Problem of the Week
Problem A and Solution
Intervals

Problem

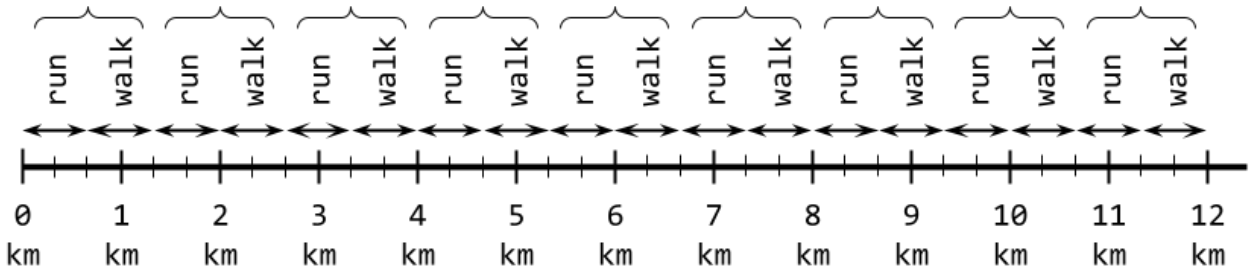
Adil completes a 12 km route using interval training. Interval training is a way to prepare for a race. One interval consists of running for $\frac{2}{3}$ of a kilometre followed by walking for $\frac{2}{3}$ of a kilometre.

(a) How many intervals will he do when completing the 12 km route?

(b) After two weeks, he changes his intervals to running for $\frac{3}{4}$ of a kilometre followed by walking for $\frac{3}{4}$ of a kilometre. At this pace, how many intervals does it take to complete the 12 km route?

Solution

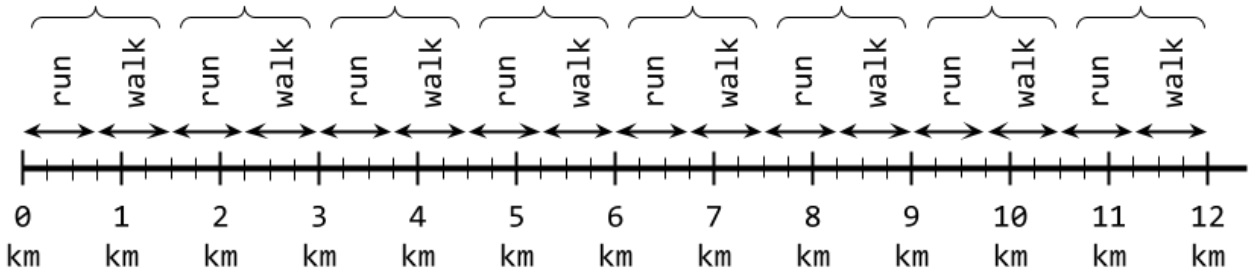
(a) One way to solve this problem is to create a number line from 0 km to 12 km, with a tick at every $\frac{1}{3}$ of a kilometre. Then, two spaces between ticks on the number line will correspond to the running portion of an interval, and two spaces between ticks on the number line will correspond to the walking portion of an interval. Thus, four spaces between ticks on the number line will correspond to one interval in the interval training. (And so, each interval covers $\frac{4}{3}$ of a kilometre.)



Counting the spaces between ticks, we determine that there are a total of 9 intervals necessary to complete the 12 km route.

We don't necessarily have to complete the whole number line to get the final answer. Notice that after 3 intervals Adil has completed 4 km. Since $4 \times 3 = 12$, Adil needs to complete 3 times that number of intervals to finish 12 km. So it takes $3 \times 3 = 9$ intervals to complete 12 km.

(b) We create a number line from 0 km to 12 km, with a tick at every $\frac{1}{4}$ of a kilometre (rather than every $\frac{1}{3}$ of a kilometre). Then, three spaces between ticks on the number line will correspond to the running portion of an interval, and three spaces between ticks on the number line will correspond to the walking portion of an interval. Thus, six spaces between ticks on the number line will correspond to one interval in the interval training.



Counting the spaces between ticks, we determine that there are a total of 8 intervals necessary to complete the 12 km route.

Again, we don't necessarily have to complete the whole number line to get the final answer. Notice that after 2 intervals Adil has completed 3 km. Since $3 \times 4 = 12$, Adil needs to complete 4 times that number of intervals to finish 12 km. So it takes $2 \times 4 = 8$ intervals to complete 12 km.