

# Problem of the Week
## Problem A and Solution
## Halloween Candy

**Problem**

After going trick-or-treating on Halloween, James, Robbie, and Ginny come home with the following treats:

James: 6 chocolate bars, 4 caramels, 6 suckers, 5 licorice

Robbie: 5 chocolate bars, 5 caramels, 3 suckers, 7 licorice

Ginny: 2 chocolate bars, 8 caramels, 4 suckers, 6 licorice

They decide to put all of their treats into a single bucket. Each day, James, Robbie, and Ginny each randomly pick 2 treats from the bucket to enjoy.

(a) Assuming that each individual treat is equally likely to be selected, on the first day, which type of treat is most likely to be picked from the bucket?

(b) Assuming that each individual treat is equally likely to be selected, on the first day, which types of treats are equally likely to be picked from the bucket?

(c) How many days must Robbie pick treats from the bucket to guarantee that he picks the same type of treat more than once?

(d) How many days must James, Robbie, and Ginny each pick from the bucket to guarantee that at least one of them has picked a piece of licorice at least once?

## Solution

(a) To determine the most likely treat to be picked, we first find the total number of each of the different types of treats.

They collected

- $6 + 5 + 2 = 13$ chocolate bars
- $4 + 5 + 8 = 17$ caramels
- $6 + 3 + 4 = 13$ suckers
- $5 + 7 + 6 = 18$ licorice

There are more pieces of licorice than any other type of treat, so licorice is the most likely treat to be selected.

(b) Using the calculations from part (a), we can determine that there are equal numbers chocolate bars and suckers. Therefore, chocolate bars and suckers are equally likely to be selected.

(c) The longest Robbie would need to wait before getting the type of treat more than once occurs when he picks different types of treats for as long as possible. Since there are four different types of treats, then let's assume Robbie picks one of each type during his first four selections.

Since he picks two treats each day, after two days Robbie will have picked four different types of treats. On Day 3, Robbie must pick a treat that has already been selected because there are no new types of treats remaining.

(d) The longest time it would take to have licorice to be selected would occur if James, Robbie, and Ginny selected all the other treats (chocolate bars, caramels, and suckers) before picking licorice. There are a total of

$$13 + 17 + 13 = 43$$

non-licorice treats.

Since each person picks 2 treats each day, a total of 6 treats are selected each day. Using skip counting,

$$6, 12, 18, 24, 30, 36, 42$$

we can determine that after 7 days, they have selected 42 treats. This means that on Day 8, they will select the 43rd treat. At this point, if no licorice has been selected, the only treats remaining must be licorice. Therefore, by Day 8, at least one of them is guaranteed to pick licorice.

## Teachers' Note

Someone could have picked licorice before Day 8. In fact, the chance of avoiding all of the licorice until the very end (without cheating) is extremely small. The probability that the first treat selected is not licorice is $\frac{43}{61}$, since there are 43 non-licorice treats and 61 treats altogether.

After that selection, there are 42 non-licorice treats and 60 treats remaining. The probability that the next treat selected is not licorice is $\frac{42}{60}$.

The probability that the third treat selected is not licorice, given that the first two treats were not licorice is $\frac{41}{59}$.

To calculate the probability that only non-licorice treats are selected before the first piece of licorice is picked, we use this pattern and combine the probabilities by multiplication like this:

$$\frac{43}{61} \times \frac{42}{60} \times \frac{41}{59} \times \cdots \times \frac{2}{20} \times \frac{1}{19} = \frac{1}{1\,312\,251\,244\,423\,350}$$

So the chance of this happening is approximately 1 in 1.312 quadrillion.