

Problem of the Week

Problem A and Solution

Step It Up

Problem

Maya builds a triangular block tower using the following pattern.

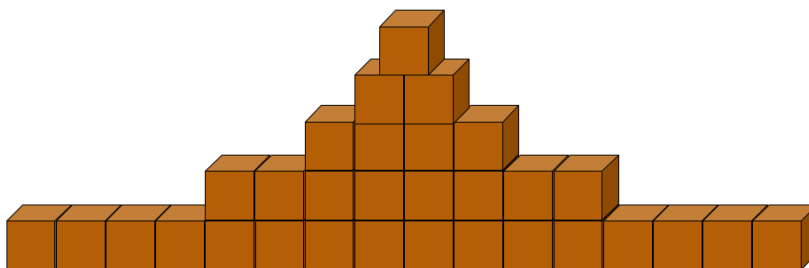
The top row has 1 block.

Every other row has double the number of blocks as the row above it.

- (a) If the tower has 5 rows, how many blocks does Maya use in total?
- (b) If you follow this pattern, how many rows do you need to use at least 200 blocks in total?

Solution

- (a) If the tower has 5 rows, we could just arrange blocks according to the pattern and count the total.



Counting the blocks gives us $1 + 2 + 4 + 8 + 16 = 31$ blocks in total.

- (b) Using this pattern we can make a table to keep track of the number of blocks in each row and the total number of blocks.

Row	Blocks in Row	Total Blocks
1	1	1
2	2	$1 + 2 = 3$
3	4	$3 + 4 = 7$
4	8	$7 + 8 = 15$
5	16	$15 + 16 = 31$
6	32	$31 + 32 = 63$
7	64	$63 + 64 = 127$
8	128	$127 + 128 = 255$

By the time we have 8 rows in our tower we will have used at least 200 blocks.



Teachers' Note

You might notice something about the total number of blocks used in this pattern. If you add 1 to the total in each row, you get a power of 2. For example, $1 + 1 = 2 = 2^1$, $3 + 1 = 4 = 2^2$, $7 + 1 = 8 = 2^3$, and so on. In general, if you add the consecutive powers of 2, starting at $2^0 = 1$, the sum is 1 less than the next power of 2. So $2^0 + 2^1 + 2^2 + 2^3 + 2^4 + 2^5 + 2^6 + 2^7 = 2^8 - 1 = 255$.

The general form of this relationship is: $2^0 + 2^1 + 2^2 + \cdots + 2^n = 2^{n+1} - 1$.