + \square + \triangle = 18$, we have $4 + 6 + \triangle = 18$, and so
 $\triangle = 18 - 10 = 8$.
Thus, $\square = 6$, $\circ = 4$, and $\triangle = 8$.
- (c) Since $\circ + \circ + \circ = 24$, we know that $3 \times \circ = 24$, and so $\circ = 24 \div 3 = 8$.
Then, since $\square \times \circ + \square = 36$, we have $\square \times 8 + \square = 36$, and so $9 \times \square = 36$.
Therefore, $\square = 36 \div 9 = 4$.
Now, since $\triangle \times \square + \triangle = 25$, we have $\triangle \times 4 + \triangle = 25$, and so $5 \times \triangle = 25$
and $\triangle = 25 \div 5 = 5$.
Thus, $\square = 4$, $\circ = 8$, and $\triangle = 5$.
- (d) Since $2 \times \triangle = 28$, we have $\triangle = 28 \div 2 = 14$.
Then, since $\triangle - 10 = \circ$, we have $14 - 10 = \circ$, and so $\circ = 4$.
Then, since $\circ + \square + \square = 10$, we have $4 + \square + \square = 10$, and so
 $\square + \square = 10 - 4 = 6$. Therefore, $\square = 6 \div 2 = 3$.
Thus, $\square = 3$, $\circ = 4$, and $\triangle = 14$.
- (e) For this problem, start with the third equation. Since $\square + \square = 18$, we have
 $2 \times \square = 18$, and so $\square = 18 \div 2 = 9$.
Then, since $\square + \triangle + \triangle = 21$, we have $9 + \triangle + \triangle = 21$, and so
 $\triangle + \triangle = 21 - 9 = 12$. Thus, $2 \times \triangle = 12$, and so $\triangle = 12 \div 2 = 6$.
Now, since $\triangle + \circ - \square = 9$, we have $6 + \circ - 9 = 9$, and so
 $\circ = 9 - 6 + 9 = 12$.
Thus, $\square = 9$, $\circ = 12$, and $\triangle = 6$.