Problem of the Week

Problems and Solutions 2025-2026

Problem A

Grade 3/4

The CENTRE for EDUCATION
in MATHEMATICS and COMPUTING
cemc.uwaterloo.ca

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Problem of the Week

Problem A

Colouring Box

Mysha was playing with a blank 10 by 10 grid on white paper.
She wants to create a pattern that follows these rules:

> The top left square of the grid is square 1.
> Mysha counts along a row from left to right. From the end of one row, she moves to the leftmost square in the row below and continues counting.
>
> Starting with the 5$^{\text{th}}$ square on the top row, she adds red to every 5$^{\text{th}}$ square.
> Starting with the 7$^{\text{th}}$ square on the top row, she adds blue to every 7$^{\text{th}}$ square.
> Starting with the 9$^{\text{th}}$ square on the top row, she adds yellow every 9$^{\text{th}}$ square.

If a square is part of more than one pattern, then the colours will mix together to make a new colour, according to the following rules:

- Red mixed with blue will make purple.

  R + B = P

- Blue mixed with yellow will make green.

  B + Y = G

- Yellow mixed with red will make orange.

  Y + R = O

(a) How many squares of each colour will there be in the grid when Mysha finishes filling in the patterns?

(b) After finishing her colouring, how many squares of the grid are still white?

Theme: Algebra

Problem of the Week

Problem A and Solution

Colouring Box

Problem

Mysha was playing with a blank 10 by 10 grid on white paper.
She wants to create a pattern that follows these rules:

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(a) How many squares of each colour will there be in the grid when Mysha finishes filling in the patterns?

(b) After finishing her colouring, how many squares of the grid are still white?

Solution

The grid below shows the result of following the pattern rules:

				R		B		Y	R
			B	R			Y		R
B				R		Y	B		R
				P	Y				R
	B			O				B	R
			Y	R	B				R
		G		R					P
	Y			R		B			R
Y			B	R					O
B				R			B	Y	R

An interactive version of the pattern can be found here:
https://openprocessing.org/sketch/2537459

(a) From the completed grid, we count the number of each colour:
16 red, 11 blue, 8 yellow, 2 purple, 1 green, 2 orange

Another way to calculate this is to use skip counting to keep track of the squares that are filled with colour and look for overlapping numbers to see where the colours purple, green, and orange will appear. We cross out the duplicates from the primary colours (red, blue, and yellow) to get the final tally of each colour.

Red squares:
5, 10, 15, 20, 25, 30, ~~35~~, 40, ~~45~~, 50, 55, 60, 65, ~~70~~, 75, 80, 85, ~~90~~, 95, 100

Blue squares: 7, 14, 21, 28, ~~35~~, 42, 49, 56, ~~63~~, ~~70~~, 77, 84, 91, 98

Yellow squares: 9, 18, 27, 36, ~~45~~, 54, ~~63~~, 72, 81, ~~90~~, 99

Purple (red and blue) squares: 35, 70

Green (blue and yellow) squares: 63

Orange (yellow and red) squares: 45, 90

Another way to calculate these totals is to consider common multiples. To start, we can calculate that there are $100 \div 5 = 20$ squares that will be coloured in red, $100 \div 7 = 14$ (remainder 2) squares that will be coloured in blue, and $100 \div 9 = 11$ (remainder 1) squares that will be coloured in yellow. Next we consider the common multiples of 5 and 7 that are less than or equal to 100. Since $5 \times 7 = 35$, then we can skip count by 35 to find these numbers, 35 and 70, for a total of 2 purple squares. Similarly the common multiples of 5 and 9 that are less than 100 are 45 and 90, for a total of 2 orange squares. Finally, the only common multiple of 7 and 9 that is less than 100 is 63, for a total of 1 green square.

Again, we can eliminate the duplicates from the primary colours count.
We eliminate 4 from the number of red squares for a total of $20 - 4 = 16$.
We eliminate 3 from the number of blue squares for a total of $14 - 3 = 11$.
We eliminate 3 from the number of yellow squares for a total of $11 - 3 = 8$.
The lowest common multiple of 5, 7, and 9 is $5 \times 7 \times 9 = 315$, which is greater than 100. So none of the squares in the grid will be filled in with all three primary colours.

(b) Since the grid is 10 by 10, there are a total of $10 \times 10 = 100$ squares. In part (a), we found a total of $16 + 11 + 8 + 2 + 1 + 2 = 40$ squares filled with some colour. This means there is a total of $100 - 40 = 60$ white squares left in the grid.

Problem of the Week
Problem A
Gathering Flowers

Julian grows five different types of flowers in his garden: lilies, crocuses, tulips, daisies, and roses. He picks some flowers from his garden and makes the following observations about the flowers he picked:

There are four more daisies than lilies.
There are twice as many roses as crocuses.
There are half as many tulips as daisies.
There are two fewer lilies than roses.
There are five crocuses.

How many of each type of flower did Julian pick? Justify your answer.

Theme: Algebra

Problem of the Week
Problem A and Solution
Gathering Flowers

Problem

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> There are twice as many roses as crocuses.
> There are half as many tulips as daisies.
> There are two fewer lilies than roses.
> There are five crocuses.

How many of each type of flower did Julian pick? Justify your answer.

Solution

Since we know there are 5 crocuses and we know that there are twice as many roses as crocuses, there must be $5 \times 2 = 10$ roses.

Since we know there are 10 roses and we know that there are 2 fewer lilies than roses, there must be $10 - 2 = 8$ lilies.

Since we know there are 8 lilies and we know that there are 4 more daisies than lilies, there must be $8 + 4 = 12$ daisies.

Since we know there are 12 daisies and we know that there are half as many tulips as daisies, there must be $12 \div 2 = 6$ tulips.

In summary Julian picked:
5 crocuses, 10 roses, 8 lilies, 12 daisies, and 6 tulips.

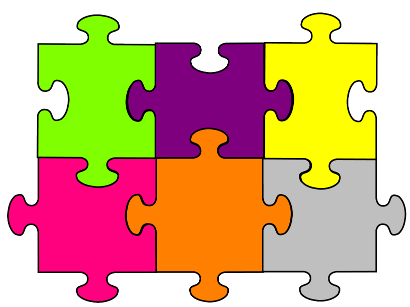
Problem of the Week
Problem A
Puzzling Predicament

Pakiza gets a 100 piece puzzle as a gift. When the puzzle is finished, the pieces form a grid, where each row and each column of the grid has the same number of pieces.

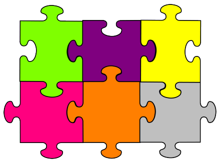
(a) How many pieces are in each row and each column of the puzzle?

(b) On Monday, Pakiza gathers all the edge and corner pieces and puts them together to form the outside edge of the puzzle. How many edge and corner pieces does the puzzle have in total?

(c) Pakiza continues to work on the puzzle a little bit every day. She places 10 pieces in the puzzle on Tuesday. On each day after that, she places one more puzzle piece than the day before, until the puzzle is complete. So, she places 11 pieces on Wednesday, 12 pieces on Thursday, and so on until all the pieces are in place. On what day of the week does Pakiza complete the puzzle? Justify your answer.



Themes: Algebra, Geometry & Measurement



Problem of the Week
Problem A and Solution
Puzzling Predicament

Problem

Pakiza gets a 100 piece puzzle as a gift. When the puzzle is finished, the pieces form a grid, where each row and each column of the grid has the same number of pieces.

(a) How many pieces are in each row and each column of the puzzle?

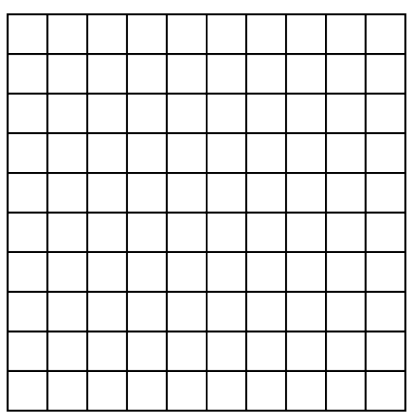
(b) On Monday, Pakiza gathers all the edge and corner pieces and puts them together to form the outside edge of the puzzle. How many edge and corner pieces does the puzzle have in total?

(c) Pakiza continues to work on the puzzle a little bit every day. She places 10 pieces in the puzzle on Tuesday. On each day after that, she places one more puzzle piece than the day before, until the puzzle is complete. So, she places 11 pieces on Wednesday, 12 pieces on Thursday, and so on until all the pieces are in place. On what day of the week does Pakiza complete the puzzle? Justify your answer.

Solution

(a) We know that each row and each column of the puzzle have the same number of pieces, and there are 100 pieces in total. Since $10 \times 10 = 100$, then each row and each column of the puzzle must contain 10 pieces.

(b) It might help to look at a 10 by 10 grid in order to determine the total number of edge and corner pieces.



There are 4 corner pieces. Not counting the corners, there are 8 edge pieces on each side of the puzzle. So the total number of edge and corner pieces is: $4 + 8 + 8 + 8 + 8 = 36$.

Note: This is a classic example of the *fencepost problem*. If we are not careful, we might think that a 10×10 grid has $4 \times 10 = 40$ squares around its edge. However, this counts each corner piece twice, since a corner piece is part of two sides of the puzzle. The fencepost problem describes a situation where you may be off-by-one when counting. It can cause problems when writing computer programs that include some repetition.

(c) We can make a table to keep track of how many pieces are filled in the puzzle over time. The first entry in the table will be on Monday when Pakiza assembles the 36 pieces to form the outside edge of the puzzle.

Day	**Number of Pieces Added**	**Total Pieces**
Monday	36	36
Tuesday	10	46
Wednesday	11	57
Thursday	12	69
Friday	13	82
Saturday	14	96

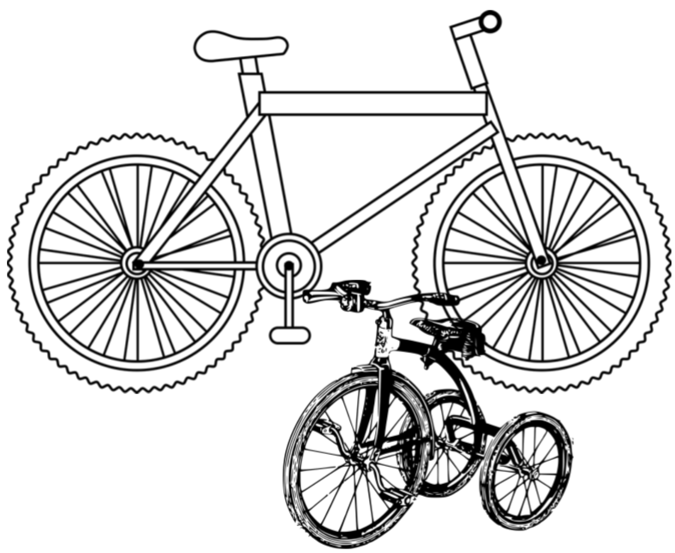
At this point, there are only 4 pieces left in the puzzle. So, we know Pakiza will complete the puzzle on the next day, which is Sunday.

Problem of the Week
Problem A
Storage Unit

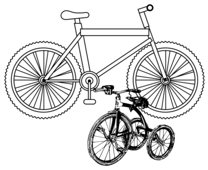
Jerry bought the contents of two large storage units at an auction for a total of \$500. One storage unit contains some bicycles and the other storage unit contains some tricycles.

(a) If each storage unit has exactly 72 wheels, how many bicycles and tricycles are there in total? Justify your answer.

(b) Jerry plans to sell each tricycle for \$10 and each bicycle for \$25. What is the maximum profit Jerry can make if he is able to sell all the bicycles and tricycles in storage for these prices? Justify your answer.



Themes: Algebra, Number Sense



Problem of the Week
Problem A and Solution
Storage Unit

Problem

Jerry bought the contents of two large storage units at an auction for a total of \$500. One storage unit contains some bicycles and the other storage unit contains some tricycles.

(a) If each storage unit has exactly 72 wheels, how many bicycles and tricycles are there in total? Justify your answer.

(b) Jerry plans to sell each tricycle for \$10 and each bicycle for \$25. What is the maximum profit Jerry can make if he is able to sell all the bicycles and tricycles in storage for these prices? Justify your answer.

Solution

(a) Since a tricycle has 3 wheels and there are 72 wheels in one of the storage units, then we can determine the number of tricycles by dividing 72 by 3. There are $72 \div 3 = 24$ tricycles in one of the storage units.

Similarly, since a bicycle has 2 wheels and there are 72 wheels in one of the storage units, then we can determine the number of bicycles by dividing 72 by 2. There are $72 \div 2 = 36$ bicycles in the other storage unit.

(b) To calculate the amount of money Jerry can make from selling the tricycles, we multiply $24 \times \$10 = \$240$.

To calculate the amount of money Jerry can make from selling the bicycles, we multiply $36 \times \$25 = \$900$. Alternatively, we can skip count by 25. A third way to figure out the amount of money is to notice that if he sells 4 bicycles, he would make \$100. Then we can use a table to figure out how much he would make selling 36 bicycles:

Bikes Sold	4	8	12	16	20	24	28	32	36
Money Made	\$100	\$200	\$300	\$400	\$500	\$600	\$700	\$800	\$900

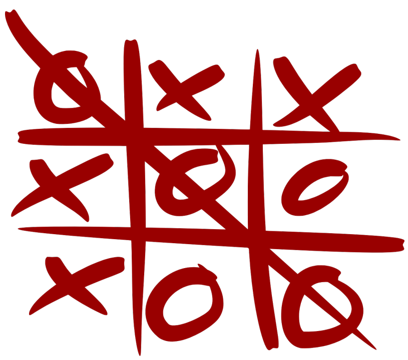
Therefore, Jerry could make $\$240 + \$900 = \$1140$ if he sells all of the bikes. We need to subtract the amount he paid for the storage unit to determine the overall profit. Therefore, the maximum profit Jerry could make by selling all the bikes is $\$1140 - \$500 = \$640$.

Problem of the Week
Problem A
Team Tournament

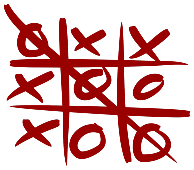
Ms. McNeil organizes a bean bag tic-tac-toe tournament for her class. She divides her class into 6 teams. When one team plays another team during the tournament, this is called a match.

(a) In the tournament, each team has to play every other team once. How many matches will each team play during the tournament? Justify your answer.

(b) How many matches are played in total during the tournament? Justify your answer.



Themes: Algebra, Data Management



Problem of the Week
Problem A and Solution
Team Tournament

Problem

Ms. McNeil organizes a bean bag tic-tac-toe tournament for her class. She divides her class into 6 teams. When one team plays another team during the tournament, this is called a match.

(a) In the tournament, each team has to play every other team once. How many matches will each team play during the tournament? Justify your answer.

(b) How many matches are played in total during the tournament? Justify your answer.

Solution

(a) Since each team has to play every other team once during the tournament, each team must play the other 5 teams. In other words, each team has 5 matches.

(b) We will call the teams Team 1, Team 2, Team 3, Team 4, Team 5, and Team 6. One way to solve this problem is to write out all the matches, being careful to make sure we do not include duplicate matches. For example, when Team 1 plays Team 2, that match is the same as saying Team 2 plays Team 1.

The possible matches are:

- Team 1 and Team 2, Team 1 and Team 3, Team 1 and Team 4, Team 1 and Team 5, Team 1 and Team 6
- Team 2 and Team 3, Team 2 and Team 4, Team 2 and Team 5, Team 2 and Team 6
- Team 3 and Team 4, Team 3 and Team 5, Team 3 and Team 6
- Team 4 and Team 5, Team 4 and Team 6
- Team 5 and Team 6

Therefore, there are 15 matches in total in the tournament.

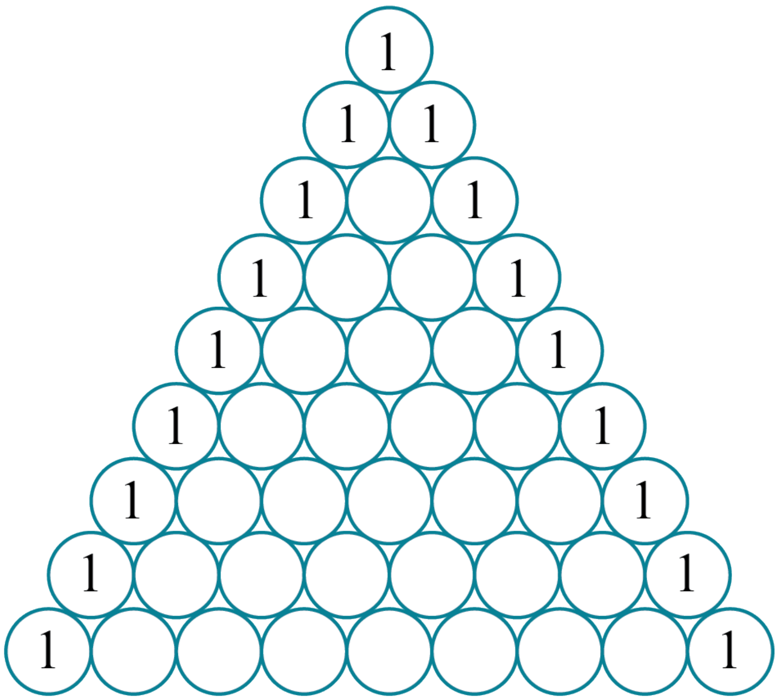
Alternatively, since each team has to play 5 matches and there are 6 teams, we can count the total number of matches in the tournament as $5+5+5+5+5+5 = 5 \times 6 = 30$. However, this calculation actually double counts matches. For example, it counts the match between Team 1 and Team 2 as two separate matches (one match from the point of view of Team 1 and one match from the point of view of Team 2). This is true for every match between two teams. So, to get the true count of the total number of matches in the tournament, we need to divide by 2 to get $30 \div 2 = 15$ matches.

Problem of the Week

Problem A

Pencil Pyramid

Juliane arranges identical cylindrical tubes to form a pyramid. She puts one pencil in the top tube and then puts one pencil in the leftmost and rightmost tubes in each of the remaining rows, as shown.



Juliane then fills the remaining tubes as follows:

Step 1: Find the topmost empty tube and add up the number of pencils in the two tubes touching it in the row above.

Step 2: Put that many pencils in the empty tube.

Step 3: Repeat Steps 1 and 2 until all tubes are filled.

(a) Complete the pyramid with the number of pencils in each tube.

(b) Can you find any patterns in the numbers in the completed pyramid? If so, explain them.

Themes: Algebra, Number Sense

Problem of the Week
Problem A and Solution
Pencil Pyramid

Problem

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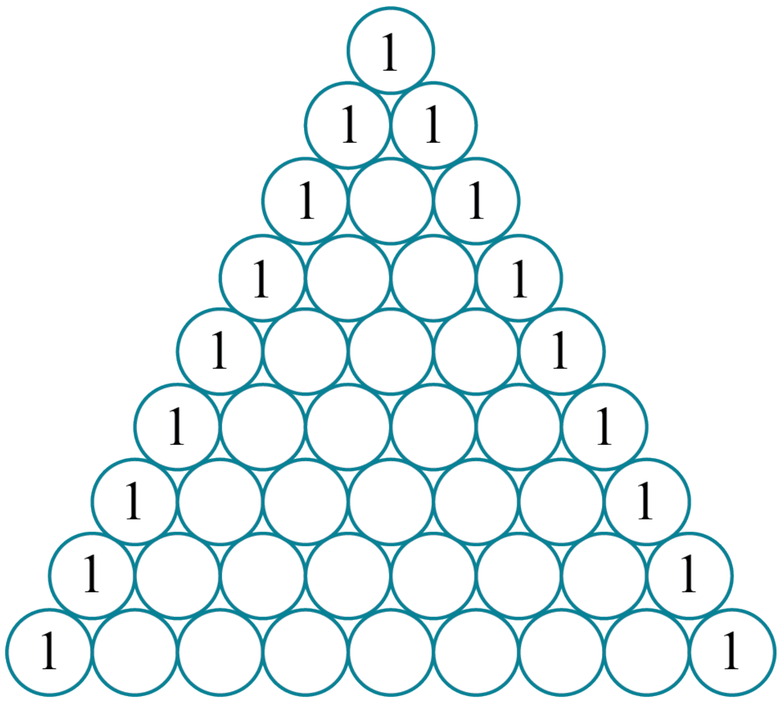

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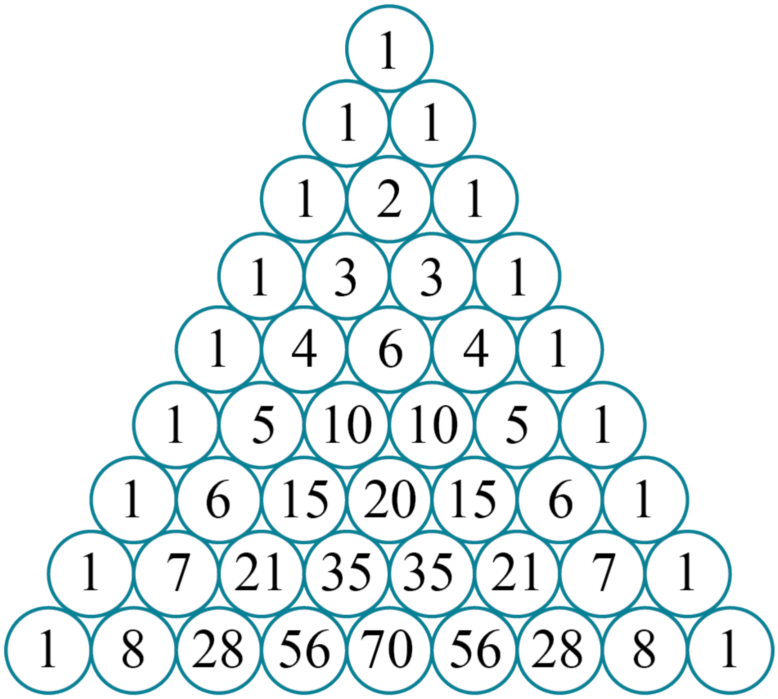
Step 3: Repeat Steps 1 and 2 until all tubes are filled.

(a) Complete the pyramid with the number of pencils in each tube.

(b) Can you find any patterns in the numbers in the completed pyramid? If so, explain them.

Solution

(a) The completed pyramid is shown.



(b) Here are some patterns you may have noticed:

- The triangle is symmetric, so the left side matches the right side.
- Starting with the leftmost 1 in the second row and moving down diagonally we see the numbers 1, 2, 3, 4, 5, 6, 7, and 8.
- Starting with the leftmost 1 in the third row and moving down diagonally we see the numbers 1, 3, 6, 10, 15, 21, and 28. The pattern rule for this sequence is to add 2, then 3, then 4, then 5, etc.
- The sum of the numbers in any row is twice the sum of the numbers in the row above it.
- If you start from any 1, follow the diagonal down, and then change directions for the last number, the sum of the numbers in the diagonal will equal the last number. For example, $1 + 6 + 21 = 28$. This is called the *Hockey Stick Identity* because if you circle the numbers used, it will look somewhat like a hockey stick.

Teacher's Notes

This problem is an exploration of *Pascal's Triangle*, which is named for French mathematician Blaise Pascal (1623 - 1662). There are lots of patterns to be found in this triangle. We listed some of the patterns in the solution, but perhaps your students were able to find others!

Pascal's Triangle is also used in algebra. The numbers in the n^{th} row provide the coefficients for the expansion of a *binomial expression* raised to the n^{th} power. The expansions for $n = 0$ to $n = 4$ are shown.

$$
\begin{aligned}
(x+y)^0 &= 1\\
(x+y)^1 &= 1x + 1y\\
(x+y)^2 &= 1x^2 + 2xy + 1y^2\\
(x+y)^3 &= 1x^3 + 3x^2y + 3xy^2 + 1y^3\\
(x+y)^4 &= 1x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + 1y^4
\end{aligned}
$$

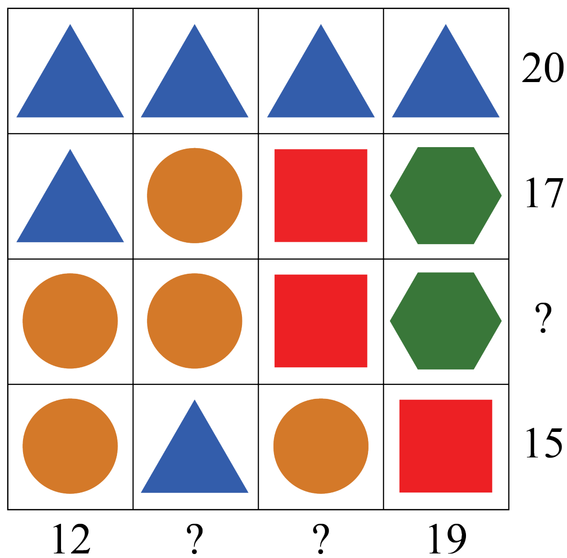
As the value of n increases, expanding the binomials manually becomes quite time consuming. Using Pascal's Triangle allows us to expand binomial expressions more efficiently.

Problem of the Week

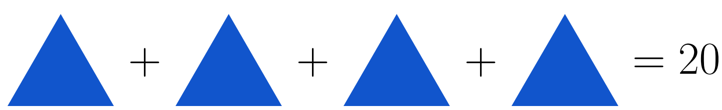
Problem A

Shape Sums

Each symbol in the grid below represents a number. The sums of the symbols in the first, second, and bottom rows and the leftmost and rightmost columns are shown.



For example, we know:



Determine all the missing sums.

Themes: Algebra, Number Sense

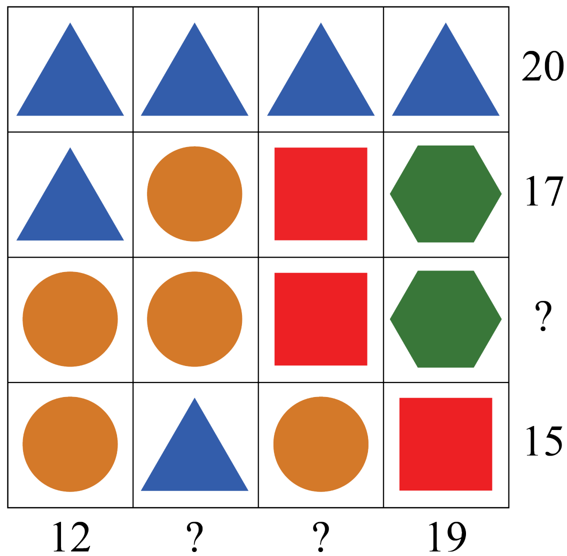
Problem of the Week

Problem A and Solution

Shape Sums

Problem

Each symbol in the grid below represents a number. The sums of the symbols in the first, second, and bottom rows and the leftmost and rightmost columns are shown.



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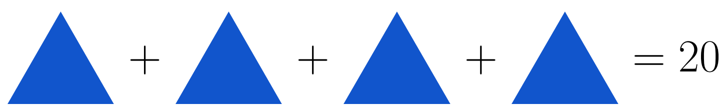


Determine all the missing sums.

Solution

There are many ways to solve this problem. Here is one approach.

Since the leftmost column has two triangles and two circles, and the column beside it also has two triangles and two circles, the sum of these columns must be the same. So the sum of the second column must be 12.

From the top row we know that the sum of four triangles is 20. So the value of one triangle must be $\frac{1}{4}$ of 20 or $20 \div 4 = 5$.

Since the value of one triangle is 5, then the sum of two triangles is $5 + 5 = 10$. The leftmost column shows the sum of two triangles and two circles is 12, so the sum of two circles is $12 - 10 = 2$. Therefore, the value of one circle must be $\frac{1}{2}$ of 2 or $2 \div 2 = 1$.

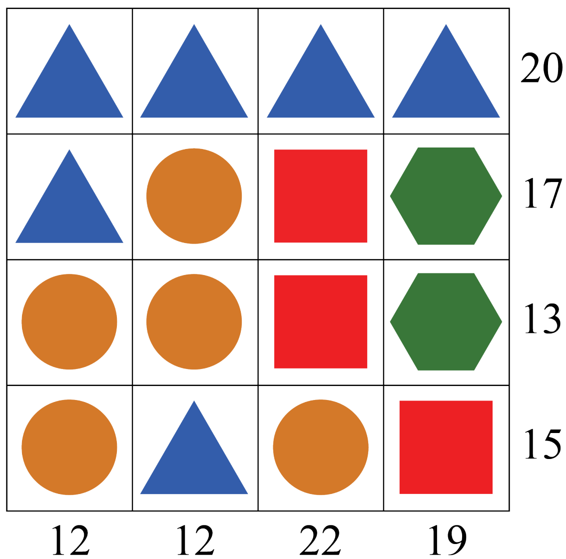
The bottom row has a sum of 15 and contains two circles, one triangle, and one square. Since we know the sum of two circles and one triangle is $2 + 5 = 7$, then the value of one square is $15 - 7 = 8$.

Now we know that the sum of the third column is $5 + 8 + 8 + 1 = 22$.

The second row has a sum of 17 and contains one triangle, one circle, one square and one hexagon. Since we know the sum of one triangle, one circle, and one square is $5 + 1 + 8 = 14$, then the value of one hexagon is $17 - 14 = 3$.

Now we know that the sum of the third row is $1 + 1 + 8 + 3 = 13$.

The completed grid is shown.



Alternatively, we can find the sum of the third row without actually finding the value of the hexagon. Notice that the second and third rows have the same shapes in the last three columns, and only the shape in the first column is different. Since we know that the value of one triangle is 5 and the value of one circle is 1, then the sum of the third row must be $5 - 1 = 4$ less than the sum of the second row. Thus, the sum of the third row is $17 - 4 = 13$.

Teacher's Notes

We could look at this problem algebraically, where we are trying to solve a *linear system of equations*. If we assign variable names to the shapes,

- t = triangle
- c = circle
- s = square
- h = hexagon

then we can describe the information given in the problem as follows:

$$\begin{aligned} 4t &= 20 \\ t + c + s + h &= 17 \\ 2c + t + s &= 15 \\ 2t + 2c &= 12 \\ t + 2h + s &= 19 \end{aligned}$$

Now, we could use algebraic techniques to find the values of these four variables.

We expect that there is a single solution to these types of problems, and it turns out that there is for this problem. However, for some problems we are not given enough information to determine specific values for each variable. For a problem with four variables, a minimum requirement to guarantee a single solution is that we need *at least* four equations. Also, those equations must be *consistent*. This means that none of the equations can contradict each other. For example, if we added the equation $t = 7$ to the system in this problem, this would contradict the first equation. A system that is *inconsistent* has no solutions. This is something that students will learn more about in higher level math courses.

Problem of the Week
Problem A
Cribbage Scoring

Cribbage is a card game that is played with a standard deck of 52 cards. There are four suits: clubs ♣, diamonds ♢, hearts ♡, and spades ♠. Each suit has 13 cards identified with either a letter or number as follows: A, 2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q, K. These letters and numbers are called the card's *rank*. Since there are four suits, there are four cards with the same rank in every deck. *Note: In this activity we will ignore the suits and just focus on the ranks.*

To determine the number of points a hand is worth, each player makes combinations using 2, 3, 4, or 5 of their cards. Cards can be used more than once in different combinations. These are the combinations that count for points:

- Each combination that totals 15 is worth 2 points. When making these combinations, an A is worth 1, a J, Q, or K is worth 10, and all number cards are worth their rank. For example:

  $6 + 9 = 15$ is worth 2 points

  $5 + Q = 15$ is worth 2 points

  $2 + 4 + 9 = 15$ is worth 2 points

- Each pair of cards that have the same rank is worth 2 points.

- Each combination of three or more cards that form a sequence of consecutive ranked cards is worth the number of cards in the sequence. For example:

  3, 4, 5 is worth 3 points

  8, 9, 10, J is worth 4 points

Which of the following hands is worth the most points?

Hand A: 5♡, 10♢, J♠, Q♣, and K♠

Hand B: A♠, 6♣, 7♢, 7♡, and 8♣

Hand C: 5♣, 5♡, 5♠, 8♠, and Q♢

Justify your answer.

Themes: Algebra, Number Sense

Problem of the Week
Problem A and Solution
Cribbage Scoring

Problem

Cribbage is a card game that is played with a standard deck of 52 cards. There are four suits: clubs ♣, diamonds ♢, hearts ♡, and spades ♠. Each suit has 13 cards identified with either a letter or number as follows: A, 2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q, K. These letters and numbers are called the card's *rank*. Since there are four suits, there are four cards with the same rank in every deck. *Note: In this activity we will ignore the suits and just focus on the ranks.*

To determine the number of points a hand is worth, each player makes combinations using 2, 3, 4, or 5 of their cards. Cards can be used more than once in different combinations. These are the combinations that count for points:

- Each combination that totals 15 is worth 2 points. When making these combinations, an A is worth 1, a J, Q, or K is worth 10, and all number cards are worth their rank. For example:

  $6 + 9 = 15$ is worth 2 points

  $5 + \text{Q} = 15$ is worth 2 points

  $2 + 4 + 9 = 15$ is worth 2 points

- Each pair of cards that have the same rank is worth 2 points.

- Each combination of three or more cards that form a sequence of consecutive ranked cards is worth the number of cards in the sequence. For example:

  3, 4, 5 is worth 3 points

  8, 9, 10, J is worth 4 points

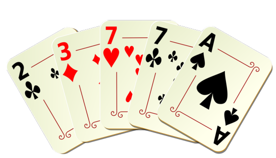
Which of the following hands is worth the most points?

Hand A: 5♡, 10♢, J♠, Q♣, and K♠

Hand B: A♠, 6♣, 7♢, 7♡, and 8♣

Hand C: 5♣, 5♡, 5♠, 8♠, and Q♢

Justify your answer.



Solution
We record the combinations in each hand that earn points in the following tables.

Hand A:

Type of Combination	**Combinations**	**Total Points**
Cards that total 15	$5\heartsuit + 10\diamondsuit$, $5\heartsuit + J\spadesuit$, $5\heartsuit + Q\clubsuit$, $5\heartsuit + K\spadesuit$	$2+2+2+2=8$
Pairs of cards with the same rank	none	0
Sequences of consecutive ranked cards	$10\diamondsuit$, $J\spadesuit$, $Q\clubsuit$, $K\spadesuit$	4

The total points in this hand is $8+4=12$.

Hand B:

Type of Combination	**Combinations**	**Total Points**
Cards that total 15	$7\diamondsuit + 8\clubsuit$, $7\heartsuit + 8\clubsuit$, $A\spadesuit + 6\clubsuit + 8\clubsuit$, $A\spadesuit + 7\diamondsuit + 7\heartsuit$	$2+2+2+2=8$
Pairs of cards with the same rank	$7\diamondsuit$ and $7\heartsuit$	2
Sequences of consecutive ranked cards	$6\clubsuit$, $7\diamondsuit$, $8\clubsuit$ $6\clubsuit$, $7\heartsuit$, $8\clubsuit$	$3+3=6$

The total points in this hand is $8+2+6=16$.

Hand C:

Type of Combination	**Combinations**	**Total Points**
Cards that total 15	$5\clubsuit + Q\diamondsuit$, $5\heartsuit + Q\diamondsuit$, $5\spadesuit + Q\diamondsuit$, $5\clubsuit + 5\heartsuit + 5\spadesuit$	$2+2+2+2=8$
Pairs of cards with the same rank	$5\clubsuit$ and $5\heartsuit$, $5\clubsuit$ and $5\spadesuit$, $5\heartsuit$ and $5\spadesuit$	$2+2+2=6$
Sequences of consecutive ranked cards	none	0

The total points in this hand is $8+6=14$.

Therefore, **Hand B** is worth the most points.

Computational Thinking (C)

Take me to the cover

Problem of the Week

Problem A

What Number Am I?

I am a 4-digit number. Use the following clues to determine my digits:

(1) My hundreds digit is the difference between the number of sides of a hexagon and the number of sides of a triangle.

(2) Reading my digits from left to right, they are in increasing order from the thousands digit to the ones digit. Also, they increase by the same amount each time.

(3) The difference between my tens digit and my ones digit is equal to the number of sides of an octagon divided by the number of right angles in a rectangle.

What number am I?

Themes: Computational Thinking, Geometry & Measurement, Number Sense

Problem of the Week

Problem A and Solution

What Number Am I?

Problem

I am a 4-digit number. Use the following clues to determine my digits:

(1) My hundreds digit is the difference between the number of sides of a hexagon and the number of sides of a triangle.

(2) Reading my digits from left to right, they are in increasing order from the thousands digit to the ones digit. Also, they increase by the same amount each time.

(3) The difference between my tens digit and my ones digit is equal to the number of sides of an octagon divided by the number of right angles in a rectangle.

What number am I?

Solution

Using Clue (1), the number of sides of a hexagon is 6 and the number of sides of a triangle is 3. The difference between these two numbers is $6 - 3 = 3$, so the hundreds digit is 3.

Using Clue (3), the number of sides of an octagon is 8 and the number of right angles in a rectangle is 4. So, the difference between the tens digit and the ones digit is $8 \div 4 = 2$.

Using Clue (2), since the digits increase by the same amount each time, then as we read the digits from left to right the numbers will increase by 2. Since the hundreds digit is 3, then the tens digit must be $3 + 2 = 5$. Since the tens digit is 5, then the ones digit must be $5 + 2 = 7$. Working backwards, since the hundreds digit is 3, then the thousands digit must be $3 - 2 = 1$.

Therefore the number is 1357.

Problem of the Week

Problem A

Follow the Rules

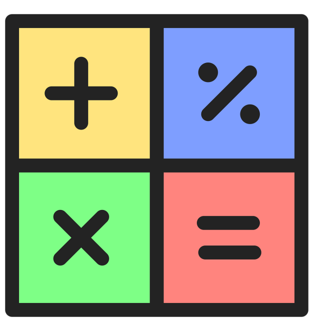
Consider the following step-by-step instructions:

1. Pick a 2-digit number.
2. If the number is even, divide it by 2. If the number is odd, add 10 to the number.
3. If the result of the calculation in step 2 is a 2-digit number, add its tens digit to its ones digit. Otherwise, add 20 to the number calculated in step 2.
4. If the result of the calculation in step 3 is a multiple of 5, divide it by 5. Otherwise, add 12 to the number calculated in step 3.
5. If the result of the calculation in step 4 is greater than 30, add 1 to it. Otherwise, subtract 1 from the number calculated in step 4.

(a) What is the result of following these instructions if you start with 25?

(b) What is the result of following these instructions if you start with 95?

(c) What is the result of following these instructions if you start with 16?



Themes: Computational Thinking, Number Sense

Problem of the Week
Problem A and Solution
Follow the Rules

Problem

Consider the following step-by-step instructions:

1. Pick a 2-digit number.
2. If the number is even, divide it by 2. If the number is odd, add 10 to the number.
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4. If the result of the calculation in step 3 is a multiple of 5, divide it by 5. Otherwise, add 12 to the number calculated in step 3.
5. If the result of the calculation in step 4 is greater than 30, add 1 to it. Otherwise, subtract 1 from the number calculated in step 4.

(a) What is the result of following these instructions if you start with 25?

(b) What is the result of following these instructions if you start with 95?

(c) What is the result of following these instructions if you start with 16?

Solution

(a) Following the steps and starting with 25 produces this sequence of numbers:

$$25, 35, 8, 20, 19$$

(b) Following the steps and starting with 95 produces this sequence of numbers:

$$95, 105, 125, 25, 24$$

(c) Following the steps and starting with 16 produces this sequence of numbers:

$$16, 8, 28, 40, 41$$

Problem of the Week
Problem A
Secret Message

Alicia and Bao want to send messages to each other without other people being able to read them. They set up a code that uses just 0s and 1s in the message. To decode the message it takes two steps:

1. Read the message, 4 digits at a time. Then look up each 4-digit code in the tables below and replace it with the given lowercase letter.

Code	0000	0001	0010	0011	0100	0101	0110	0111
Letter	a	b	c	d	e	f	g	h

Code	1000	1001	1010	1011	1100	1101	1110	1111
Letter	i	j	k	l	m	n	o	p

2. Read the new message, two consecutive letters at a time. Then look up each pair of lowercase letters in the tables below and replace them with the given uppercase letter. The word formed by these uppercase letters is the secret message.

Lowercase Letters	eb	ec	ed	ee	ef	eg	eh	ei	ij	ek	el	em	en
Uppercase Letter	A	B	C	D	E	F	G	H	I	J	K	L	M

Lowercase Letters	eo	ep	fa	fb	fc	fd	fe	ff	fg	fh	fi	fj	fk
Uppercase Letter	N	O	P	Q	R	S	T	U	V	W	X	Y	Z

For example, if Alicia sent the code 0100111101001011, after the first step the message becomes epel, and after the second step Bao knows the message is OK.

Decode the following message:

0101001001000101010100110101000001000101010000110101010100

Try writing and sending secret messages of your own.

Theme: Computational Thinking

Problem of the Week
Problem A and Solution
Secret Message

Problem

Alicia and Bao want to send messages to each other without other people being able to read them. They set up a code that uses just `0`s and `1`s in the message. To decode the message it takes two steps:

1. Read the message, 4 digits at a time. Then look up each 4-digit code in the tables below and replace it with the given lowercase letter.

Code	0000	0001	0010	0011	0100	0101	0110	0111
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Code	1000	1001	1010	1011	1100	1101	1110	1111
Letter	i	j	k	l	m	n	o	p

2. Read the new message, two consecutive letters at a time. Then look up each pair of lowercase letters in the tables below and replace them with the given uppercase letter. The word formed by these uppercase letters is the secret message.

Lowercase Letters	eb	ec	ed	ee	ef	eg	eh	ei	ij	ek	el	em	en
Uppercase Letter	A	B	C	D	E	F	G	H	I	J	K	L	M

Lowercase Letters	eo	ep	fa	fb	fc	fd	fe	ff	fg	fh	fi	fj	fk
Uppercase Letter	N	O	P	Q	R	S	T	U	V	W	X	Y	Z

For example, if Alicia sent the code `0100111101001011`, after the first step the message becomes `epel`, and after the second step Bao knows the message is `OK`.

Decode the following message:

`0101001001000101010100110101000001000101010000110101 0100`

Try writing and sending secret messages of your own.

Solution

The first step in decoding the message is to convert each group of 4 digits into a single lowercase letter. This is shown in the following tables.

Code	0101	0010	0100	0101	0101	0011	0101	0000	0100	0101
Letter	f	c	e	f	f	d	f	a	e	f

Code	0100	0011	0101	0100
Letter	e	d	f	e

Next we take each pair of lowercase letters and decode them into a single uppercase letter, as shown below.

Lowercase Letters	fc	ef	fd	fa	ef	ed	fe
Uppercase Letter	R	E	S	P	E	C	T

So the secret message is `RESPECT`.

Teacher's Note

This process of decoding a sequence of `0`s and `1`s is actually very close to the way computers store and interpret simple text. Each character on an English language keyboard has a numerical value associated with it known as its *ASCII code*. That ASCII code is represented by an 8-bit code, where a *bit* is either a `0` or `1`. Inside a computer, simple text data is stored as a sequence of `0`s and `1`s.

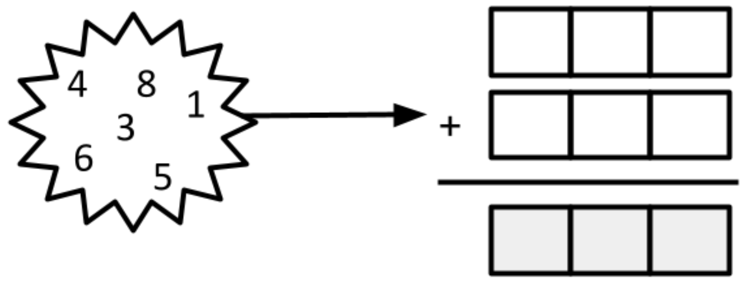
Another way an ASCII code is represented is by its 2-digit hexadecimal code. Each group of four bits is equivalent to a single digit in the hexadecimal number system. There are 16 digits in the hexadecimal number system (note that there are 10 digits in our decimal number system). The only difference between how ASCII codes work and this problem is that the hexadecimal digit values are: `0, 1, 2, 3, 4, 5, 6, 7, 8, 9, a, b, c, d, e, f`. This problem uses lowercase letters to represent the hexadecimal digits to avoid confusion with the bits of the secret code and actual hexadecimal digits.

Problem of the Week

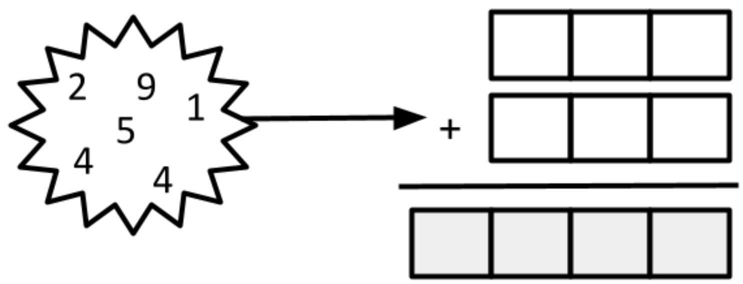
Problem A

Just Filling In

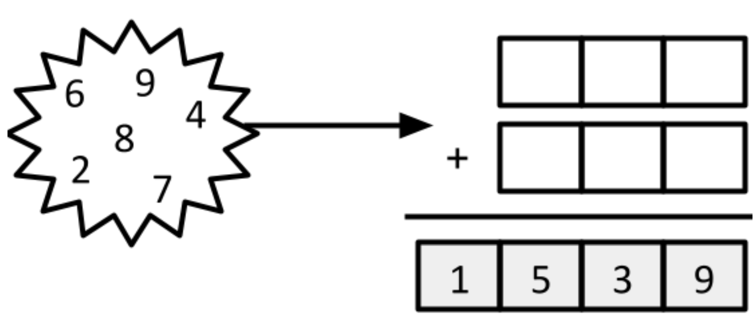
(a) Arrange the numbers 4, 8, 1, 6, 3, and 5 to form two 3-digit numbers with the *smallest* possible sum.



(b) Arrange the numbers 2, 9, 1, 4, 5, and 4 to form two 3-digit numbers with the *largest* possible sum.



(c) Arrange the numbers 6, 9, 4, 2, 8, and 7 to form two 3-digit numbers that add to 1539.



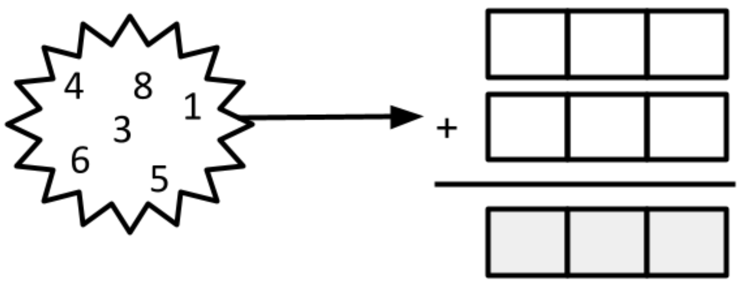
For each part, how many different solutions can you find?

Themes: Computational Thinking, Number Sense

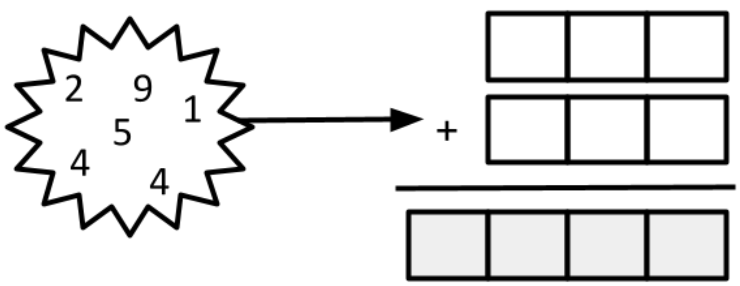
Problem of the Week
Problem A and Solution
Just Filling In

Problem

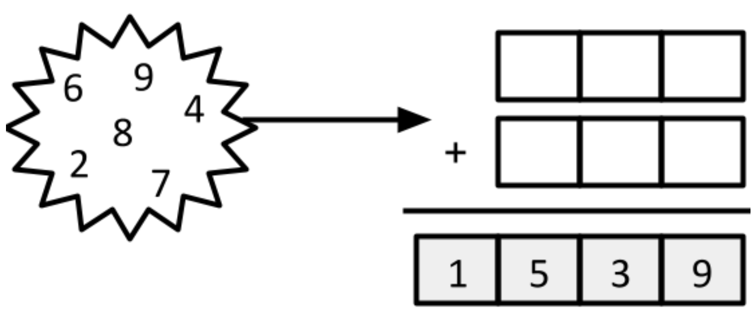
(a) Arrange the numbers 4, 8, 1, 6, 3, and 5 to form two 3-digit numbers with the *smallest* possible sum.



(b) Arrange the numbers 2, 9, 1, 4, 5, and 4 to form two 3-digit numbers with the *largest* possible sum.



(c) Arrange the numbers 6, 9, 4, 2, 8, and 7 to form two 3-digit numbers that add to 1539.



For each part, how many different solutions can you find?

Solution

(a) The strategy for creating the smallest (or largest) sum comes from recognizing that the hundreds digit has the most influence on the size of a number, and the ones digit has the least influence on the size of a number.

When we are choosing the digits for the smallest sum, we want the smallest numbers in the hundreds column, the next smallest numbers in the tens column, and the largest numbers in the ones column. So, in our answer we want 1 and 3 in the hundreds column, 4 and 5 in the tens column, and 6 and 8 in the ones column.

When adding two numbers, the order of the numbers does not affect the sum. Thus, the digits in each column can be arranged in 2 ways: with either the larger digit on top or the smaller digit on top. Since there are three columns in the sum, there are a total of $2 \times 2 \times 2 = 8$ solutions. The 8 solutions are shown.

$$\begin{array}{r}146\\+358\\\hline\end{array}\quad\begin{array}{r}148\\+356\\\hline\end{array}\quad\begin{array}{r}156\\+348\\\hline\end{array}\quad\begin{array}{r}158\\+346\\\hline\end{array}\quad\begin{array}{r}346\\+158\\\hline\end{array}\quad\begin{array}{r}348\\+156\\\hline\end{array}\quad\begin{array}{r}356\\+148\\\hline\end{array}\quad\begin{array}{r}358\\+146\\\hline\end{array}$$

In all arrangements, the sum is 504.

(b) Using the strategy from part (a), we can arrange the numbers to find the largest possible sum. This time, we want the largest numbers, 5 and 9, in the hundreds columns and the smallest numbers, 1 and 2 in the ones columns. The remaining numbers are the same digit 4 and 4.

When adding two numbers, the order of the numbers does not affect the sum. Thus, the digits in the hundreds and ones columns can each be arranged in 2 ways: with either the larger digit on top or the smaller digit on top. Since the digits in the tens column are the same, they can be arranged in only 1 way. Therefore, there are $2 \times 2 \times 1 = 4$ possible solutions in this case. The 4 solutions are shown.

$$\begin{array}{r}541\\+942\\\hline\end{array}\quad\begin{array}{r}542\\+941\\\hline\end{array}\quad\begin{array}{r}941\\+542\\\hline\end{array}\quad\begin{array}{r}942\\+541\\\hline\end{array}$$

In all arrangements, the sum is 1483.

(c) We need some logic to narrow down the choices in this example. Since we know the ones digit of the sum is 9, then the ones digits for each of the two numbers being added must have a total of 9 or 19. It is not possible to add two single digits to get 19, so the sum must be 9. The only possible combinations given the digits we have are $2 + 7$ or $7 + 2$.

In the tens column, we need two numbers that add to 3 or 13. The sum of $6 + 7 = 13$, but we have already used 7 in the ones column. There is only one pair of the remaining digits we could use: $4 + 9 = 13$ or $9 + 4 = 13$. This means we carry 1 into the hundreds column, so the sum of the two digits in the hundreds column must be 14.

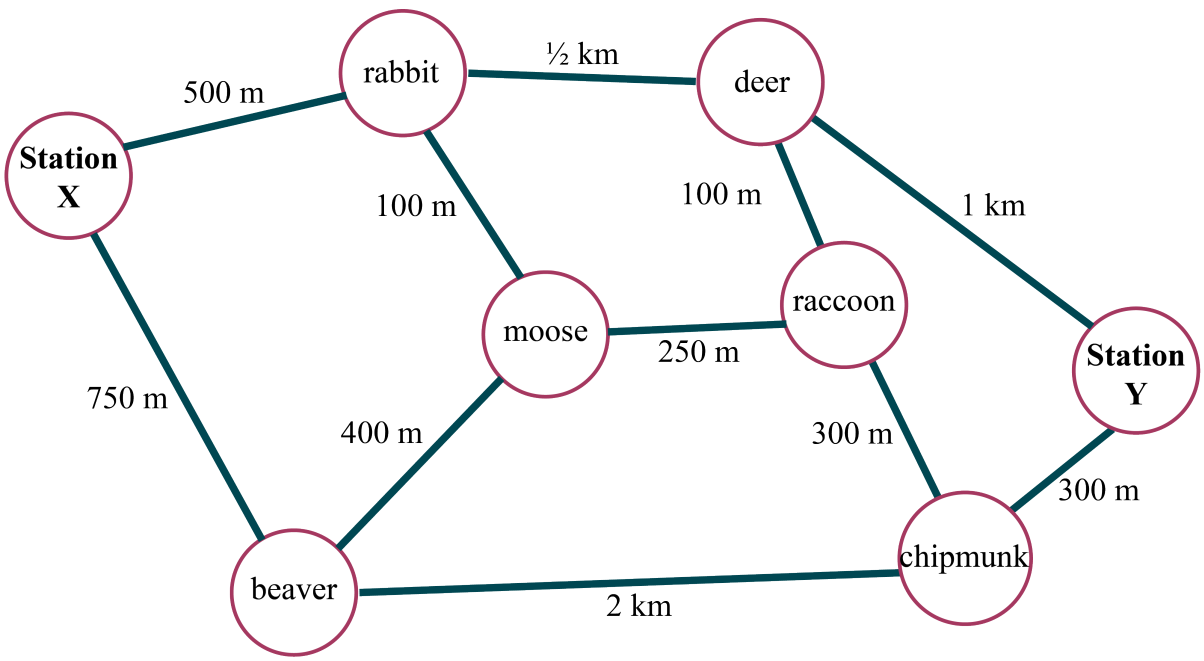
The remaining digits are 6 and 8, and fortunately the sum of these digits is 14.

As in part (a), we can arrange these digits in 8 different ways. The 8 solutions are shown.

$$\begin{array}{r}642\\+897\\\hline\end{array}\quad\begin{array}{r}647\\+892\\\hline\end{array}\quad\begin{array}{r}692\\+847\\\hline\end{array}\quad\begin{array}{r}697\\+842\\\hline\end{array}\quad\begin{array}{r}842\\+697\\\hline\end{array}\quad\begin{array}{r}847\\+692\\\hline\end{array}\quad\begin{array}{r}892\\+647\\\hline\end{array}\quad\begin{array}{r}897\\+642\\\hline\end{array}$$

Problem of the Week
Problem A
Park Paths

Arrowhead Provincial Park has various trails that people can follow between Ranger Station X and Ranger Station Y. There are trail markers at each intersection which are named after animals that may be seen in the park. The map below shows the trails and the distances between markers. The map is not to scale.



Three hikers walked from **Station X** to **Station Y** along different paths:

Path A: Station X → rabbit → deer → Station Y
Path B: Station X → beaver → chipmunk → Station Y
Path C: Station X → rabbit → moose → raccoon → deer → Station Y

(a) Who walked the furthest? Justify your answer.

(b) Find a path that is shorter than any of the three paths the hikers took.

Challenge: What is the shortest path from **Station X** to **Station Y**?

Themes: Computational Thinking, Geometry & Measurement

Problem of the Week
Problem A and Solution
Park Paths

Problem

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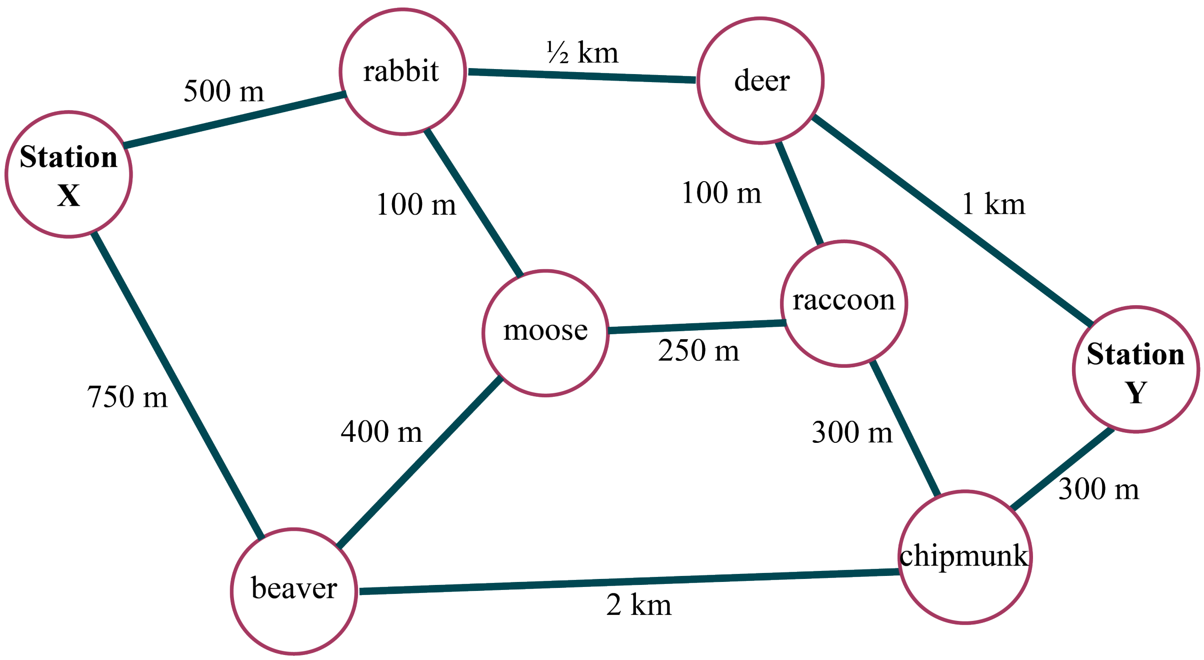

Three hikers walked from **Station X** to **Station Y** along different paths:

Path A: Station X → rabbit → deer → Station Y
Path B: Station X → beaver → chipmunk → Station Y
Path C: Station X → rabbit → moose → raccoon → deer → Station Y

(a) Who walked the furthest? Justify your answer.

(b) Find a path that is shorter than any of the three paths the hikers took.

Challenge: What is the shortest path from **Station X** to **Station Y**?

Solution

One way to solve this problem is to convert all the distances to metres. We know that 1 km is equal to 1000 m. So, 2 km is equal to 2000 m and $\frac{1}{2}$ km is equal to $1000 \div 2 = 500$ m.

(a) **Path A** is $500 + 500 + 1000 = 2000$ m or 2 km long.

Path B is $750 + 2000 + 300 = 3050$ m long.

Path C is $500 + 100 + 250 + 100 + 1000 = 1950$ m long.

The hiker that follows **Path B** walks the furthest.

(b) Answers will vary. Possible answers include:

$$\text{Station X} \to \text{rabbit} \to \text{deer} \to \text{raccoon} \to \text{chipmunk} \to \text{Station Y}$$

The distance of this path is $500 + 500 + 100 + 300 + 300 = 1700$ m.

$$\text{Station X} \to \text{rabbit} \to \text{moose} \to \text{raccoon} \to \text{chipmunk} \to \text{Station Y}$$

The distance of this path is $500 + 100 + 250 + 300 + 300 = 1450$ m.

Challenge Solution:

One way to try to find the shortest distance from **Station X** to **Station Y** is to look for the paths that have the fewest stops. Both **Path A** and **Path B** only have two stops between the starting point and the ending point. There are no possible paths that have only one stop between **Station X** and **Station Y**, so **Path A** and **Path B** have the fewest stops possible. However, as we already calculated, neither of these two paths can possibly be the shortest path since **Path C** is shorter. So this strategy for finding the shortest path does not work in general.

Another strategy for searching for the shortest path is called the *greedy algorithm*. With this strategy, at each intersection we pick the path with the shortest distance leaving that intersection. **Path C** follows this strategy.

At **Station X**, we follow the path that is 500 m instead of the path that is 750 m. At the **rabbit** intersection, we follow the path that is 100 m instead of the path that is $\frac{1}{2}$ km. At the **moose** intersection, we follow the path that is 250 m instead of the path that is 400 m. At the **raccoon** intersection, we follow the path that is 100 m instead of the path that is 300 m. At the **deer** intersection, we follow the path that is 1 km instead of the path that leads us back to an intersection we have already visited. We see that this strategy finds a path that is shorter than the other two paths that are listed, but is this the shortest possible path?

In this problem, we can use trial and error to find at least one other path that is shorter than **Path C**. So the greedy algorithm strategy for finding the shortest path does not work in general.

The next question is, how do we find the shortest path and know that there are no paths that are shorter? We could use trial and error and list all possible paths from **Station X** to **Station Y**. Then we choose the shortest path from all possible paths. However, this can get tricky and it is easy to miss one or more paths. There is a strategy that is used in higher mathematics called *Dijkstra's Algorithm*. We won't go through the details of this algorithm here, but the result of following this strategy leads us to the shortest path from **Station X** to **Station Y**:

$$\text{Station X} \to \text{rabbit} \to \text{moose} \to \text{raccoon} \to \text{chipmunk} \to \text{Station Y}$$

The distance of this path is $500 + 100 + 250 + 300 + 300 = 1450$ m.

Problem of the Week
Problem A
Tents Situation

Four families are going camping. The families are each of a different size, with 3, 4, 5, or 6 family members, and each have a different coloured tent.

Using the clues below, determine the family size and tent colour for each family.

(1) The Boivin family uses a yellow tent.

(2) The family with the blue tent has the fewest family members.

(3) The Kerr family does not have the most people in their family and they are not in the red tent.

(4) The family in the yellow tent has the most members in their family.

(5) The family in the red tent has one fewer family member than the family in the yellow tent.

(6) The Omanovic family is not in the green tent nor in the blue tent, and they are not the family with the fewest members.

(7) The green tent has the second lowest number of family members.

(8) The Banerjee family has half the number of family members as another family.

Theme: Computational Thinking

Problem of the Week

Problem A and Solution

Tents Situation

Problem

Four families are going camping. The families are each of a different size, with 3, 4, 5, or 6 family members, and each have a different coloured tent.

Using the clues below, determine the family size and tent colour for each family.

(1) The Boivin family uses a yellow tent.

(2) The family with the blue tent has the fewest family members.

(3) The Kerr family does not have the most people in their family and they are not in the red tent.

(4) The family in the yellow tent has the most members in their family.

(5) The family in the red tent has one fewer family member than the family in the yellow tent.

(6) The Omanovic family is not in the green tent nor in the blue tent, and they are not the family with the fewest members.

(7) The green tent has the second lowest number of family members.

(8) The Banerjee family has half the number of family members as another family.

Solution

Here is one way to match the families with the number of family members and the tent colours.

Clue (1) tells us that the Boivin family uses the yellow tent. Then, from Clue (4), we know that the Boivin family has the most members in their family, and so has 6 family members.

Thus, the Boivin family has 6 family members and uses the yellow tent.

From Clue (5), we know that the family in the red tent has one fewer family member than the family in the yellow tent. Since the family in the yellow tent has 6 family members, the family in the red tent has 5 family members. From Clue (6), the Omanovic family is not in the green tent nor in the blue tent. Since the Boivin family is in the yellow tent, then the Omanovic family is not in the yellow tent. So the Omanovic family must be in the red tent.

Thus, the Omanovic family has 5 family members and is in the red tent.

Clue (8) tells us that the Banerjee family has 3 family members, because 3 is the only number that is half of another number of family members. Thus, the Banerjee family has the fewest family members. Clue (2) then tells us that the Banerjee family is in the blue tent.

Thus, the Banerjee family has 3 family members and is in the blue tent.

This means that the Kerr family must have 4 family members and be in the green tent.

So we have deduced the following matches:

The Boivin family has 6 members and a yellow tent.

The Omanovic family has 5 members and a red tent.

The Kerr family has 4 members and a green tent.

The Banerjee family has 3 members and a blue tent.

Once we think we have our final answer, we should double check that all of the clue statements are true. Note that our solution did not use Clue (3) or Clue (7), however these clues match our final answer.

Data Management (D)

Take me to the cover

Problem of the Week
Problem A
Comparing Communications

Several students compared how many emails they had sent during the previous week.

Number of Emails Sent	Number of Students
0	12
1	19
2	11
3	8
4	14
5	2
6	15

(a) How many students sent fewer than 2 emails?

(b) How many students sent more than 4 emails?

(c) Separate the students into two groups so that all the students in one group sent more emails than all the students in the other group, and the number of students in each group are as close as possible.

Themes: Data Management, Number Sense

Problem of the Week

Problem A and Solution

Comparing Communications

Problem

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1	19
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(a) How many students sent fewer than 2 emails?

(b) How many students sent more than 4 emails?

(c) Separate the students into two groups so that all the students in one group sent more emails than all the students in the other group, and the number of students in each group are as close as possible.

Solution

We can use the table information to determine the answers.

(a) The numbers 0 and 1 are less than 2. So the total number of students who sent fewer than 2 emails is $12 + 19 = 31$.

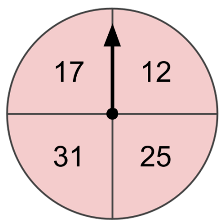
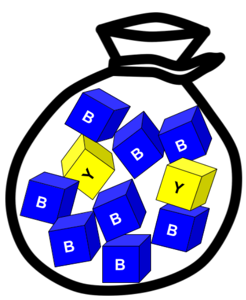
(b) The numbers 5 and 6 are more than 4. So the total number of students who sent more than 4 emails is $2 + 15 = 17$.

(c) The total number of students is $12 + 19 + 11 + 8 + 14 + 2 + 15 = 81$. If the number of students in each group are as close as possible, then one group would have 40 students and the other would have 41. This is our target.

We notice that the number of students who sent fewer than 3 emails is $12 + 19 + 11 = 42$. This is quite close to 41. In fact, it is as close as we can get to 41 because the number of students who sent 3 emails is 8 and the number of students who sent 2 emails is 11, so adding 8 or subtracting 11 from 42 would take us further away from 41. Thus, one group contains the 42 students who sent fewer than 3 emails and the other group contains the $81 - 42 = 39$ students who sent 3 or more emails.

Problem of the Week
Problem A
Probably

Think about the probability related to the following situations. Identify each one as *"certain"*, *"likely"*, *"equally likely"*, *"unlikely"*, or *"impossible"*. Justify your answers.

Event	**Probability**
Given the spinner below, what is the probability the outcome of spinning is an odd number rather than an even number? 	
If today is Thursday, what is the probability that tomorrow is Friday rather than another day of the week?	
If you ask someone to pick a whole number between 1 and 10, what is the probability the number they choose is greater than 8 and less than 5?	
If you flip a fair two-sided coin that has a moose on one side and a canoe on the other side, what is the probability it lands with the moose side up rather than the canoe side up?	
If you pick a cube from the bag below without being able to see inside the bag, what is the probability that the cube you pick is yellow rather than blue? 	

Theme: Data Management

Problem of the Week
Problem A and Solution
Probably

Problem
Think about the probability related to the following situations. Identify each one as *"certain"*, *"likely"*, *"equally likely"*, *"unlikely"*, or *"impossible"*. Justify your answers.

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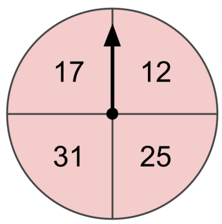
Given the spinner below, what is the probability the outcome of spinning is an odd number rather than an even number? 	
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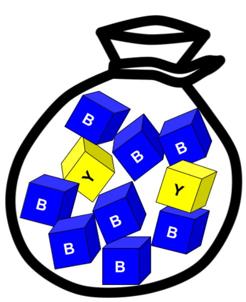
If you pick a cube from the bag below without being able to see inside the bag, what is the probability that the cube you pick is yellow rather than blue? 	

Solution

Given the spinner, it is *likely* that the outcome will be an odd number since there are 3 times as many odd numbers as even numbers on the spinner and it appears each area for a number on the spinner is approximately the same size.

If today is Thursday, it is *certain* that tomorrow is Friday rather than another day of the week. Days of the week have a set sequence and Friday always follows Thursday.

If you ask someone to pick a whole number between 1 and 10, it is *impossible* that the number they choose is greater than 8 and less than 5. Since $5 < 8$, numbers that are less than 5 cannot also be greater than 8 and numbers that are greater than 8 cannot also be less than 5.

If you flip a fair two-sided coin that has a moose on one side and a canoe on the other side it is *equally likely* it lands with the moose side up rather than the canoe side up. Since there are only two sides to the coin, and it is is a fair coin, over time the coin should land on each side approximately the same number of times.

If you pick a cube from the bag, without being able to see inside the bag, it is *unlikely* that the cube you pick is yellow rather than blue. The number of yellow cubes (2) is a much smaller than the number of blue cubes (8) in the bag. If you are unable to see the colours of the cubes, then it is much less likely that you will pick a yellow cube rather than a blue cube randomly from the bag.

Problem of the Week

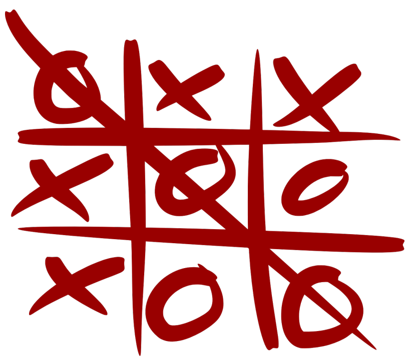
Problem A

Team Tournament

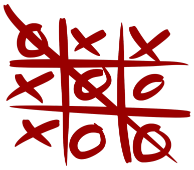
Ms. McNeil organizes a bean bag tic-tac-toe tournament for her class. She divides her class into 6 teams. When one team plays another team during the tournament, this is called a match.

(a) In the tournament, each team has to play every other team once. How many matches will each team play during the tournament? Justify your answer.

(b) How many matches are played in total during the tournament? Justify your answer.



Themes: Algebra, Data Management



Problem of the Week
Problem A and Solution
Team Tournament

Problem

Ms. McNeil organizes a bean bag tic-tac-toe tournament for her class. She divides her class into 6 teams. When one team plays another team during the tournament, this is called a match.

(a) In the tournament, each team has to play every other team once. How many matches will each team play during the tournament? Justify your answer.

(b) How many matches are played in total during the tournament? Justify your answer.

Solution

(a) Since each team has to play every other team once during the tournament, each team must play the other 5 teams. In other words, each team has 5 matches.

(b) We will call the teams Team 1, Team 2, Team 3, Team 4, Team 5, and Team 6. One way to solve this problem is to write out all the matches, being careful to make sure we do not include duplicate matches. For example, when Team 1 plays Team 2, that match is the same as saying Team 2 plays Team 1.

The possible matches are:

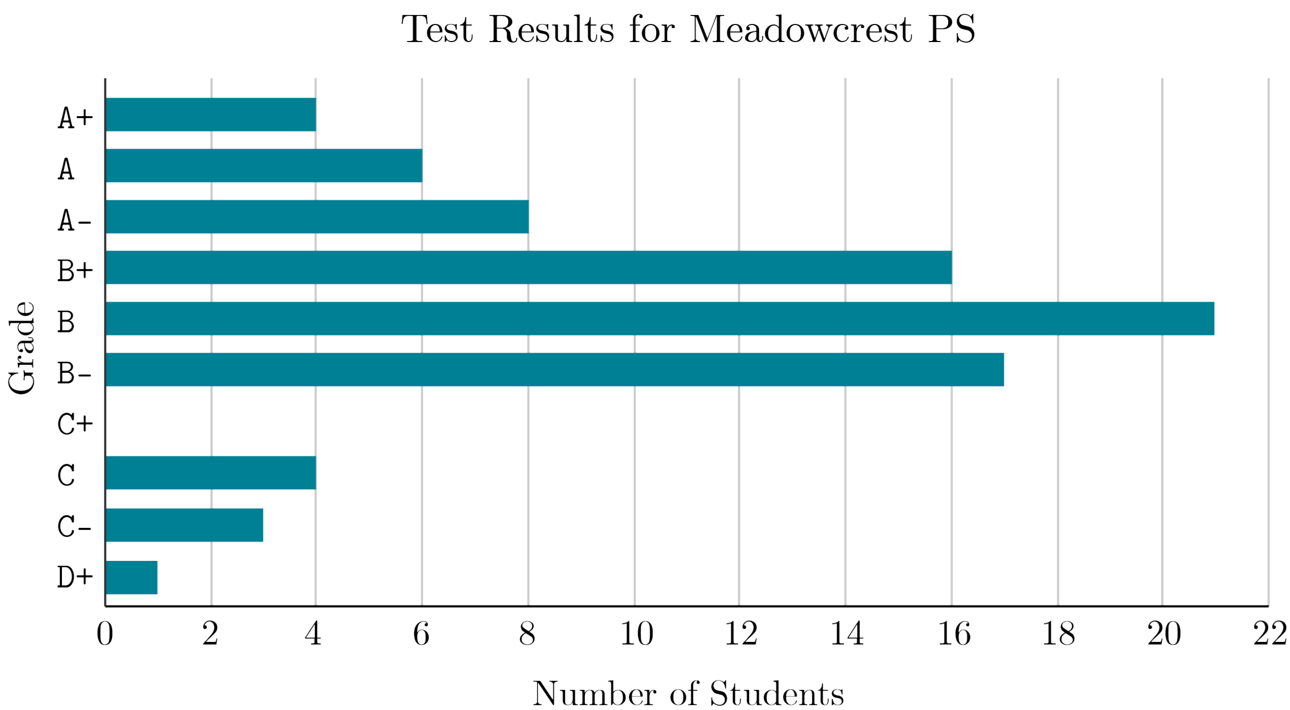
- Team 1 and Team 2, Team 1 and Team 3, Team 1 and Team 4, Team 1 and Team 5, Team 1 and Team 6
- Team 2 and Team 3, Team 2 and Team 4, Team 2 and Team 5, Team 2 and Team 6
- Team 3 and Team 4, Team 3 and Team 5, Team 3 and Team 6
- Team 4 and Team 5, Team 4 and Team 6
- Team 5 and Team 6

Therefore, there are 15 matches in total in the tournament.

Alternatively, since each team has to play 5 matches and there are 6 teams, we can count the total number of matches in the tournament as $5 + 5 + 5 + 5 + 5 + 5 = 5 \times 6 = 30$. However, this calculation actually double counts matches. For example, it counts the match between Team 1 and Team 2 as two separate matches (one match from the point of view of Team 1 and one match from the point of view of Team 2). This is true for every match between two teams. So, to get the true count of the total number of matches in the tournament, we need to divide by 2 to get $30 \div 2 = 15$ matches.

Problem of the Week
Problem A
Test Results

The students in Meadowcrest Public School wrote a standardized math test. The bar chart below shows the results of the test. Reading the grades from top to bottom in the bar chart, the grades appear from highest to lowest. In other words, `A+` is the highest grade and `D+` is the lowest grade.



(a) How many students wrote the test?

(b) What is the mode of the test results?
(The mode is the score that occurs the most often.)

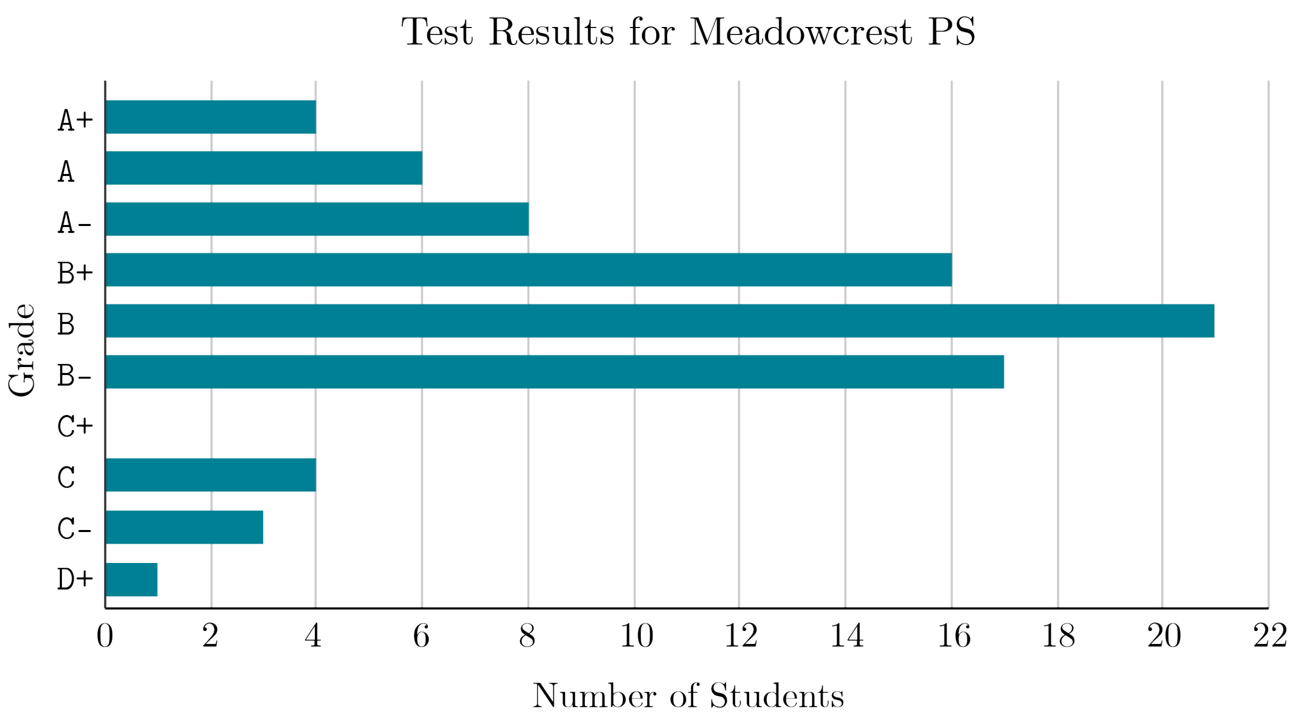
(c) If a grade of `B-` or higher means meeting or exceeding expectations, how many students are meeting or exceeding expectations?

Theme: Data Management

Problem of the Week
Problem A and Solution
Test Results

Problem

The students in Meadowcrest Public School wrote a standardized math test. The bar chart below shows the results of the test. Reading the grades from top to bottom in the bar chart, the grades appear from highest to lowest. In other words, `A+` is the highest grade and `D+` is the lowest grade.



(a) How many students wrote the test?

(b) What is the mode of the test results?
(The mode is the score that occurs the most often.)

(c) If a grade of `B-` or higher means meeting or exceeding expectations, how many students are meeting or exceeding expectations?

Solution

(a) We can read the bar graph to determine how many students earned each grade on the test. The following table summarizes the results in the bar graph:

Grade	A+	A	A-	B+	B	B-	C+	C	C-	D+
Number of Students	4	6	8	16	21	17	0	4	3	1

So the total number of students who wrote the test is:
$4 + 6 + 8 + 16 + 21 + 17 + 0 + 4 + 3 + 1 = 80$

(b) The mode of the data matches the longest bar in the graph. In this case the mode is B since that is the grade with the longest bar.

(c) If we only consider the grades of B- or higher, we can add together the number of students who achieved grades of A+, A, A-, B+, B, and B-. This gives $4 + 6 + 8 + 16 + 21 + 17 = 72$.

Alternatively, we can subtract the number of students who achieved grades below B- from the total we found in part (a). This gives $80 - 4 - 3 - 1 = 72$.

Problem of the Week

Problem A

Birthday Mode

Jaylen and Julia are in different classes at school. They both take a survey of the birth month of their classmates. They record each birthday month as a number, with January = 1, February = 2, March = 3, and so on.

Here are their two data sets:

Jaylen: 4, 5, 4, 8, 9, 3, 5, 6, 3, 4, 11, 6, 5, 12, 10, 12, 3, 1, 2

Julia: 12, 3, 4, 3, 5, 7, 8, 3, 6, 4, 11, 11, 6, 5, 3, 9, 10, 10, 9, 3, 5

A new student comes into the school and joins one of their classes. After adding the new student's birthday information to the survey results, Jaylen and Julia notice that the modes of the two data sets are the same. (The mode of a data set is the value that appears most often.)

In what month was the new student born, and whose class did they enter?

Theme: Data Management

Problem of the Week
Problem A and Solution
Birthday Mode

Problem

Jaylen and Julia are in different classes at school. They both take a survey of the birth month of their classmates. They record each birthday month as a number, with January = 1, February = 2, March = 3, and so on.

Here are their two data sets:

Jaylen: 4, 5, 4, 8, 9, 3, 5, 6, 3, 4, 11, 6, 5, 12, 10, 12, 3, 1, 2

Julia: 12, 3, 4, 3, 5, 7, 8, 3, 6, 4, 11, 11, 6, 5, 3, 9, 10, 10, 9, 3, 5

A new student comes into the school and joins one of their classes. After adding the new student's birthday information to the survey results, Jaylen and Julia notice that the modes of the two data sets are the same. (The mode of a data set is the value that appears most often.)

In what month was the new student born, and whose class did they enter?

Solution

We start by finding the mode of each of the data sets. To do so, we can make a table of the frequencies of each value in the data set:

Jaylen's data:

Month:	1	2	3	4	5	6	7	8	9	10	11	12
Frequency:	1	1	3	3	3	2	0	1	1	1	1	2

Julia's data:

Month:	1	2	3	4	5	6	7	8	9	10	11	12
Frequency:	0	0	5	2	3	2	1	1	2	2	2	1

Before the new student arrived, the mode of Julia's data was 3, since that value appears the most often (5 times).

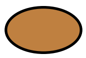
There is a three-way tie for the mode of Jaylen's data, since the values 3, 4, and 5 all appear 3 times in the data set.

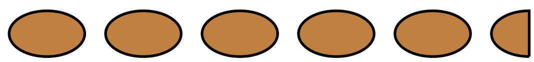
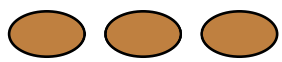
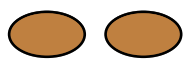
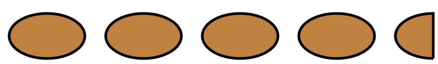
Since the modes of the two data sets are the same after adding only one more value to the data, then the two data sets must both have a mode of 3. If the new student joins Jaylen's class, and the student's birth month is March, then there will be 4 students that have March as their birthday month. With this update, the mode in Jaylen's data set is 3, the same as the mode of Julia's data set.

Note that the mode is the same for each data set, but the actual frequency of the mode value is different in the two classes.

Problem of the Week
Problem A
Missing Cookies

Janelle and Jean-Paul baked 70 cookies for a school bake sale. They baked five different kinds of cookies and there was an even number of each kind of cookie. When Janelle and Jean-Paul got to school, they realized that they were missing the *sugar* cookies.

Below is a pictograph of the cookies that they brought to the school.
Each  represents 4 cookies.

Cookie Type	**Number of Cookies**
Chocolate Chip	
Oatmeal Raisin	
Gingerbread	
Sugar	
Coconut Macaroon	

How many *sugar* cookies had they forgotten at home? Complete the pictograph and explain your answer.

Theme: Data Management

Problem of the Week

Problem A and Solution

Missing Cookies

Problem

Janelle and Jean-Paul baked 70 cookies for a school bake sale. They baked five different kinds of cookies and there was an even number of each kind of cookie. When Janelle and Jean-Paul got to school, they realized that they were missing the *sugar* cookies.

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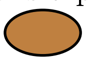
Each  represents 4 cookies.

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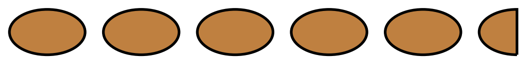
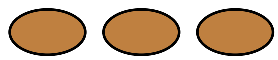
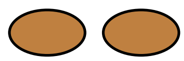
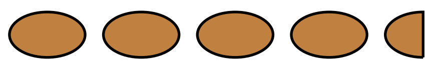
Chocolate Chip	
Oatmeal Raisin	
Gingerbread	
Sugar	
Coconut Macaroon	

How many *sugar* cookies had they forgotten at home? Complete the pictograph and explain your answer.

Solution

We start by counting the known number of cookies represented in the pictograph. Since each full oval represents four cookies, then a half oval must represent half as many cookies, or two cookies.

- *Chocolate Chip:* There are five full ovals, which represent $5 \times 4 = 20$ cookies. There is one half oval, which represents 2 cookies. So there are a total of $20 + 2 = 22$ chocolate chip cookies.
- *Oatmeal Raisin:* There are three full ovals, which represent $3 \times 4 = 12$ oatmeal raisin cookies.
- *Gingerbread:* There are two full ovals, which represents $2 \times 4 = 8$ gingerbread cookies.

- *Coconut Macaroon:* There are four full ovals, which represent a total of $4 \times 4 = 16$ coconut macaroon cookies. There is one half oval, which represents 2 cookies. So there are a total of $16 + 2 = 18$ coconut macaroons.

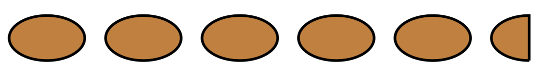
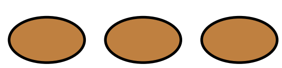
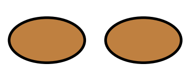
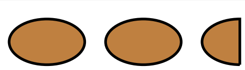
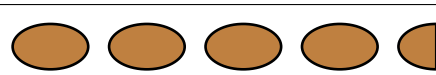
From this, the known total number of cookies is $22 + 12 + 8 + 18 = 60$.

Another way we could have computed the known number of cookies is by counting up all of the full ovals, for a total of 14, and counting up all of the half ovals, for a total of 2. The total number of cookies represented by full ovals is $14 \times 4 = 56$. The total number of cookies represented by half ovals is $2 \times 2 = 4$. Thus, the total number of cookies is $56 + 4 = 60$.

Since Janelle and Jean-Paul baked a total of 70 cookies, but we can only account for 60 of them, then there are $70 - 60 = 10$ cookies missing. These must be our *sugar* cookies.

We can figure out how many extra full ovals we need in our pictograph by dividing the number of sugar cookies we need to represent by 4, and getting the quotient of that calculation. Since $10 \div 4 = 2$ with a remainder 2, then we need 2 full ovals and 1 half oval to represent this number of sugar cookies.

The completed pictograph is shown.

Cookie Type	**Number of Cookies**
Chocolate Chip	
Oatmeal Raisin	
Gingerbread	
Sugar	
Coconut Macaroon	

Geometry & Measurement (G)

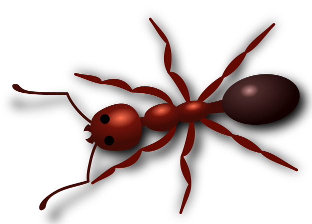
Take me to the cover

Problem of the Week
Problem A
The Metric Argument

Millie, Kai, Mircea and Candice were debating about which metric units they liked the most. Millie thought millimeters were best, Kai loved kilometers, Mircea loved meters, and Candice's favourite was centimeters.

Who should measure each of the following lengths with their favourite unit, if they want to keep each value between 2 and 50, inclusive?

Length Being Measured	**Measurer**
The length of an ant	
The length of a bus	
The length of a marathon	
The length of an adult foot	
The length of a new pencil	
The length of an olympic swimming pool	
The length of a human baby tooth	



Theme: Geometry & Measurement

Problem of the Week
Problem A and Solution
The Metric Argument

Problem

Millie, Kai, Mircea and Candice were debating about which metric units they liked the most. Millie thought millimeters were best, Kai loved kilometers, Mircea loved meters, and Candice's favourite was centimeters.

Who should measure each of the following lengths with their favourite unit, if they want to keep each value between 2 and 50, inclusive?

Solution

In order to keep the measurement numbers between 2 and 50, inclusive, here is the completed table, along with an approximate measurement for each:

Length Being Measured	**Measurer**
The length of an ant	Millie (6 mm)
The length of a bus	Mircea (12 m)
The length of a marathon	Kai (42 km)
The length of an adult foot	Candice (25 cm)
The length of a new pencil	Candice (18 cm)
The length of an olympic swimming pool	Mircea (50 m)
The length of a human baby tooth	Millie (9 mm)

Problem of the Week

Problem A

What Number Am I?

I am a 4-digit number. Use the following clues to determine my digits:

(1) My hundreds digit is the difference between the number of sides of a hexagon and the number of sides of a triangle.

(2) Reading my digits from left to right, they are in increasing order from the thousands digit to the ones digit. Also, they increase by the same amount each time.

(3) The difference between my tens digit and my ones digit is equal to the number of sides of an octagon divided by the number of right angles in a rectangle.

What number am I?

Themes: Computational Thinking, Geometry & Measurement, Number Sense

Problem of the Week

Problem A and Solution

What Number Am I?

Problem

I am a 4-digit number. Use the following clues to determine my digits:

(1) My hundreds digit is the difference between the number of sides of a hexagon and the number of sides of a triangle.

(2) Reading my digits from left to right, they are in increasing order from the thousands digit to the ones digit. Also, they increase by the same amount each time.

(3) The difference between my tens digit and my ones digit is equal to the number of sides of an octagon divided by the number of right angles in a rectangle.

What number am I?

Solution

Using Clue (1), the number of sides of a hexagon is 6 and the number of sides of a triangle is 3. The difference between these two numbers is $6 - 3 = 3$, so the hundreds digit is 3.

Using Clue (3), the number of sides of an octagon is 8 and the number of right angles in a rectangle is 4. So, the difference between the tens digit and the ones digit is $8 \div 4 = 2$.

Using Clue (2), since the digits increase by the same amount each time, then as we read the digits from left to right the numbers will increase by 2. Since the hundreds digit is 3, then the tens digit must be $3 + 2 = 5$. Since the tens digit is 5, then the ones digit must be $5 + 2 = 7$. Working backwards, since the hundreds digit is 3, then the thousands digit must be $3 - 2 = 1$.

Therefore the number is 1357.

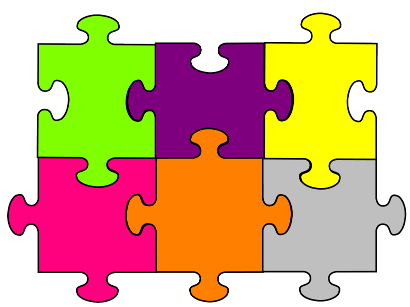
Problem of the Week
Problem A
Puzzling Predicament

Pakiza gets a 100 piece puzzle as a gift. When the puzzle is finished, the pieces form a grid, where each row and each column of the grid has the same number of pieces.

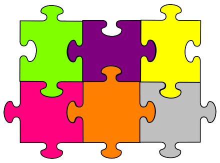
(a) How many pieces are in each row and each column of the puzzle?

(b) On Monday, Pakiza gathers all the edge and corner pieces and puts them together to form the outside edge of the puzzle. How many edge and corner pieces does the puzzle have in total?

(c) Pakiza continues to work on the puzzle a little bit every day. She places 10 pieces in the puzzle on Tuesday. On each day after that, she places one more puzzle piece than the day before, until the puzzle is complete. So, she places 11 pieces on Wednesday, 12 pieces on Thursday, and so on until all the pieces are in place. On what day of the week does Pakiza complete the puzzle? Justify your answer.



Themes: Algebra, Geometry & Measurement



Problem of the Week

Problem A and Solution

Puzzling Predicament

Problem

Pakiza gets a 100 piece puzzle as a gift. When the puzzle is finished, the pieces form a grid, where each row and each column of the grid has the same number of pieces.

(a) How many pieces are in each row and each column of the puzzle?

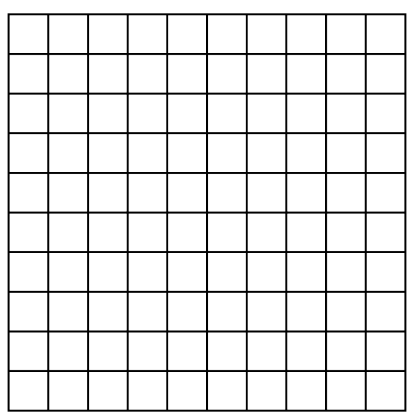
(b) On Monday, Pakiza gathers all the edge and corner pieces and puts them together to form the outside edge of the puzzle. How many edge and corner pieces does the puzzle have in total?

(c) Pakiza continues to work on the puzzle a little bit every day. She places 10 pieces in the puzzle on Tuesday. On each day after that, she places one more puzzle piece than the day before, until the puzzle is complete. So, she places 11 pieces on Wednesday, 12 pieces on Thursday, and so on until all the pieces are in place. On what day of the week does Pakiza complete the puzzle? Justify your answer.

Solution

(a) We know that each row and each column of the puzzle have the same number of pieces, and there are 100 pieces in total. Since $10 \times 10 = 100$, then each row and each column of the puzzle must contain 10 pieces.

(b) It might help to look at a 10 by 10 grid in order to determine the total number of edge and corner pieces.



There are 4 corner pieces. Not counting the corners, there are 8 edge pieces on each side of the puzzle. So the total number of edge and corner pieces is: $4 + 8 + 8 + 8 + 8 = 36$.

Note: This is a classic example of the *fencepost problem*. If we are not careful, we might think that a 10×10 grid has $4 \times 10 = 40$ squares around its edge. However, this counts each corner piece twice, since a corner piece is part of two sides of the puzzle. The fencepost problem describes a situation where you may be off-by-one when counting. It can cause problems when writing computer programs that include some repetition.

(c) We can make a table to keep track of how many pieces are filled in the puzzle over time. The first entry in the table will be on Monday when Pakiza assembles the 36 pieces to form the outside edge of the puzzle.

Day	**Number of Pieces Added**	**Total Pieces**
Monday	36	36
Tuesday	10	46
Wednesday	11	57
Thursday	12	69
Friday	13	82
Saturday	14	96

At this point, there are only 4 pieces left in the puzzle. So, we know Pakiza will complete the puzzle on the next day, which is Sunday.

Problem of the Week
Problem A
Cupcake Bonanza

Archie, Lin, and Umar are making vanilla cupcakes for a bake sale. They need to make 8 dozen cupcakes using the recipe below.

Recipe to make 1 dozen vanilla cupcakes:

- 250 grams flour
- $2\frac{1}{2}$ grams salt
- 8 grams baking powder
- $\frac{1}{4}$ cup of butter
- 150 grams sugar
- 2 eggs
- 240 mL milk
- 5 mL vanilla

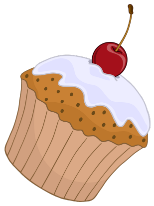
(a) How much of each ingredient do they need to make 8 dozen cupcakes?

(b) They have enough salt, baking powder, and vanilla already. They do not have any of the other ingredients. When they go to the store, they see the ingredients they need are sold in packages in the following sizes:

- Flour is sold in 1 kg bags.
- Butter is sold in 1 cup blocks.
- Sugar is sold in 2 kg bags.
- Eggs are sold by the dozen.
- Milk is sold in 1 L cartons.

How many of each package of the ingredients do they need to buy to have enough to make all of the cupcakes?

Themes: Geometry & Measurement, Number Sense



Problem of the Week
Problem A and Solution
Cupcake Bonanza

Problem

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(a) How much of each ingredient do they need to make 8 dozen cupcakes?

(b) They have enough salt, baking powder, and vanilla already. They do not have any of the other ingredients. When they go to the store, they see the ingredients they need are sold in packages in the following sizes:

- Flour is sold in 1 kg bags.
- Butter is sold in 1 cup blocks.
- Sugar is sold in 2 kg bags.
- Eggs are sold by the dozen.
- Milk is sold in 1 L cartons.

How many of each package of the ingredients do they need to buy to have enough to make all of the cupcakes?

Solution

(a) Since the recipe makes 1 dozen cupcakes, to calculate how much of each ingredient we need to make 8 dozen cupcakes we must multiply the amount of each ingredient by 8. For the larger numbers, we can do this calculation by skip counting or multiple additions.

For example, calculating the amount of flour required we could
skip count: 250, 500, 750, 1000, 1250, 1500, 1750, 2000,
or do multiple additions: $250 + 250 + 250 + 250 + 250 + 250 + 250 + 250 = 2000$.

Another way we could calculate 8 times some number is to double the value 3 times. Notice that if we double the number 1, we get 2. If we double the number 2, we get 4. If we double the number 4, we get 8. If we follow this same procedure with any starting number, the result will be 8 times the original value. For example, if we double 250, we get 500. If we double 500, we get 1000. If we double 1000, we get 2000.

Using any of these procedures, we determine that $8 \times 250 = 2000$.

The total amount of each ingredient needed to make 8 dozen cupcakes is below.

- Flour: $250 \times 8 = 2000$ g. Since 1000 g = 1 kg, then 2000 g = 2 kg is needed.
- Salt: $2\frac{1}{2} + 2\frac{1}{2} = 5$ grams, so 2 dozen cupcakes require 5 grams. Thus, 8 dozen cupcakes require $4 \times 5 = 20$ grams.
- Baking powder: $8 \times 8 = 64$ grams required.
- Butter: $\frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} = 1$ cup, so 4 dozen cupcakes require 1 cup. Thus, 8 dozen cupcakes require 2 cups.
- Sugar: $150 \times 8 = 1200$ grams required.
- Eggs: $2 \times 8 = 16$ eggs required.
- Milk: $240 \times 8 = 1920$ mL required.
- Vanilla: $5 \times 8 = 40$ mL required.

(b) We can use the total amounts calculated in part (a) to determine how many packages of each ingredient they must buy.

- Since flour is sold in 1 kg bags and they need 2 kg of flour, then they need 2 bags of flour.
- Since butter is sold in 1 cup blocks, and they need 2 cups of butter, then they need 2 blocks of butter.
- Since sugar is sold in 2 kg bags, and 2 kg = 2000 g, then they need 1 bag of sugar to have at least 1200 g of sugar.
- Since eggs are sold by the dozen, and there are 12 eggs in 1 dozen and 24 eggs in 2 dozen, then they need 2 cartons of eggs to have at least 16 eggs.
- Since milk is sold in 1 L cartons, and since 1 L = 1000 mL and so 2 L = 2000 mL, then they need 2 cartons of milk.

Problem of the Week
Problem A
Fund Raising

Flynn's class is raising money for a tree-planting charity by recycling electronics. They have found a local company that will give them \$5 for each pound of cell phone e-waste and \$2 for each pound of computer e-waste.

The school has gathered the following electronics for recycling, but has weighed them in grams instead of pounds.

Cell Phones	**Computers**
10 uPhones: 200 g each	5 Bonobo laptops: 800 g each
5 Mixel phones: 100 g each	7 uPads: 500 g each

Knowing that 1 pound is about 454 g, estimate how much money Flynn's class will make. Justify your answer.

Themes: Geometry & Measurement, Number Sense

Problem of the Week
Problem A and Solution
Fund Raising

Problem

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Knowing that 1 pound is about 454 g, estimate how much money Flynn's class will make. Justify your answer.

Solution

Solutions may vary, possibly resulting in different estimations. We will show two different approaches.

Since 1 pound is about 454 g, we will approximate 1 pound as 500 g. Then 2 pounds is approximately 1000 g.

Solution 1

In this solution, we find the total mass of each type of item and then estimate this in pounds before finding the total mass of all items.

Phones:

- uPhones: Each uPhone is 200 g, so 10 uPhones are $10 \times 200 = 2000$ g. Since 1000 g is approximately 2 pounds, then 2000 g is approximately 4 pounds.
- Mixel phones: Each Mixel phone is 100 g, so 5 Mixel phones are $5 \times 100 = 500$ g, which is approximately 1 pound.

This is approximately $4 + 1 = 5$ pounds of cell phone waste. The cell phone e-waste is worth approximately $\$5 \times 5 = \$25$.

Computers:

- Bonobo laptops: Each laptop is 800 g, so 5 laptops are $5 \times 800 = 4000$ g. Since 1000 g is approximately 2 pounds, then 4000 g is approximately $4 \times 2 = 8$ pounds.
- uPads: Each uPad is 500 g, which is approximately 1 pound, so 7 uPads are approximately 7 pounds.

This is approximately $8 + 7 = 15$ pounds of computer waste. The computer e-waste is worth approximately $\$2 \times 15 = \$30$. Therefore the total value is approximately $\$25 + \$30 = \$55$.

Solution 2

In this solution, we will find the total mass of computer waste and the total mass of cell phone waste, and then estimate these totals in pounds.

Phones:

- uPhones: Each uPhone is 200 g, so 10 uPhones are $10 \times 200 = 2000$ g.
- Mixel phones: Each Mixel phone is 100 g, so 5 Mixel phones are $5 \times 100 = 500$ g.

This is $2000 + 500 = 2500$ g. Since 500 g is approximately 1 pound and 1000 g is approximately 2 pounds, then 2500 g is approximately $2 + 2 + 1 = 5$ pounds. The cell phone e-waste is worth approximately $\$5 \times 5 = \$25$.

Computers:

- Bonobo laptops: Each laptop is 800 g, so 5 laptops are $5 \times 800 = 4000$ g.
- uPads: Each uPad is 500 g, so 7 uPads are $7 \times 500 = 3500$ g.

This is $4000 + 3500 = 7500$ g. Since 500 g is approximately 1 pound and 1000 g is approximately 2 pounds, then 7500 g is approximately $2 \times 7 + 1 = 15$ pounds. The computer e-waste is worth approximately $\$2 \times 15 = \$30$. Therefore the total value is approximately $\$25 + \$30 = \$55$.

Problem of the Week
Problem A
Zookeeper

Zoey works at the city's zoo feeding and caring for the animals. Her schedule is the same every morning. She starts work at 7:00 a.m. The table below lists the sequence of morning activities, along with the duration of each activity. The durations include the time it takes to travel to each spot.

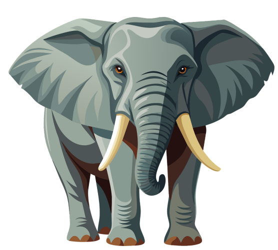
Activity	**Duration of Activity**
Team meeting	$\frac{1}{2}$ hour
Feeding the elephants	30 minutes
Moving tropical birds to outside habitat	45 minutes
Filling the monkey feed bins with fruit and nuts	30 minutes
Break	15 minutes
Changing the hay for the camels	45 minutes
Feeding the walruses and seals	$\frac{3}{4}$ hour
Washing down the rhino stalls	$\frac{1}{2}$ hour
Lunch break	half an hour

(a) What time does Zoey start feeding the elephants?

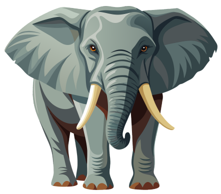
(b) What time does the monkey feeding start?

(c) What time does Zoey's break end?

(d) How long is it from the time the team meeting starts to the time Zoey's lunch break ends?



Themes: Geometry & Measurement, Number Sense



Problem of the Week
Problem A and Solution
Zookeeper

Problem

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Lunch break	half an hour

(a) What time does Zoey start feeding the elephants?

(b) What time does the monkey feeding start?

(c) What time does Zoey's break end?

(d) How long is it from the time the team meeting starts to the time Zoey's lunch break ends?

Solution

We add two columns to the table, one with the start time of the activity and one with the end time. From this table, we can answer the questions.

Activity	**Duration of Activity**	**Start Time**	**End Time**
Team meeting	$\frac{1}{2}$ hour	7:00	7:30
Feeding the elephants	30 minutes	7:30	8:00
Moving tropical birds to outside habitat	45 minutes	8:00	8:45
Filling the monkey feed bins with fruit/nuts	30 minutes	8:45	9:15
Break	15 minutes	9:15	9:30
Changing the hay for the camels	45 minutes	9:30	10:15
Feeding the walruses and seals	$\frac{3}{4}$ hour	10:15	11:00
Washing down the rhino stalls	$\frac{1}{2}$ hour	11:00	11:30
Lunch break	half an hour	11:30	12:00

(a) Zoey starts feeding the elephants at 7:30 a.m.

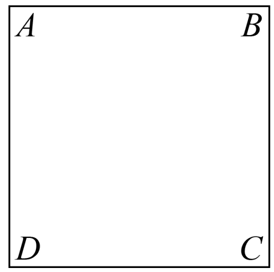
(b) The monkey feeding starts at 8:45 a.m.

(c) Zoey's break ends at 9:30 a.m.

(d) Since the lunch break ends at 12:00 p.m., it is 5 hours after the team meeting starts.

Problem of the Week
Problem A
Paper Folding

Start with a square piece of paper, and label the corners A, B, C, and D, starting with the top-left corner and moving clockwise.



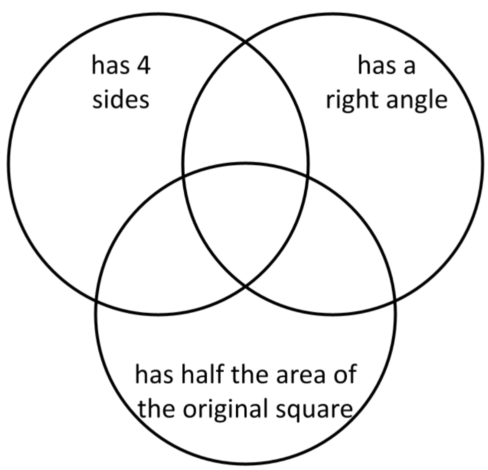
(a) Fold the paper so that corner A touches corner C. What shape did you make?

(b) Open the paper up so that you are back to the square, but with a diagonal crease. Fold the paper so that the edge between corners A and B lines up with the diagonal crease. Then fold the paper so that the edge between corners C and D lines up with the diagonal crease. What shape did you make?

(c) Open the paper up so that you are back to the square. Fold the paper so that corner D touches corner A and corner C touches corner B. What shape did you make?

(d) Continuing from the shape you made in part (c), fold the paper so that corners B and C touch the fold line. What shape did you make?

(e) Fill in the Venn diagram below with the letters a, b, c, and d, representing the shape you made in each part.



Theme: Geometry & Measurement

Problem of the Week
Problem A and Solution
Paper Folding

Problem

Start with a square piece of paper, and label the corners A, B, C, and D, starting with the top-left corner and moving clockwise.

(a) Fold the paper so that corner A touches corner C. What shape did you make?

(b) Open the paper up so that you are back to the square, but with a diagonal crease. Fold the paper so that the edge between corners A and B lines up with the diagonal crease. Then fold the paper so that the edge between corners C and D lines up with the diagonal crease. What shape did you make?

(c) Open the paper up so that you are back to the square. Fold the paper so that corner D touches corner A and corner C touches corner B. What shape did you make?

(d) Continuing from the shape you made in part (c), fold the paper so that corners B and C touch the fold line. What shape did you make?

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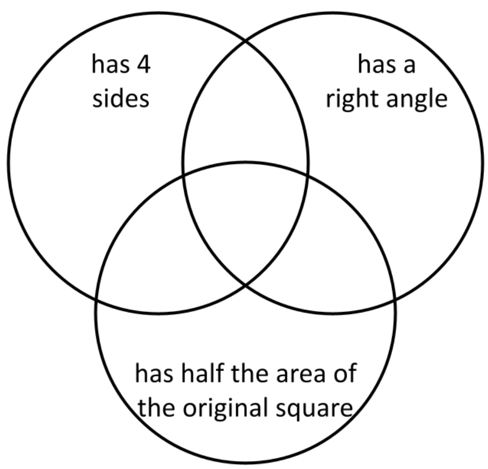
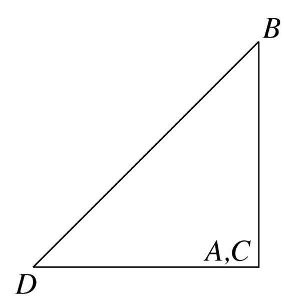
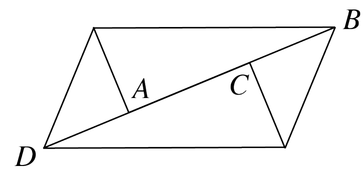


Solution

(a) Since the angle at corners A and C is a right angle, the shape made is a right-angled triangle.

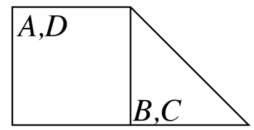


(b) Since the pairs of opposite sides are parallel and equal in length, the shape made is a parallelogram.



(c) Since the pairs of opposite sides are parallel and equal in length, and all angles are right angles, the shape made is a rectangle.

(d) Since one pair of opposite sides is parallel, the shape made is a trapezoid. To be more specific, we can call it a right trapezoid since it contains two right angles.



(e) We will look at each shape one by one to decide where it should go in the Venn diagram.

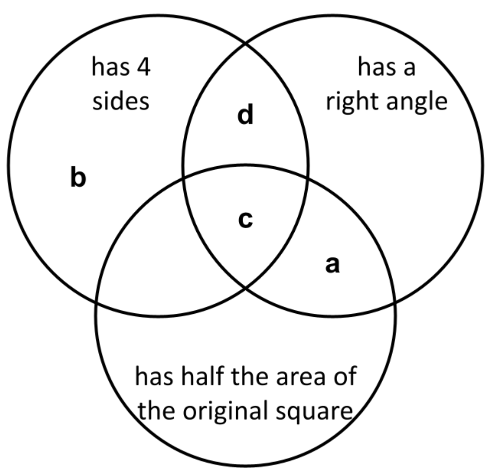
In part (a), we folded the paper in half along the diagonal between corners B and D, so the shape made has half the area of the original square. The shape has a right angle, but has 3 sides, not 4. So in the Venn diagram, this shape would go in the overlap between the circle labeled "has a right angle" and the circle labeled "has half the area of the original square".

In part (b), the paper is not folded in half, so the shape made does not have half the area of the original square. The shape has 4 sides but does not have a right angle. So in the Venn diagram, this shape would go in the circle labeled "has 4 sides".

In part (c), we folded the paper in half along the middle line between the top and bottom sides. The shape has 4 sides and 4 right angles. So in the Venn diagram, this shape would go in the overlap between all three circles.

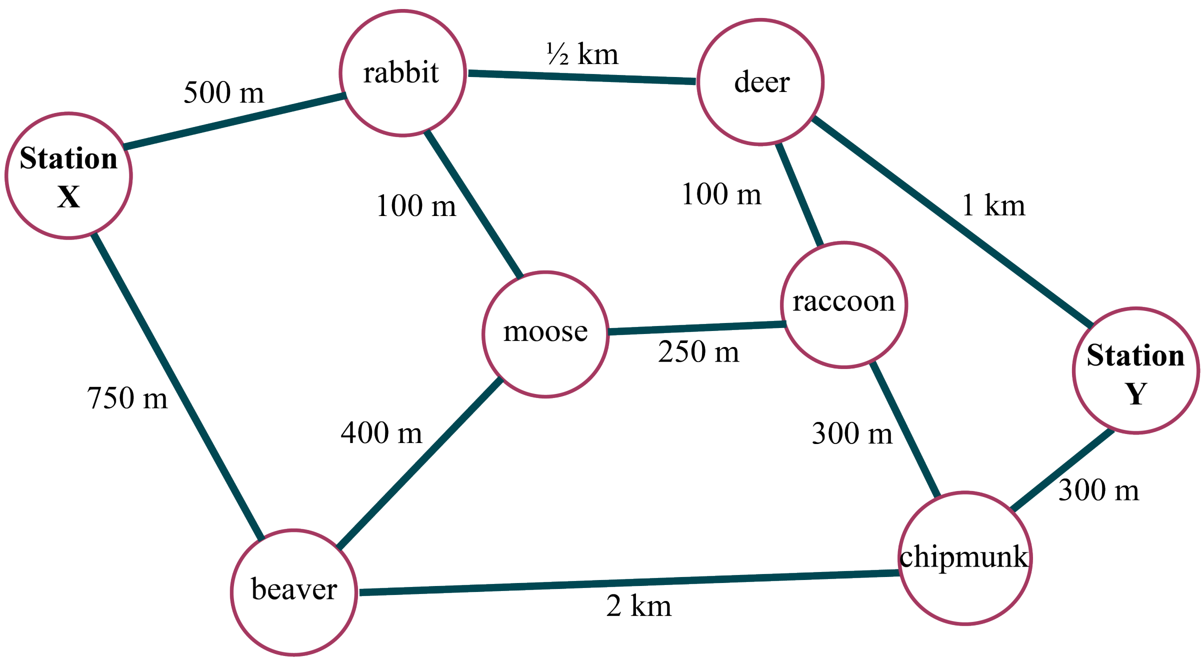
In part (d), we started with the paper folded in half and then folded it further, so the shape made does not have half the area of the original square. The shape has 4 sides and 2 right angles. So in the Venn diagram, this shape would go in the overlap between the circle labeled "has 4 sides" and the circle labeled "has a right angle".

The completed Venn diagram is shown.



Problem of the Week
Problem A
Park Paths

Arrowhead Provincial Park has various trails that people can follow between Ranger Station X and Ranger Station Y. There are trail markers at each intersection which are named after animals that may be seen in the park. The map below shows the trails and the distances between markers. The map is not to scale.



Three hikers walked from **Station X** to **Station Y** along different paths:

Path A: Station X → rabbit → deer → Station Y
Path B: Station X → beaver → chipmunk → Station Y
Path C: Station X → rabbit → moose → raccoon → deer → Station Y

(a) Who walked the furthest? Justify your answer.

(b) Find a path that is shorter than any of the three paths the hikers took.

Challenge: What is the shortest path from **Station X** to **Station Y**?

Themes: Computational Thinking, Geometry & Measurement

Problem of the Week
Problem A and Solution
Park Paths

Problem

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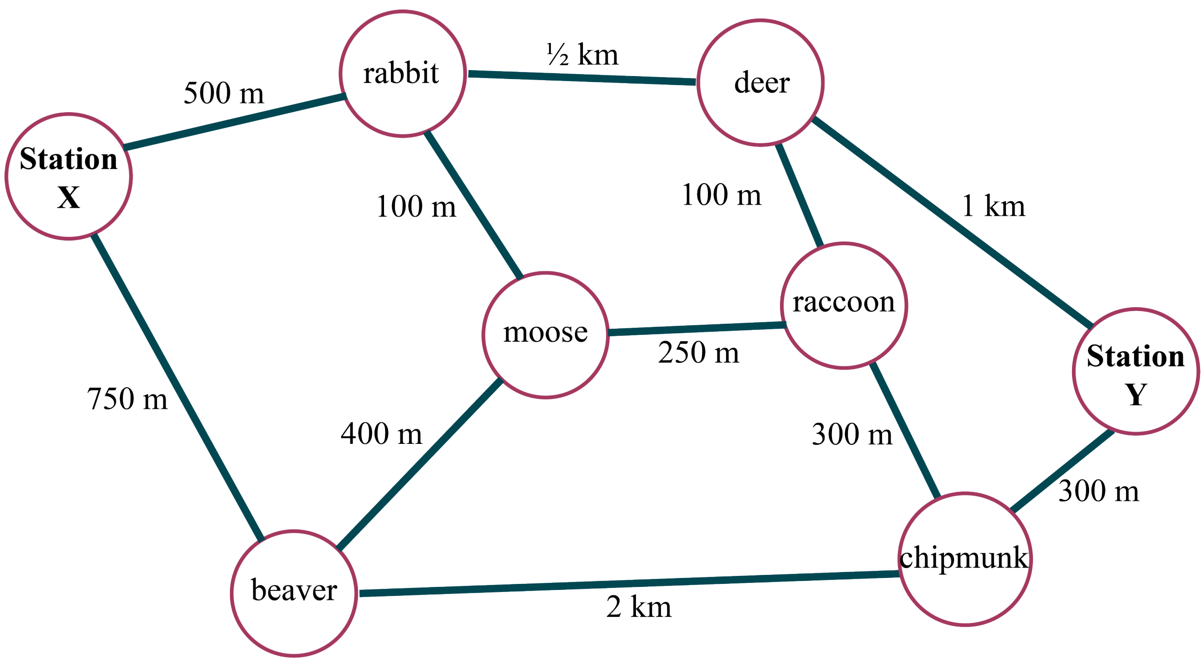

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Path A: Station X → rabbit → deer → Station Y
Path B: Station X → beaver → chipmunk → Station Y
Path C: Station X → rabbit → moose → raccoon → deer → Station Y

(a) Who walked the furthest? Justify your answer.

(b) Find a path that is shorter than any of the three paths the hikers took.

Challenge: What is the shortest path from **Station X** to **Station Y**?

Solution

One way to solve this problem is to convert all the distances to metres. We know that 1 km is equal to 1000 m. So, 2 km is equal to 2000 m and $\frac{1}{2}$ km is equal to $1000 \div 2 = 500$ m.

(a) **Path A** is $500 + 500 + 1000 = 2000$ m or 2 km long.

Path B is $750 + 2000 + 300 = 3050$ m long.

Path C is $500 + 100 + 250 + 100 + 1000 = 1950$ m long.

The hiker that follows **Path B** walks the furthest.

(b) Answers will vary. Possible answers include:

$$\text{Station X} \to \text{rabbit} \to \text{deer} \to \text{raccoon} \to \text{chipmunk} \to \text{Station Y}$$

The distance of this path is $500 + 500 + 100 + 300 + 300 = 1700$ m.

$$\text{Station X} \to \text{rabbit} \to \text{moose} \to \text{raccoon} \to \text{chipmunk} \to \text{Station Y}$$

The distance of this path is $500 + 100 + 250 + 300 + 300 = 1450$ m.

Challenge Solution:

One way to try to find the shortest distance from **Station X** to **Station Y** is to look for the paths that have the fewest stops. Both **Path A** and **Path B** only have two stops between the starting point and the ending point. There are no possible paths that have only one stop between **Station X** and **Station Y**, so **Path A** and **Path B** have the fewest stops possible. However, as we already calculated, neither of these two paths can possibly be the shortest path since **Path C** is shorter. So this strategy for finding the shortest path does not work in general.

Another strategy for searching for the shortest path is called the *greedy algorithm*. With this strategy, at each intersection we pick the path with the shortest distance leaving that intersection. **Path C** follows this strategy.

At **Station X**, we follow the path that is 500 m instead of the path that is 750 m. At the **rabbit** intersection, we follow the path that is 100 m instead of the path that is $\frac{1}{2}$ km. At the **moose** intersection, we follow the path that is 250 m instead of the path that is 400 m. At the **raccoon** intersection, we follow the path that is 100 m instead of the path that is 300 m. At the **deer** intersection, we follow the path that is 1 km instead of the path that leads us back to an intersection we have already visited. We see that this strategy finds a path that is shorter than the other two paths that are listed, but is this the shortest possible path?

In this problem, we can use trial and error to find at least one other path that is shorter than **Path C**. So the greedy algorithm strategy for finding the shortest path does not work in general.

The next question is, how do we find the shortest path and know that there are no paths that are shorter? We could use trial and error and list all possible paths from **Station X** to **Station Y**. Then we choose the shortest path from all possible paths. However, this can get tricky and it is easy to miss one or more paths. There is a strategy that is used in higher mathematics called *Dijkstra's Algorithm*. We won't go through the details of this algorithm here, but the result of following this strategy leads us to the shortest path from **Station X** to **Station Y**:

$$\text{Station X} \to \text{rabbit} \to \text{moose} \to \text{raccoon} \to \text{chipmunk} \to \text{Station Y}$$

The distance of this path is $500 + 100 + 250 + 300 + 300 = 1450$ m.

Number Sense (N)

Take me to the cover

Problem of the Week

Problem A

Difference of Numbers

Consider the digits in the box below:

2	5	6	1

(a) What is the largest number that uses each digit exactly once?

(b) What is the smallest number that uses each digit exactly once?

(c) Find the difference between the number you found in part (a) and the number you found in part (b).

Theme: Number Sense

Problem of the Week

Problem A and Solution

Difference of Numbers

Problem

Consider the digits in the box below:

2	5	6	1

(a) What is the largest number that uses each digit exactly once?

(b) What is the smallest number that uses each digit exactly once?

(c) Find the difference between the number you found in part (a) and the number you found in part (b).

Solution

(a) To create the largest number, the digits should be arranged in decreasing order with the largest digit in the thousands place and the smallest digit in the ones place. That number is 6521.

(b) To create the smallest number, the digits should be arranged in increasing order with the smallest digit in the thousands place and the largest digit in the ones place. That number is 1256.

(c) The difference between the numbers is $6521 - 1256 = 5265$.

Problem of the Week

Problem A

What Number Am I?

I am a 4-digit number. Use the following clues to determine my digits:

(1) My hundreds digit is the difference between the number of sides of a hexagon and the number of sides of a triangle.

(2) Reading my digits from left to right, they are in increasing order from the thousands digit to the ones digit. Also, they increase by the same amount each time.

(3) The difference between my tens digit and my ones digit is equal to the number of sides of an octagon divided by the number of right angles in a rectangle.

What number am I?

Themes: Computational Thinking, Geometry & Measurement, Number Sense

Problem of the Week
Problem A and Solution
What Number Am I?

Problem

I am a 4-digit number. Use the following clues to determine my digits:

(1) My hundreds digit is the difference between the number of sides of a hexagon and the number of sides of a triangle.

(2) Reading my digits from left to right, they are in increasing order from the thousands digit to the ones digit. Also, they increase by the same amount each time.

(3) The difference between my tens digit and my ones digit is equal to the number of sides of an octagon divided by the number of right angles in a rectangle.

What number am I?

Solution

Using Clue (1), the number of sides of a hexagon is 6 and the number of sides of a triangle is 3. The difference between these two numbers is $6 - 3 = 3$, so the hundreds digit is 3.

Using Clue (3), the number of sides of an octagon is 8 and the number of right angles in a rectangle is 4. So, the difference between the tens digit and the ones digit is $8 \div 4 = 2$.

Using Clue (2), since the digits increase by the same amount each time, then as we read the digits from left to right the numbers will increase by 2. Since the hundreds digit is 3, then the tens digit must be $3 + 2 = 5$. Since the tens digit is 5, then the ones digit must be $5 + 2 = 7$. Working backwards, since the hundreds digit is 3, then the thousands digit must be $3 - 2 = 1$.

Therefore the number is 1357.

Problem of the Week

Problem A

Comparing Communications

Several students compared how many emails they had sent during the previous week.

Number of Emails Sent	Number of Students
0	12
1	19
2	11
3	8
4	14
5	2
6	15

(a) How many students sent fewer than 2 emails?

(b) How many students sent more than 4 emails?

(c) Separate the students into two groups so that all the students in one group sent more emails than all the students in the other group, and the number of students in each group are as close as possible.

Themes: Data Management, Number Sense

Problem of the Week
Problem A and Solution
Comparing Communications

Problem

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Number of Emails Sent	Number of Students
0	12
1	19
2	11
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6	15

(a) How many students sent fewer than 2 emails?

(b) How many students sent more than 4 emails?

(c) Separate the students into two groups so that all the students in one group sent more emails than all the students in the other group, and the number of students in each group are as close as possible.

Solution

We can use the table information to determine the answers.

(a) The numbers 0 and 1 are less than 2. So the total number of students who sent fewer than 2 emails is $12 + 19 = 31$.

(b) The numbers 5 and 6 are more than 4. So the total number of students who sent more than 4 emails is $2 + 15 = 17$.

(c) The total number of students is $12 + 19 + 11 + 8 + 14 + 2 + 15 = 81$. If the number of students in each group are as close as possible, then one group would have 40 students and the other would have 41. This is our target.

We notice that the number of students who sent fewer than 3 emails is $12 + 19 + 11 = 42$. This is quite close to 41. In fact, it is as close as we can get to 41 because the number of students who sent 3 emails is 8 and the number of students who sent 2 emails is 11, so adding 8 or subtracting 11 from 42 would take us further away from 41. Thus, one group contains the 42 students who sent fewer than 3 emails and the other group contains the $81 - 42 = 39$ students who sent 3 or more emails.

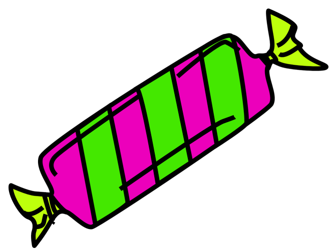
Problem of the Week

Problem A

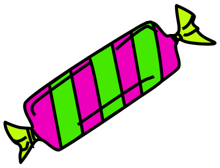
Candy Crunch

Carissa bought a box of candy to share with her friends. However, when she opened the box there was much less candy than she expected. The box says it contains a total of 420 grams of candy. In the box, there are two types of candies: Sour Sassies and Caramellos. Each Sour Sassy has a mass of 15 grams and each Caramello has a mass of 10 grams.

If she counts 10 Sour Sassies and 25 Caramellos in the box, was the information on the box correct? Justify your answer.



Theme: Number Sense



Problem of the Week
Problem A and Solution
Candy Crunch

Problem

Carissa bought a box of candy to share with her friends. However, when she opened the box there was much less candy than she expected. The box says it contains a total of 420 grams of candy. In the box, there are two types of candies: Sour Sassies and Caramellos. Each Sour Sassy has a mass of 15 grams and each Caramello has a mass of 10 grams.

If she counts 10 Sour Sassies and 25 Caramellos in the box, was the information on the box correct? Justify your answer.

Solution

Since the Sour Sassies each have a mass of 15 grams and there are 10 Sour Sassies, we can use skip counting to calculate the total mass of the Sour Sassies:

$$15, 30, 45, 60, 75, 90, 105, 120, 135, 150$$

Alternatively, we can multiply to determine the total mass of the Sour Sassies is $10 \times 15 = 150$ grams.

Since the Caramellos each have a mass of 10 grams and there are 25 Caramellos, we can use skip counting to calculate the total mass of the Caramellos:

$$10, 20, 30, 40, 50, 60, 70, 80, 90, 100, 110, 120, 130, 140, 150,$$

$$160, 170, 180, 190, 200, 210, 220, 230, 240, 250$$

Alternatively, we can multiply to determine the total mass of the Caramellos is $25 \times 10 = 250$ grams.

The total mass of both candies is $250 + 150 = 400$ grams. This is less than the mass shown on the box, so the information on the box is incorrect.

Problem of the Week

Problem A

Bussing Business

Ms. Oakley is taking her 20 students on a field trip to the Science Centre. The costs are as follows:

- It costs $440 for bus transportation to and from the Science Centre.
- The entrance fee to the Science Centre is $20 per student.
  There is no charge for Ms. Oakley.
- The cost to watch the IMAX movie is $7 per student.
  There is no charge for Ms. Oakley.

The budget for this field trip is $50 per student. Do they have enough money in the budget for all students to watch the IMAX movie while they are at the Science Centre? Justify your answer.

Problem of the Week
Problem A and Solution
Bussing Business

Problem

Ms. Oakley is taking her 20 students on a field trip to the Science Centre. The costs are as follows:

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The budget for this field trip is \$50 per student. Do they have enough money in the budget for all students to watch the IMAX movie while they are at the Science Centre? Justify your answer.

Solution

Solution 1

One way to solve this problem is to calculate the total budget available, and the total cost for each component of the trip. Since there are 20 students and the budget for each student is \$50, then the total budget available is $20 \times 50 = \$1000$.

The cost of transportation for all students is \$440.
The cost for the entrance fee for all students is $20 \times \$20 = \$400$.
The cost for all students to watch the IMAX movie is $20 \times \$7 = \$140$.

Therefore the total cost of the trip, including the IMAX movie, would be $440 + 400 + 140 = \$980$. Since this is less than the \$1000 budget, they have enough money to watch the IMAX movie.

Solution 2

Another way to approach this problem is to calculate a cost per student for the trip. Since there are 20 students, then each student's share of the cost of the bus is $440 \div 20 = \$22$. Therefore, the total cost of the trip per student, including the IMAX movie, would be: $22 + 20 + 7 = \$49$. This is less than the budget of \$50 for each student, so they have enough money to watch the IMAX movie.

Problem of the Week

Problem A

Food Bank Challenge

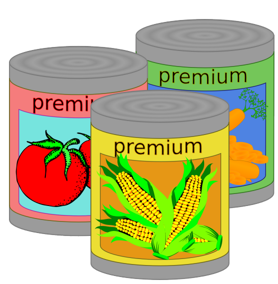
Two classes at Whitefish Bay Elementary School took on a challenge to gather donations for a local food bank. Different types of food are worth different amounts of points in the challenge.

- 5-point foods: canned meat, canned veggies, canned fruit, lentils, rice
- 4-point foods: cereal, crackers, canned beans, canned fish, pasta sauce
- 3-point foods: nuts, peanut butter, granola bars
- 2-point foods: chips, cookies
- 1-point foods: all other non-perishable food items

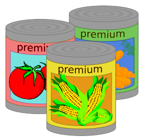
The table below lists the donations made by each class.

Class A	Class B
6 cans of tuna fish	3 boxes of rice
5 jars of peanut butter	5 packages of granola bars
2 bags of lentils	4 cans of peaches
3 cans of peas	2 packages of almonds
2 bags of chips	7 boxes of cereal
4 boxes of pasta	2 packages of cookies
4 jars of pasta sauce	3 boxes of powdered milk

Which class won the challenge? Justify your answer.



Theme: Number Sense



Problem of the Week

Problem A and Solution

Food Bank Challenge

Problem

Two classes at Whitefish Bay Elementary School took on a challenge to gather donations for a local food bank. Different types of food are worth different amounts of points in the challenge.

- 5-point foods: canned meat, canned veggies, canned fruit, lentils, rice
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The table below lists the donations made by each class.

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3 cans of peas	2 packages of almonds
2 bags of chips	7 boxes of cereal
4 boxes of pasta	2 packages of cookies
4 jars of pasta sauce	3 boxes of powdered milk

Which class won the challenge? Justify your answer.

Solution

For each class, we calculate how many points they earn for each type of food donated, and summarize in a table.

Class A:

Donation	Points per Unit	Total Points
6 cans of tuna fish	4	$6 \times 4 = 24$
5 jars of peanut butter	3	$5 \times 3 = 15$
2 bags of lentils	5	$2 \times 5 = 10$
3 cans of peas	5	$3 \times 5 = 15$
2 bags of chips	2	$2 \times 2 = 4$
4 boxes of pasta	1	$4 \times 1 = 4$
4 jars of pasta sauce	4	$4 \times 4 = 16$

Class B:

Donation	Points per Unit	Total Points
3 boxes of rice	5	$3 \times 5 = 15$
5 packages of granola bars	3	$5 \times 3 = 15$
4 cans of peaches	5	$4 \times 5 = 20$
2 packages of almonds	3	$2 \times 3 = 6$
7 boxes of cereal	4	$7 \times 4 = 28$
2 packages of cookies	2	$2 \times 2 = 4$
3 boxes of powdered milk	1	$3 \times 1 = 3$

Now we can add the total points for each class, to determine that Class A made $24 + 15 + 10 + 15 + 4 + 4 + 16 = 88$ points, and Class B made $15 + 15 + 20 + 6 + 28 + 4 + 3 = 91$ points.

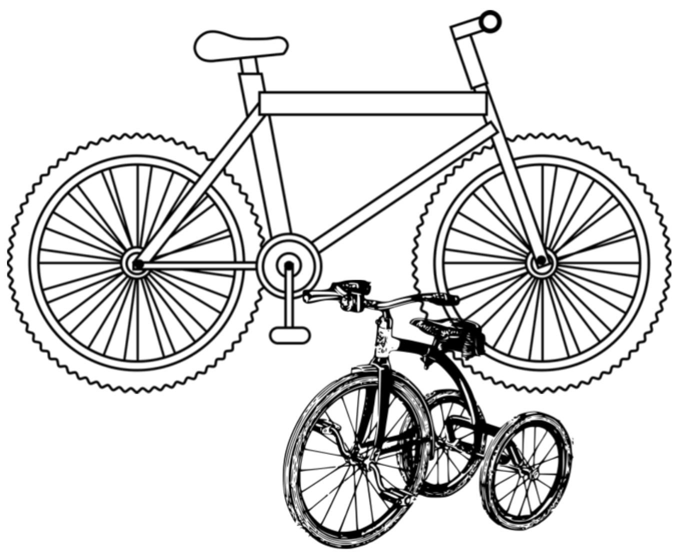
Therefore, **Class B** won the challenge.

Problem of the Week
Problem A
Storage Unit

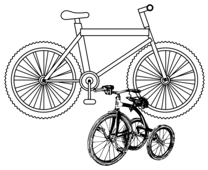
Jerry bought the contents of two large storage units at an auction for a total of \$500. One storage unit contains some bicycles and the other storage unit contains some tricycles.

(a) If each storage unit has exactly 72 wheels, how many bicycles and tricycles are there in total? Justify your answer.

(b) Jerry plans to sell each tricycle for \$10 and each bicycle for \$25. What is the maximum profit Jerry can make if he is able to sell all the bicycles and tricycles in storage for these prices? Justify your answer.



Themes: Algebra, Number Sense



Problem of the Week
Problem A and Solution
Storage Unit

Problem

Jerry bought the contents of two large storage units at an auction for a total of \$500. One storage unit contains some bicycles and the other storage unit contains some tricycles.

(a) If each storage unit has exactly 72 wheels, how many bicycles and tricycles are there in total? Justify your answer.

(b) Jerry plans to sell each tricycle for \$10 and each bicycle for \$25. What is the maximum profit Jerry can make if he is able to sell all the bicycles and tricycles in storage for these prices? Justify your answer.

Solution

(a) Since a tricycle has 3 wheels and there are 72 wheels in one of the storage units, then we can determine the number of tricycles by dividing 72 by 3. There are $72 \div 3 = 24$ tricycles in one of the storage units.

Similarly, since a bicycle has 2 wheels and there are 72 wheels in one of the storage units, then we can determine the number of bicycles by dividing 72 by 2. There are $72 \div 2 = 36$ bicycles in the other storage unit.

(b) To calculate the amount of money Jerry can make from selling the tricycles, we multiply $24 \times \$10 = \$240$.

To calculate the amount of money Jerry can make from selling the bicycles, we multiply $36 \times \$25 = \$900$. Alternatively, we can skip count by 25. A third way to figure out the amount of money is to notice that if he sells 4 bicycles, he would make \$100. Then we can use a table to figure out how much he would make selling 36 bicycles:

Bikes Sold	4	8	12	16	20	24	28	32	36
Money Made	\$100	\$200	\$300	\$400	\$500	\$600	\$700	\$800	\$900

Therefore, Jerry could make $\$240 + \$900 = \$1140$ if he sells all of the bikes. We need to subtract the amount he paid for the storage unit to determine the overall profit. Therefore, the maximum profit Jerry could make by selling all the bikes is $\$1140 - \$500 = \$640$.

Problem of the Week
Problem A
Intervals

Adil completes a 12 km route using interval training. Interval training is a way to prepare for a race. One interval consists of running for $\frac{2}{3}$ of a kilometre followed by walking for $\frac{2}{3}$ of a kilometre.

(a) How many intervals will he do when completing the 12 km route?

(b) After two weeks, he changes his intervals to running for $\frac{3}{4}$ of a kilometre followed by walking for $\frac{3}{4}$ of a kilometre. At this pace, how many intervals does it take to complete the 12 km route?

Theme: Number Sense

Problem of the Week
Problem A and Solution
Intervals

Problem

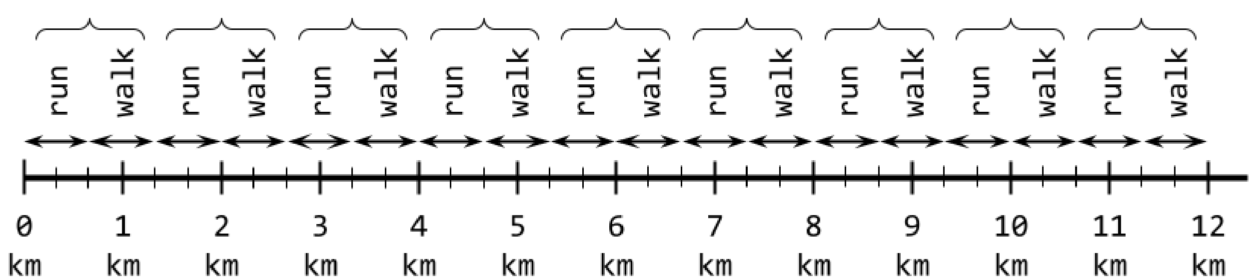
Adil completes a 12 km route using interval training. Interval training is a way to prepare for a race. One interval consists of running for $\frac{2}{3}$ of a kilometre followed by walking for $\frac{2}{3}$ of a kilometre.

(a) How many intervals will he do when completing the 12 km route?

(b) After two weeks, he changes his intervals to running for $\frac{3}{4}$ of a kilometre followed by walking for $\frac{3}{4}$ of a kilometre. At this pace, how many intervals does it take to complete the 12 km route?

Solution

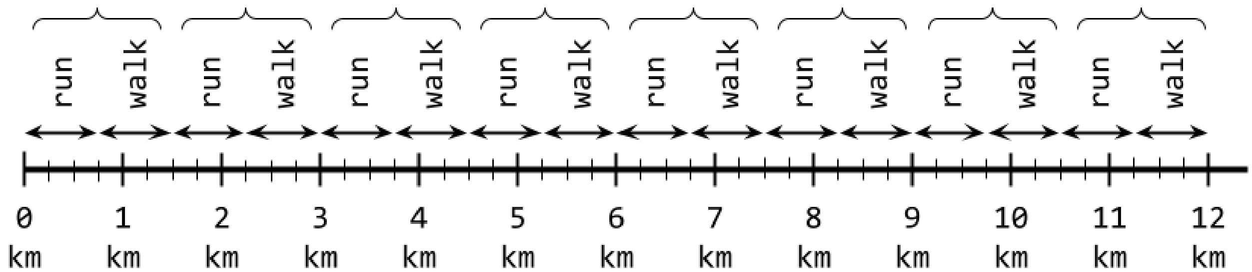
(a) One way to solve this problem is to create a number line from 0 km to 12 km, with a tick at every $\frac{1}{3}$ of a kilometre. Then, two spaces between ticks on the number line will correspond to the running portion of an interval, and two spaces between ticks on the number line will correspond to the walking portion of an interval. Thus, four spaces between ticks on the number line will correspond to one interval in the interval training. (And so, each interval covers $\frac{4}{3}$ of a kilometre.)



Counting the spaces between ticks, we determine that there are a total of 9 intervals necessary to complete the 12 km route.

We don't necessarily have to complete the whole number line to get the final answer. Notice that after 3 intervals Adil has completed 4 km. Since $4 \times 3 = 12$, Adil needs to complete 3 times that number of intervals to finish 12 km. So it takes $3 \times 3 = 9$ intervals to complete 12 km.

(b) We create a number line from 0 km to 12 km, with a tick at every $\frac{1}{4}$ of a kilometre (rather than every $\frac{1}{3}$ of a kilometre). Then, three spaces between ticks on the number line will correspond to the running portion of an interval, and three spaces between ticks on the number line will correspond to the walking portion of an interval. Thus, six spaces between ticks on the number line will correspond to one interval in the interval training.



Counting the spaces between ticks, we determine that there are a total of 8 intervals necessary to complete the 12 km route.

Again, we don't necessarily have to complete the whole number line to get the final answer. Notice that after 2 intervals Adil has completed 3 km. Since $3 \times 4 = 12$, Adil needs to complete 4 times that number of intervals to finish 12 km. So it takes $2 \times 4 = 8$ intervals to complete 12 km.

Problem of the Week

Problem A

Follow the Rules

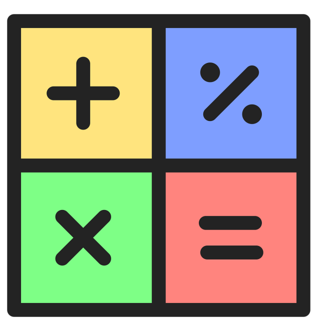
Consider the following step-by-step instructions:

1. Pick a 2-digit number.
2. If the number is even, divide it by 2. If the number is odd, add 10 to the number.
3. If the result of the calculation in step 2 is a 2-digit number, add its tens digit to its ones digit. Otherwise, add 20 to the number calculated in step 2.
4. If the result of the calculation in step 3 is a multiple of 5, divide it by 5. Otherwise, add 12 to the number calculated in step 3.
5. If the result of the calculation in step 4 is greater than 30, add 1 to it. Otherwise, subtract 1 from the number calculated in step 4.

(a) What is the result of following these instructions if you start with 25?

(b) What is the result of following these instructions if you start with 95?

(c) What is the result of following these instructions if you start with 16?



Themes: Computational Thinking, Number Sense

Problem of the Week
Problem A and Solution
Follow the Rules

Problem

Consider the following step-by-step instructions:

1. Pick a 2-digit number.
2. If the number is even, divide it by 2. If the number is odd, add 10 to the number.
3. If the result of the calculation in step 2 is a 2-digit number, add its tens digit to its ones digit. Otherwise, add 20 to the number calculated in step 2.
4. If the result of the calculation in step 3 is a multiple of 5, divide it by 5. Otherwise, add 12 to the number calculated in step 3.
5. If the result of the calculation in step 4 is greater than 30, add 1 to it. Otherwise, subtract 1 from the number calculated in step 4.

(a) What is the result of following these instructions if you start with 25?

(b) What is the result of following these instructions if you start with 95?

(c) What is the result of following these instructions if you start with 16?

Solution

(a) Following the steps and starting with 25 produces this sequence of numbers:

$$25, 35, 8, 20, 19$$

(b) Following the steps and starting with 95 produces this sequence of numbers:

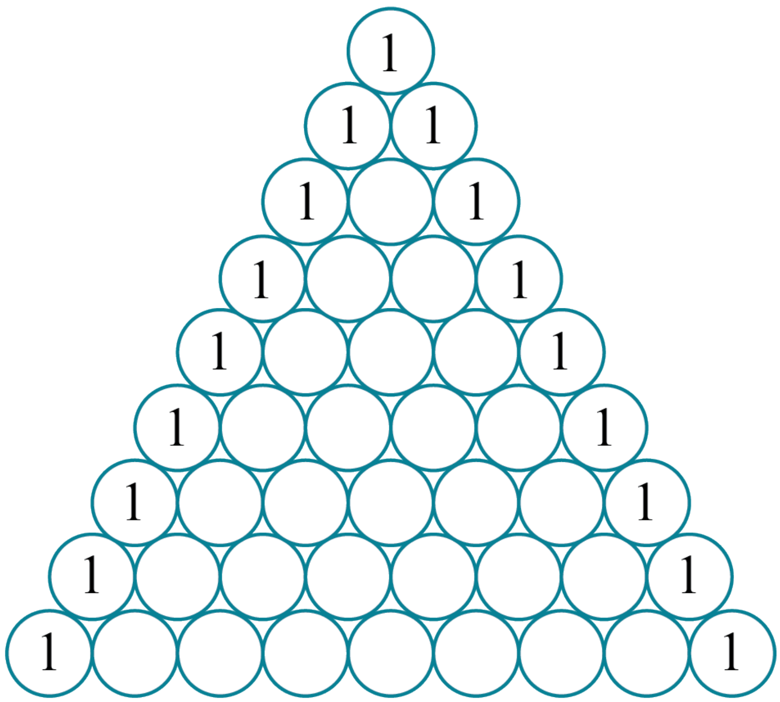
$$95, 105, 125, 25, 24$$

(c) Following the steps and starting with 16 produces this sequence of numbers:

$$16, 8, 28, 40, 41$$

Problem of the Week
Problem A
Pencil Pyramid

Juliane arranges identical cylindrical tubes to form a pyramid. She puts one pencil in the top tube and then puts one pencil in the leftmost and rightmost tubes in each of the remaining rows, as shown.



Juliane then fills the remaining tubes as follows:

Step 1: Find the topmost empty tube and add up the number of pencils in the two tubes touching it in the row above.

Step 2: Put that many pencils in the empty tube.

Step 3: Repeat Steps 1 and 2 until all tubes are filled.

(a) Complete the pyramid with the number of pencils in each tube.

(b) Can you find any patterns in the numbers in the completed pyramid? If so, explain them.

Themes: Algebra, Number Sense

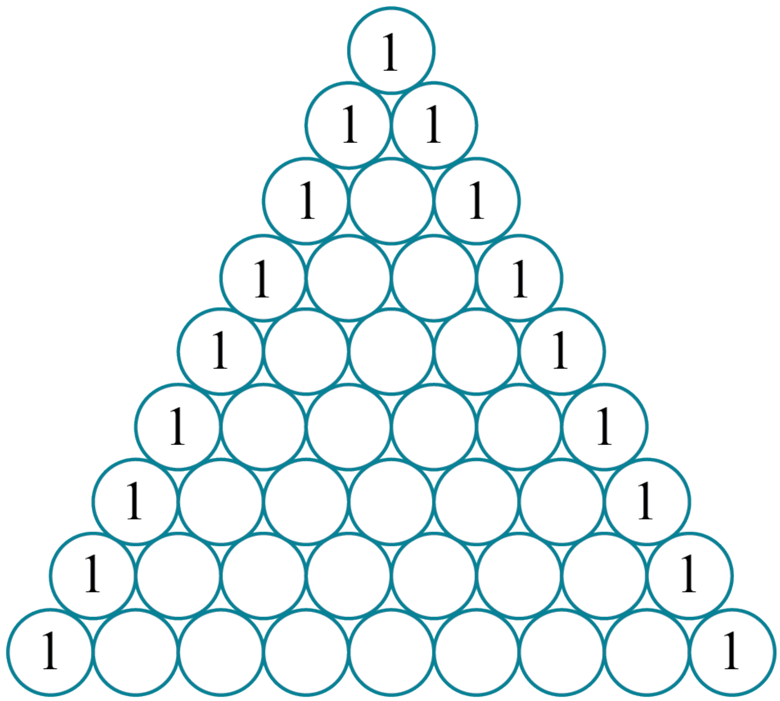
Problem of the Week

Problem A and Solution

Pencil Pyramid

Problem

Juliane arranges identical cylindrical tubes to form a pyramid. She puts one pencil in the top tube and then puts one pencil in the leftmost and rightmost tubes in each of the remaining rows, as shown.



Juliane then fills the remaining tubes as follows:

Step 1: Find the topmost empty tube and add up the number of pencils in the two tubes touching it in the row above.

Step 2: Put that many pencils in the empty tube.

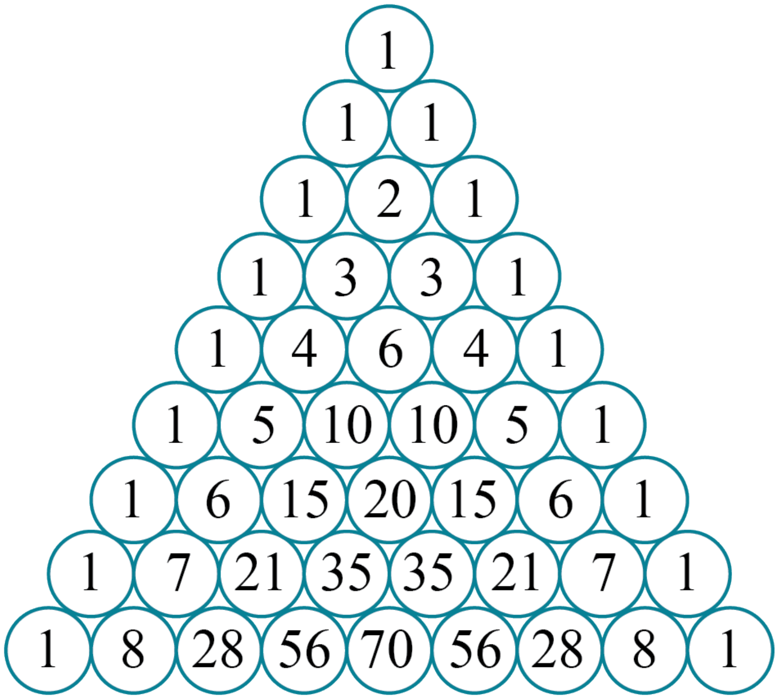
Step 3: Repeat Steps 1 and 2 until all tubes are filled.

(a) Complete the pyramid with the number of pencils in each tube.

(b) Can you find any patterns in the numbers in the completed pyramid? If so, explain them.

Solution

(a) The completed pyramid is shown.



(b) Here are some patterns you may have noticed:

- The triangle is symmetric, so the left side matches the right side.
- Starting with the leftmost 1 in the second row and moving down diagonally we see the numbers 1, 2, 3, 4, 5, 6, 7, and 8.
- Starting with the leftmost 1 in the third row and moving down diagonally we see the numbers 1, 3, 6, 10, 15, 21, and 28. The pattern rule for this sequence is to add 2, then 3, then 4, then 5, etc.
- The sum of the numbers in any row is twice the sum of the numbers in the row above it.
- If you start from any 1, follow the diagonal down, and then change directions for the last number, the sum of the numbers in the diagonal will equal the last number. For example, $1 + 6 + 21 = 28$. This is called the *Hockey Stick Identity* because if you circle the numbers used, it will look somewhat like a hockey stick.

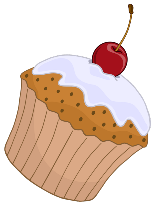
Teacher's Notes

This problem is an exploration of *Pascal's Triangle*, which is named for French mathematician Blaise Pascal (1623 - 1662). There are lots of patterns to be found in this triangle. We listed some of the patterns in the solution, but perhaps your students were able to find others!

Pascal's Triangle is also used in algebra. The numbers in the n^{th} row provide the coefficients for the expansion of a *binomial expression* raised to the n^{th} power. The expansions for $n = 0$ to $n = 4$ are shown.

$$\begin{aligned}
(x+y)^0 &= 1\\
(x+y)^1 &= 1x + 1y\\
(x+y)^2 &= 1x^2 + 2xy + 1y^2\\
(x+y)^3 &= 1x^3 + 3x^2y + 3xy^2 + 1y^3\\
(x+y)^4 &= 1x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + 1y^4
\end{aligned}$$

As the value of n increases, expanding the binomials manually becomes quite time consuming. Using Pascal's Triangle allows us to expand binomial expressions more efficiently.



Problem of the Week
Problem A
Cupcake Bonanza

Archie, Lin, and Umar are making vanilla cupcakes for a bake sale. They need to make 8 dozen cupcakes using the recipe below.

Recipe to make 1 dozen vanilla cupcakes:

- 250 grams flour
- $2\frac{1}{2}$ grams salt
- 8 grams baking powder
- $\frac{1}{4}$ cup of butter
- 150 grams sugar
- 2 eggs
- 240 mL milk
- 5 mL vanilla

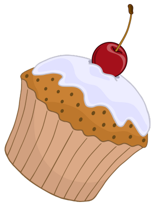
(a) How much of each ingredient do they need to make 8 dozen cupcakes?

(b) They have enough salt, baking powder, and vanilla already. They do not have any of the other ingredients. When they go to the store, they see the ingredients they need are sold in packages in the following sizes:

- Flour is sold in 1 kg bags.
- Butter is sold in 1 cup blocks.
- Sugar is sold in 2 kg bags.
- Eggs are sold by the dozen.
- Milk is sold in 1 L cartons.

How many of each package of the ingredients do they need to buy to have enough to make all of the cupcakes?

Themes: Geometry & Measurement, Number Sense



Problem of the Week
Problem A and Solution
Cupcake Bonanza

Problem

Archie, Lin, and Umar are making vanilla cupcakes for a bake sale. They need to make 8 dozen cupcakes using the recipe below.

Recipe to make 1 dozen vanilla cupcakes:

- 250 grams flour
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- 8 grams baking powder
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- 150 grams sugar
- 2 eggs
- 240 mL milk
- 5 mL vanilla

(a) How much of each ingredient do they need to make 8 dozen cupcakes?

(b) They have enough salt, baking powder, and vanilla already. They do not have any of the other ingredients. When they go to the store, they see the ingredients they need are sold in packages in the following sizes:

- Flour is sold in 1 kg bags.
- Butter is sold in 1 cup blocks.
- Sugar is sold in 2 kg bags.
- Eggs are sold by the dozen.
- Milk is sold in 1 L cartons.

How many of each package of the ingredients do they need to buy to have enough to make all of the cupcakes?

Solution

(a) Since the recipe makes 1 dozen cupcakes, to calculate how much of each ingredient we need to make 8 dozen cupcakes we must multiply the amount of each ingredient by 8. For the larger numbers, we can do this calculation by skip counting or multiple additions.

For example, calculating the amount of flour required we could
skip count: 250, 500, 750, 1000, 1250, 1500, 1750, 2000,
or do multiple additions: $250 + 250 + 250 + 250 + 250 + 250 + 250 + 250 = 2000$.

Another way we could calculate 8 times some number is to double the value 3 times. Notice that if we double the number 1, we get 2. If we double the number 2, we get 4. If we double the number 4, we get 8. If we follow this same procedure with any starting number, the result will be 8 times the original value. For example, if we double 250, we get 500. If we double 500, we get 1000. If we double 1000, we get 2000.

Using any of these procedures, we determine that $8 \times 250 = 2000$.

The total amount of each ingredient needed to make 8 dozen cupcakes is below.

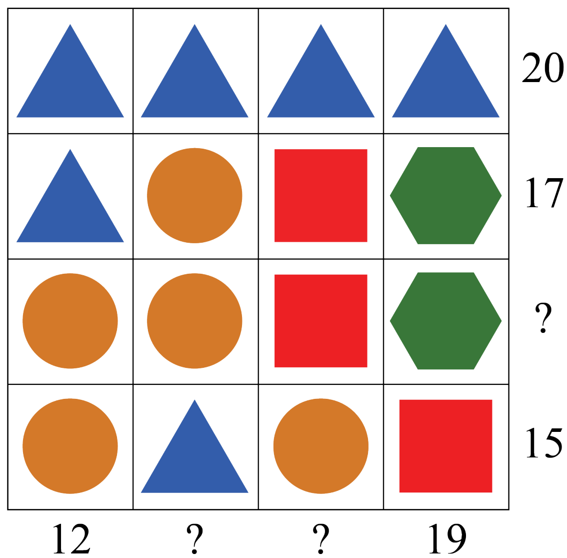
- Flour: $250 \times 8 = 2000$ g. Since 1000 g = 1 kg, then 2000 g = 2 kg is needed.
- Salt: $2\frac{1}{2} + 2\frac{1}{2} = 5$ grams, so 2 dozen cupcakes require 5 grams. Thus, 8 dozen cupcakes require $4 \times 5 = 20$ grams.
- Baking powder: $8 \times 8 = 64$ grams required.
- Butter: $\frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} = 1$ cup, so 4 dozen cupcakes require 1 cup. Thus, 8 dozen cupcakes require 2 cups.
- Sugar: $150 \times 8 = 1200$ grams required.
- Eggs: $2 \times 8 = 16$ eggs required.
- Milk: $240 \times 8 = 1920$ mL required.
- Vanilla: $5 \times 8 = 40$ mL required.

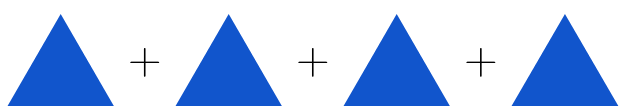
(b) We can use the total amounts calculated in part (a) to determine how many packages of each ingredient they must buy.

- Since flour is sold in 1 kg bags and they need 2 kg of flour, then they need 2 bags of flour.
- Since butter is sold in 1 cup blocks, and they need 2 cups of butter, then they need 2 blocks of butter.
- Since sugar is sold in 2 kg bags, and 2 kg = 2000 g, then they need 1 bag of sugar to have at least 1200 g of sugar.
- Since eggs are sold by the dozen, and there are 12 eggs in 1 dozen and 24 eggs in 2 dozen, then they need 2 cartons of eggs to have at least 16 eggs.
- Since milk is sold in 1 L cartons, and since 1 L = 1000 mL and so 2 L = 2000 mL, then they need 2 cartons of milk.

Problem of the Week
Problem A
Shape Sums

Each symbol in the grid below represents a number. The sums of the symbols in the first, second, and bottom rows and the leftmost and rightmost columns are shown.



For example, we know:  $= 20$

Determine all the missing sums.

Themes: Algebra, Number Sense

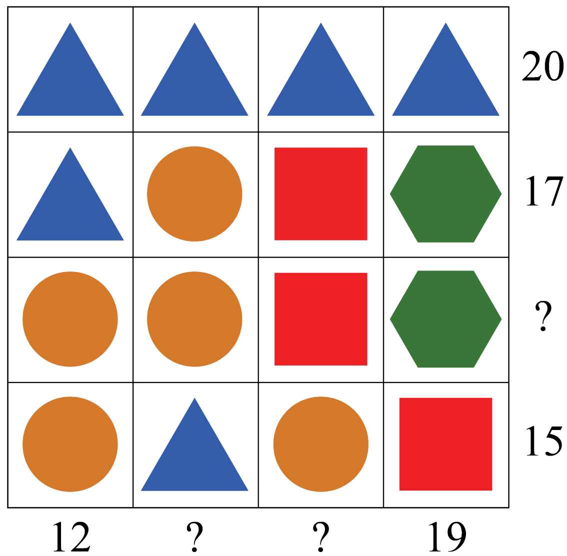
Problem of the Week

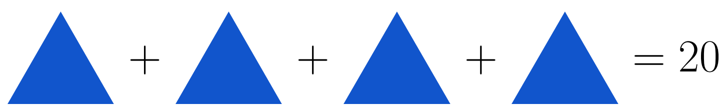
Problem A and Solution

Shape Sums

Problem

Each symbol in the grid below represents a number. The sums of the symbols in the first, second, and bottom rows and the leftmost and rightmost columns are shown.



For example, we know: 

Determine all the missing sums.

Solution

There are many ways to solve this problem. Here is one approach.

Since the leftmost column has two triangles and two circles, and the column beside it also has two triangles and two circles, the sum of these columns must be the same. So the sum of the second column must be 12.

From the top row we know that the sum of four triangles is 20. So the value of one triangle must be $\frac{1}{4}$ of 20 or $20 \div 4 = 5$.

Since the value of one triangle is 5, then the sum of two triangles is $5 + 5 = 10$. The leftmost column shows the sum of two triangles and two circles is 12, so the sum of two circles is $12 - 10 = 2$. Therefore, the value of one circle must be $\frac{1}{2}$ of 2 or $2 \div 2 = 1$.

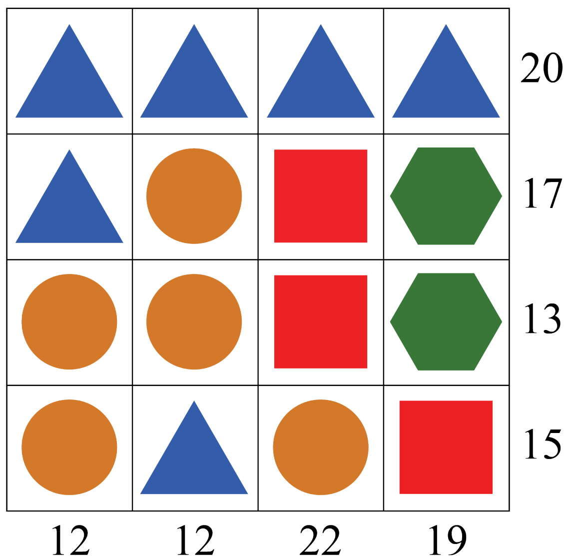
The bottom row has a sum of 15 and contains two circles, one triangle, and one square. Since we know the sum of two circles and one triangle is $2 + 5 = 7$, then the value of one square is $15 - 7 = 8$.

Now we know that the sum of the third column is $5 + 8 + 8 + 1 = 22$.

The second row has a sum of 17 and contains one triangle, one circle, one square and one hexagon. Since we know the sum of one triangle, one circle, and one square is $5 + 1 + 8 = 14$, then the value of one hexagon is $17 - 14 = 3$.

Now we know that the sum of the third row is $1 + 1 + 8 + 3 = 13$.

The completed grid is shown.



Alternatively, we can find the sum of the third row without actually finding the value of the hexagon. Notice that the second and third rows have the same shapes in the last three columns, and only the shape in the first column is different. Since we know that the value of one triangle is 5 and the value of one circle is 1, then the sum of the third row must be $5 - 1 = 4$ less than the sum of the second row. Thus, the sum of the third row is $17 - 4 = 13$.

Teacher's Notes

We could look at this problem algebraically, where we are trying to solve a *linear system of equations*. If we assign variable names to the shapes,

- t = triangle
- c = circle
- s = square
- h = hexagon

then we can describe the information given in the problem as follows:

$$\begin{aligned} 4t &= 20 \\ t + c + s + h &= 17 \\ 2c + t + s &= 15 \\ 2t + 2c &= 12 \\ t + 2h + s &= 19 \end{aligned}$$

Now, we could use algebraic techniques to find the values of these four variables.

We expect that there is a single solution to these types of problems, and it turns out that there is for this problem. However, for some problems we are not given enough information to determine specific values for each variable. For a problem with four variables, a minimum requirement to guarantee a single solution is that we need *at least* four equations. Also, those equations must be *consistent*. This means that none of the equations can contradict each other. For example, if we added the equation $t = 7$ to the system in this problem, this would contradict the first equation. A system that is *inconsistent* has no solutions. This is something that students will learn more about in higher level math courses.

Problem of the Week

Problem A

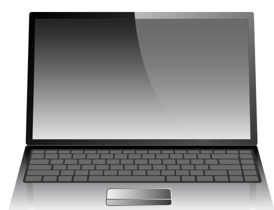
Fund Raising

Flynn's class is raising money for a tree-planting charity by recycling electronics. They have found a local company that will give them \$5 for each pound of cell phone e-waste and \$2 for each pound of computer e-waste.

The school has gathered the following electronics for recycling, but has weighed them in grams instead of pounds.

Cell Phones	**Computers**
10 uPhones: 200 g each	5 Bonobo laptops: 800 g each
5 Mixel phones: 100 g each	7 uPads: 500 g each

Knowing that 1 pound is about 454 g, estimate how much money Flynn's class will make. Justify your answer.



Themes: Geometry & Measurement, Number Sense

Problem of the Week
Problem A and Solution
Fund Raising

Problem

Flynn's class is raising money for a tree-planting charity by recycling electronics. They have found a local company that will give them \$5 for each pound of cell phone e-waste and \$2 for each pound of computer e-waste.

The school has gathered the following electronics for recycling, but has weighed them in grams instead of pounds.

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Knowing that 1 pound is about 454 g, estimate how much money Flynn's class will make. Justify your answer.

Solution

Solutions may vary, possibly resulting in different estimations. We will show two different approaches.

Since 1 pound is about 454 g, we will approximate 1 pound as 500 g. Then 2 pounds is approximately 1000 g.

Solution 1

In this solution, we find the total mass of each type of item and then estimate this in pounds before finding the total mass of all items.

Phones:

- uPhones: Each uPhone is 200 g, so 10 uPhones are $10 \times 200 = 2000$ g. Since 1000 g is approximately 2 pounds, then 2000 g is approximately 4 pounds.
- Mixel phones: Each Mixel phone is 100 g, so 5 Mixel phones are $5 \times 100 = 500$ g, which is approximately 1 pound.

This is approximately $4 + 1 = 5$ pounds of cell phone waste. The cell phone e-waste is worth approximately $\$5 \times 5 = \$25$.

Computers:

- Bonobo laptops: Each laptop is 800 g, so 5 laptops are $5 \times 800 = 4000$ g. Since 1000 g is approximately 2 pounds, then 4000 g is approximately $4 \times 2 = 8$ pounds.
- uPads: Each uPad is 500 g, which is approximately 1 pound, so 7 uPads are approximately 7 pounds.

This is approximately $8 + 7 = 15$ pounds of computer waste. The computer e-waste is worth approximately $\$2 \times 15 = \$30$. Therefore the total value is approximately $\$25 + \$30 = \$55$.

Solution 2
In this solution, we will find the total mass of computer waste and the total mass of cell phone waste, and then estimate these totals in pounds.

Phones:

- uPhones: Each uPhone is 200 g, so 10 uPhones are $10 \times 200 = 2000$ g.
- Mixel phones: Each Mixel phone is 100 g, so 5 Mixel phones are $5 \times 100 = 500$ g.

This is $2000 + 500 = 2500$ g. Since 500 g is approximately 1 pound and 1000 g is approximately 2 pounds, then 2500 g is approximately $2 + 2 + 1 = 5$ pounds. The cell phone e-waste is worth approximately $\$5 \times 5 = \$25$.

Computers:

- Bonobo laptops: Each laptop is 800 g, so 5 laptops are $5 \times 800 = 4000$ g.
- uPads: Each uPad is 500 g, so 7 uPads are $7 \times 500 = 3500$ g.

This is $4000 + 3500 = 7500$ g. Since 500 g is approximately 1 pound and 1000 g is approximately 2 pounds, then 7500 g is approximately $2 \times 7 + 1 = 15$ pounds. The computer e-waste is worth approximately $\$2 \times 15 = \$30$. Therefore the total value is approximately $\$25 + \$30 = \$55$.

Problem of the Week

Problem A

Zookeeper

Zoey works at the city's zoo feeding and caring for the animals. Her schedule is the same every morning. She starts work at 7:00 a.m. The table below lists the sequence of morning activities, along with the duration of each activity. The durations include the time it takes to travel to each spot.

Activity	**Duration of Activity**
Team meeting	$\frac{1}{2}$ hour
Feeding the elephants	30 minutes
Moving tropical birds to outside habitat	45 minutes
Filling the monkey feed bins with fruit and nuts	30 minutes
Break	15 minutes
Changing the hay for the camels	45 minutes
Feeding the walruses and seals	$\frac{3}{4}$ hour
Washing down the rhino stalls	$\frac{1}{2}$ hour
Lunch break	half an hour

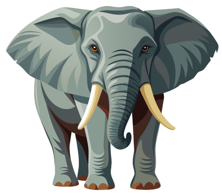
(a) What time does Zoey start feeding the elephants?

(b) What time does the monkey feeding start?

(c) What time does Zoey's break end?

(d) How long is it from the time the team meeting starts to the time Zoey's lunch break ends?

Themes: Geometry & Measurement, Number Sense



Problem of the Week

Problem A and Solution

Zookeeper

Problem

Zoey works at the city's zoo feeding and caring for the animals. Her schedule is the same every morning. She starts work at 7:00 a.m. The table below lists the sequence of morning activities, along with the duration of each activity. The durations include the time it takes to travel to each spot.

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(a) What time does Zoey start feeding the elephants?

(b) What time does the monkey feeding start?

(c) What time does Zoey's break end?

(d) How long is it from the time the team meeting starts to the time Zoey's lunch break ends?

Solution

We add two columns to the table, one with the start time of the activity and one with the end time. From this table, we can answer the questions.

Activity	**Duration of Activity**	**Start Time**	**End Time**
Team meeting	$\frac{1}{2}$ hour	7:00	7:30
Feeding the elephants	30 minutes	7:30	8:00
Moving tropical birds to outside habitat	45 minutes	8:00	8:45
Filling the monkey feed bins with fruit/nuts	30 minutes	8:45	9:15
Break	15 minutes	9:15	9:30
Changing the hay for the camels	45 minutes	9:30	10:15
Feeding the walruses and seals	$\frac{3}{4}$ hour	10:15	11:00
Washing down the rhino stalls	$\frac{1}{2}$ hour	11:00	11:30
Lunch break	half an hour	11:30	12:00

(a) Zoey starts feeding the elephants at 7:30 a.m.

(b) The monkey feeding starts at 8:45 a.m.

(c) Zoey's break ends at 9:30 a.m.

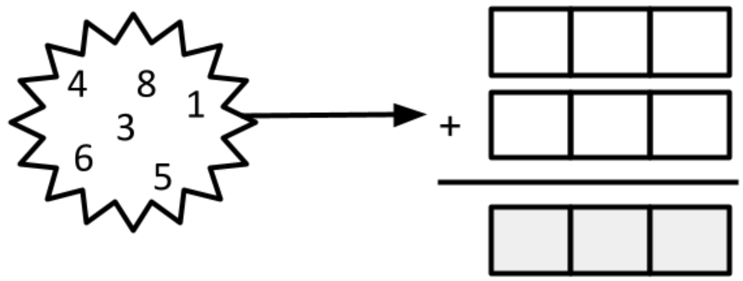
(d) Since the lunch break ends at 12:00 p.m., it is 5 hours after the team meeting starts.

Problem of the Week

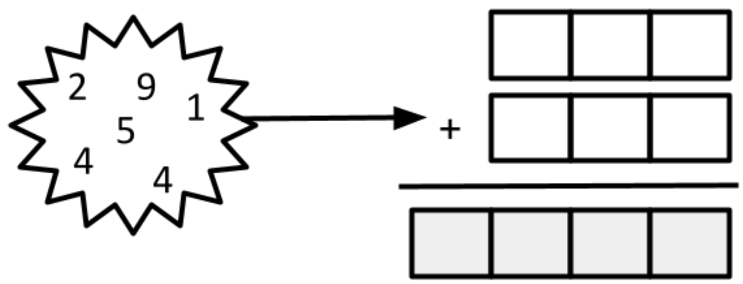
Problem A

Just Filling In

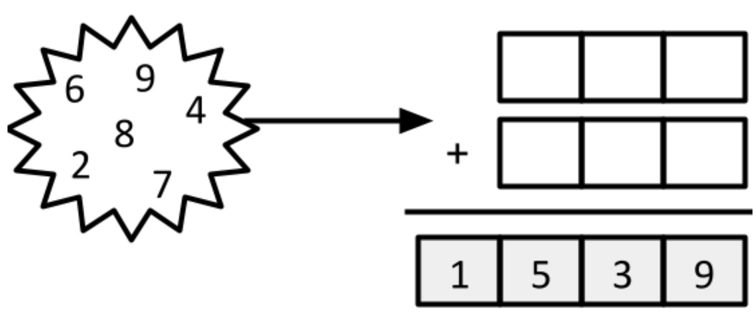
(a) Arrange the numbers 4, 8, 1, 6, 3, and 5 to form two 3-digit numbers with the *smallest* possible sum.



(b) Arrange the numbers 2, 9, 1, 4, 5, and 4 to form two 3-digit numbers with the *largest* possible sum.



(c) Arrange the numbers 6, 9, 4, 2, 8, and 7 to form two 3-digit numbers that add to 1539.



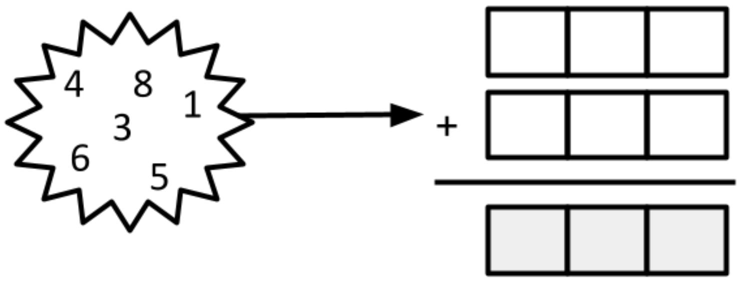
For each part, how many different solutions can you find?

Themes: Computational Thinking, Number Sense

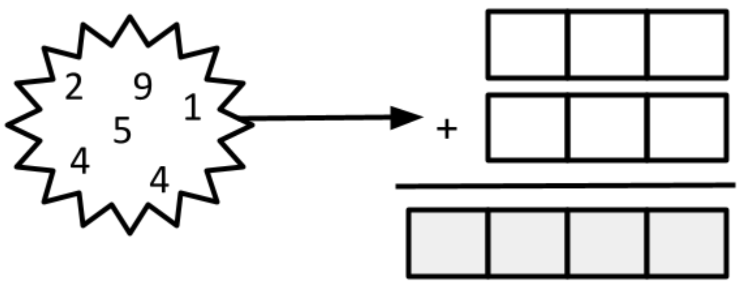
Problem of the Week
Problem A and Solution
Just Filling In

Problem

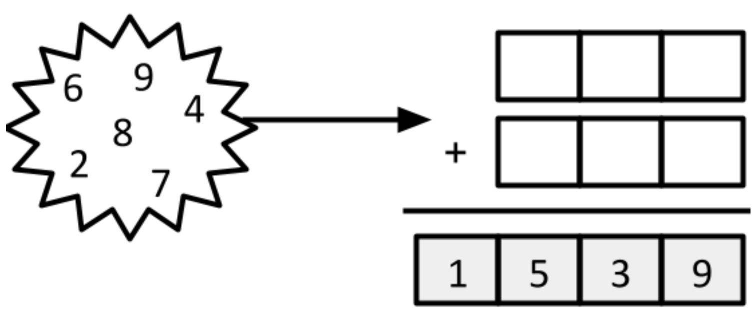
(a) Arrange the numbers 4, 8, 1, 6, 3, and 5 to form two 3-digit numbers with the *smallest* possible sum.



(b) Arrange the numbers 2, 9, 1, 4, 5, and 4 to form two 3-digit numbers with the *largest* possible sum.



(c) Arrange the numbers 6, 9, 4, 2, 8, and 7 to form two 3-digit numbers that add to 1539.



For each part, how many different solutions can you find?

Solution

(a) The strategy for creating the smallest (or largest) sum comes from recognizing that the hundreds digit has the most influence on the size of a number, and the ones digit has the least influence on the size of a number.

When we are choosing the digits for the smallest sum, we want the smallest numbers in the hundreds column, the next smallest numbers in the tens column, and the largest numbers in the ones column. So, in our answer we want 1 and 3 in the hundreds column, 4 and 5 in the tens column, and 6 and 8 in the ones column.

When adding two numbers, the order of the numbers does not affect the sum. Thus, the digits in each column can be arranged in 2 ways: with either the larger digit on top or the smaller digit on top. Since there are three columns in the sum, there are a total of $2 \times 2 \times 2 = 8$ solutions. The 8 solutions are shown.

$$\begin{array}{r}146\\+358\\\hline\end{array}\quad\begin{array}{r}148\\+356\\\hline\end{array}\quad\begin{array}{r}156\\+348\\\hline\end{array}\quad\begin{array}{r}158\\+346\\\hline\end{array}\quad\begin{array}{r}346\\+158\\\hline\end{array}\quad\begin{array}{r}348\\+156\\\hline\end{array}\quad\begin{array}{r}356\\+148\\\hline\end{array}\quad\begin{array}{r}358\\+146\\\hline\end{array}$$

In all arrangements, the sum is 504.

(b) Using the strategy from part (a), we can arrange the numbers to find the largest possible sum. This time, we want the largest numbers, 5 and 9, in the hundreds columns and the smallest numbers, 1 and 2 in the ones columns. The remaining numbers are the same digit 4 and 4.

When adding two numbers, the order of the numbers does not affect the sum. Thus, the digits in the hundreds and ones columns can each be arranged in 2 ways: with either the larger digit on top or the smaller digit on top. Since the digits in the tens column are the same, they can be arranged in only 1 way. Therefore, there are $2 \times 2 \times 1 = 4$ possible solutions in this case. The 4 solutions are shown.

$$\begin{array}{r}541\\+942\\\hline\end{array}\quad\begin{array}{r}542\\+941\\\hline\end{array}\quad\begin{array}{r}941\\+542\\\hline\end{array}\quad\begin{array}{r}942\\+541\\\hline\end{array}$$

In all arrangements, the sum is 1483.

(c) We need some logic to narrow down the choices in this example. Since we know the ones digit of the sum is 9, then the ones digits for each of the two numbers being added must have a total of 9 or 19. It is not possible to add two single digits to get 19, so the sum must be 9. The only possible combinations given the digits we have are $2 + 7$ or $7 + 2$.

In the tens column, we need two numbers that add to 3 or 13. The sum of $6 + 7 = 13$, but we have already used 7 in the ones column. There is only one pair of the remaining digits we could use: $4 + 9 = 13$ or $9 + 4 = 13$. This means we carry 1 into the hundreds column, so the sum of the two digits in the hundreds column must be 14.

The remaining digits are 6 and 8, and fortunately the sum of these digits is 14.

As in part (a), we can arrange these digits in 8 different ways. The 8 solutions are shown.

$$\begin{array}{r}642\\+897\\\hline\end{array}\quad\begin{array}{r}647\\+892\\\hline\end{array}\quad\begin{array}{r}692\\+847\\\hline\end{array}\quad\begin{array}{r}697\\+842\\\hline\end{array}\quad\begin{array}{r}842\\+697\\\hline\end{array}\quad\begin{array}{r}847\\+692\\\hline\end{array}\quad\begin{array}{r}892\\+647\\\hline\end{array}\quad\begin{array}{r}897\\+642\\\hline\end{array}$$

Problem of the Week
Problem A
Sorting Socks

Jessie is organizing her sock drawer by arranging her socks in rows.

She notices that if she puts 5 pairs of socks in each row, there will be one row with only 1 pair of socks. Also, if she puts 3 pairs of socks in each row, there will be one row with only 2 pairs of socks.

If Jessie has fewer than 60 pairs of socks, what is the maximum number of socks she could have?

Theme: Number Sense

Problem of the Week
Problem A and Solution
Sorting Socks

Problem

Jessie is organizing her sock drawer by arranging her socks in rows.

She notices that if she puts 5 pairs of socks in each row, there will be one row with only 1 pair of socks. Also, if she puts 3 pairs of socks in each row, there will be one row with only 2 pairs of socks.

If Jessie has fewer than 60 pairs of socks, what is the maximum number of socks she could have?

Solution

One way to solve this problem is to use skip counting. Since there are extra socks in one of the rows, we count up to but not including 60.

If she puts 5 pairs of socks in each row, then the total number of socks in the filled rows could be:

$$5, 10, 15, 20, 25, 30, 35, 40, 45, 50, \text{ or } 55.$$

Since there is another row with only 1 pair of socks, the total number of socks for each of these options is:

$$6, 11, 16, 21, 26, 31, 36, 41, 46, 51, \text{ or } 56.$$

If she puts 3 pairs of socks in each row, then the total number of socks in the filled rows could be:

$$3, 6, 9, 12, 15, 18, 21, 24, 27, 30, 33, 36, 39, 42, 45, 48, 51, 54, \text{ or } 57.$$

Since there is another row with only 2 pairs of socks, the total number of socks for each of these options is:

$$5, 8, 11, 14, 17, 20, 23, 26, 29, 32, 35, 38, 41, 44, 47, 50, 53, 56, \text{ or } 59.$$

We need to find the largest common number in the two lists containing the possible total number of socks. This number is 56.

Therefore, the maximum total number of socks Jessie could have is 56.

Problem of the Week
Problem A
Cribbage Scoring

Cribbage is a card game that is played with a standard deck of 52 cards. There are four suits: clubs ♣, diamonds ♢, hearts ♡, and spades ♠. Each suit has 13 cards identified with either a letter or number as follows: A, 2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q, K. These letters and numbers are called the card's *rank*. Since there are four suits, there are four cards with the same rank in every deck. *Note: In this activity we will ignore the suits and just focus on the ranks.*

To determine the number of points a hand is worth, each player makes combinations using 2, 3, 4, or 5 of their cards. Cards can be used more than once in different combinations. These are the combinations that count for points:

- Each combination that totals 15 is worth 2 points. When making these combinations, an A is worth 1, a J, Q, or K is worth 10, and all number cards are worth their rank. For example:

  $6 + 9 = 15$ is worth 2 points

  $5 + Q = 15$ is worth 2 points

  $2 + 4 + 9 = 15$ is worth 2 points

- Each pair of cards that have the same rank is worth 2 points.

- Each combination of three or more cards that form a sequence of consecutive ranked cards is worth the number of cards in the sequence. For example:

  3, 4, 5 is worth 3 points

  8, 9, 10, J is worth 4 points

Which of the following hands is worth the most points?

Hand A: 5♡, 10♢, J♠, Q♣, and K♠

Hand B: A♠, 6♣, 7♢, 7♡, and 8♣

Hand C: 5♣, 5♡, 5♠, 8♠, and Q♢

Justify your answer.

Themes: Algebra, Number Sense

Problem of the Week
Problem A and Solution
Cribbage Scoring

Problem

Cribbage is a card game that is played with a standard deck of 52 cards. There are four suits: clubs $\clubsuit$, diamonds $\diamondsuit$, hearts $\heartsuit$, and spades $\spadesuit$. Each suit has 13 cards identified with either a letter or number as follows: A, 2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q, K. These letters and numbers are called the card's *rank*. Since there are four suits, there are four cards with the same rank in every deck. *Note: In this activity we will ignore the suits and just focus on the ranks.*

To determine the number of points a hand is worth, each player makes combinations using 2, 3, 4, or 5 of their cards. Cards can be used more than once in different combinations. These are the combinations that count for points:

- Each combination that totals 15 is worth 2 points. When making these combinations, an A is worth 1, a J, Q, or K is worth 10, and all number cards are worth their rank. For example:

  $6 + 9 = 15$ is worth 2 points

  $5 + \text{Q} = 15$ is worth 2 points

  $2 + 4 + 9 = 15$ is worth 2 points

- Each pair of cards that have the same rank is worth 2 points.

- Each combination of three or more cards that form a sequence of consecutive ranked cards is worth the number of cards in the sequence. For example:

  3, 4, 5 is worth 3 points

  8, 9, 10, J is worth 4 points

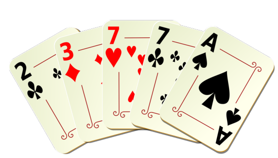
Which of the following hands is worth the most points?

Hand A: 5$\heartsuit$, 10$\diamondsuit$, J$\spadesuit$, Q$\clubsuit$, and K$\spadesuit$

Hand B: A$\spadesuit$, 6$\clubsuit$, 7$\diamondsuit$, 7$\heartsuit$, and 8$\clubsuit$

Hand C: 5$\clubsuit$, 5$\heartsuit$, 5$\spadesuit$, 8$\spadesuit$, and Q$\diamondsuit$

Justify your answer.



Solution

We record the combinations in each hand that earn points in the following tables.

Hand A:

Type of Combination	**Combinations**	**Total Points**
Cards that total 15	5♡+ 10♢, 5♡+ J♠, 5♡+ Q♣, 5♡+ K♠	$2+2+2+2=8$
Pairs of cards with the same rank	none	0
Sequences of consecutive ranked cards	10♢, J♠, Q♣, K♠	4

The total points in this hand is $8+4=12$.

Hand B:

Type of Combination	**Combinations**	**Total Points**
Cards that total 15	7♢+ 8♣, 7♡+ 8♣, A♠+ 6♣+ 8♣, A♠+ 7♢+ 7♡	$2+2+2+2=8$
Pairs of cards with the same rank	7♢ and 7♡	2
Sequences of consecutive ranked cards	6♣, 7♢, 8♣ 6♣, 7♡, 8♣	$3+3=6$

The total points in this hand is $8+2+6=16$.

Hand C:

Type of Combination	**Combinations**	**Total Points**
Cards that total 15	5♣+ Q♢, 5♡+ Q♢, 5♠+ Q♢, 5♣+ 5♡+ 5♠	$2+2+2+2=8$
Pairs of cards with the same rank	5♣ and 5♡, 5♣ and 5♠, 5♡ and 5♠	$2+2+2=6$
Sequences of consecutive ranked cards	none	0

The total points in this hand is $8+6=14$.

Therefore, **Hand B** is worth the most points.