

Problem of the Week
Problem A and Solution
Party Pattern

Problem

Students at Dryden Elementary School are hanging decorations for an end of year celebration. They have a variety of decorations available in different shapes and colours. The shapes are: circles, stars, hearts, and diamonds. Each shape comes in three colours: red, blue, and yellow. The students create a fun pattern that repeats in both colour and shape.

- The colour pattern repeats in the order: red, blue, yellow.
- The shape pattern repeats in the order: circle, star, heart, diamond.

They hang decorations following both of these patterns.

(a) If they start with a red circle, what will be the colour and shape of the 10^{th} decoration?

(b) If the students hang 24 decorations, how many of each colour and each shape will they hang?

(c) How many of each colour and each shape will they need if they hang 240 decorations? Justify your answer.

Solution

(a) One way to keep track of the pattern is to make a table. The first row of the table shows the repeating pattern of the colours. The second row of the table shows the repeating pattern of the shapes. To keep things more compact, we use an uppercase letter to represent the colour (R-red, B-blue, Y-yellow), and a lowercase letter to represent the shape (c-circle, s-star, h-heart, d-diamond). The first 10 decorations will be:

Decoration Number	1	2	3	4	5	6	7	8	9	10
Colour	R	B	Y	R	B	Y	R	B	Y	R
Shape	c	s	h	d	c	s	h	d	c	s

From this table we can determine that the first decoration is a red circle, the second decoration is a blue star and the 10^{th} decoration is a red star.

(b) Since the pattern of colours and shapes repeats consistently, we know that there will be an equal number of each colour (as long as the total number decorations is a multiple of 3), and there will be an equal number of each shape (as long as the total number decorations is a multiple of 4).

Since 24 is a multiple of 3, we know $\frac{1}{3}$ of the decorations are red, $\frac{1}{3}$ are blue, and $\frac{1}{3}$ are yellow. Either by dividing $24 \div 3 = 8$ or separating 24 tokens into three equal groups, we can determine that there will be 8 of each colour decoration.

Similarly, since 24 is a multiple of 4, we know $\frac{1}{4}$ of the decorations are circles, $\frac{1}{4}$ of the decorations are stars, $\frac{1}{4}$ of the decorations are hearts, and $\frac{1}{4}$ of the decorations are diamonds. By dividing, we determine that $24 \div 4 = 6$ and so there will be 6 of each shape in the 24 decorations.

(c) We could do a similar calculation as above to figure out how many of each shape and colour there will be for 240 decorations. Another way to determine the result would be to recognize that $240 = 24 \times 10$. Since we know that there are 8 of each colour and 6 of each shape for 24 decorations, then there would be 10 times as many of each colour and shape for 240 decorations. This means there would be 80 of each colour and 60 of each shape.

NOTE: This strategy only works here because 24 is a multiple of both 3 and 4, so there were an equal number of each shape and each colour with 24 decorations.