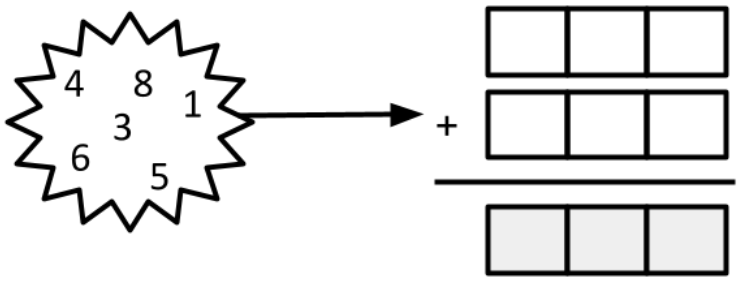
Problem of the Week

Problem A and Solution

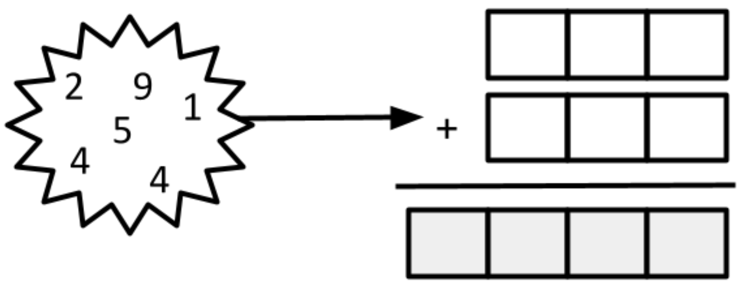
Just Filling In

Problem

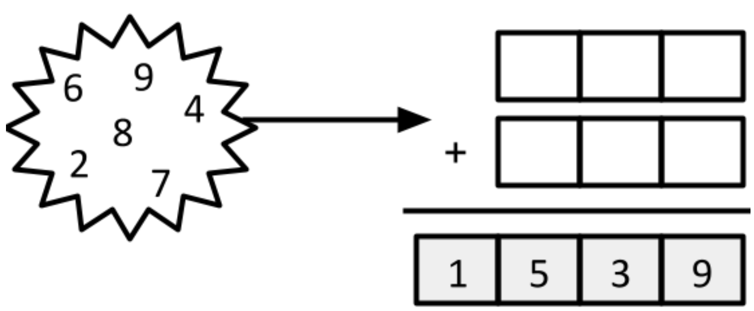
(a) Arrange the numbers 4, 8, 1, 6, 3, and 5 to form two 3-digit numbers with the *smallest* possible sum.



(b) Arrange the numbers 2, 9, 1, 4, 5, and 4 to form two 3-digit numbers with the *largest* possible sum.



(c) Arrange the numbers 6, 9, 4, 2, 8, and 7 to form two 3-digit numbers that add to 1539.



For each part, how many different solutions can you find?

Solution

(a) The strategy for creating the smallest (or largest) sum comes from recognizing that the hundreds digit has the most influence on the size of a number, and the ones digit has the least influence on the size of a number.

When we are choosing the digits for the smallest sum, we want the smallest numbers in the hundreds column, the next smallest numbers in the tens column, and the largest numbers in the ones column. So, in our answer we want 1 and 3 in the hundreds column, 4 and 5 in the tens column, and 6 and 8 in the ones column.

When adding two numbers, the order of the numbers does not affect the sum. Thus, the digits in each column can be arranged in 2 ways: with either the larger digit on top or the smaller digit on top. Since there are three columns in the sum, there are a total of $2 \times 2 \times 2 = 8$ solutions. The 8 solutions are shown.

$$\begin{array}{r} 146 \\ +358 \\ \hline \end{array} \quad \begin{array}{r} 148 \\ +356 \\ \hline \end{array} \quad \begin{array}{r} 156 \\ +348 \\ \hline \end{array} \quad \begin{array}{r} 158 \\ +346 \\ \hline \end{array} \quad \begin{array}{r} 346 \\ +158 \\ \hline \end{array} \quad \begin{array}{r} 348 \\ +156 \\ \hline \end{array} \quad \begin{array}{r} 356 \\ +148 \\ \hline \end{array} \quad \begin{array}{r} 358 \\ +146 \\ \hline \end{array}$$

In all arrangements, the sum is 504.

(b) Using the strategy from part (a), we can arrange the numbers to find the largest possible sum. This time, we want the largest numbers, 5 and 9, in the hundreds columns and the smallest numbers, 1 and 2 in the ones columns. The remaining numbers are the same digit 4 and 4.

When adding two numbers, the order of the numbers does not affect the sum. Thus, the digits in the hundreds and ones columns can each be arranged in 2 ways: with either the larger digit on top or the smaller digit on top. Since the digits in the tens column are the same, they can be arranged in only 1 way. Therefore, there are $2 \times 2 \times 1 = 4$ possible solutions in this case. The 4 solutions are shown.

$$\begin{array}{r} 541 \\ +942 \\ \hline \end{array} \quad \begin{array}{r} 542 \\ +941 \\ \hline \end{array} \quad \begin{array}{r} 941 \\ +542 \\ \hline \end{array} \quad \begin{array}{r} 942 \\ +541 \\ \hline \end{array}$$

In all arrangements, the sum is 1483.

(c) We need some logic to narrow down the choices in this example. Since we know the ones digit of the sum is 9, then the ones digits for each of the two numbers being added must have a total of 9 or 19. It is not possible to add two single digits to get 19, so the sum must be 9. The only possible combinations given the digits we have are $2+7$ or $7+2$.

In the tens column, we need two numbers that add to 3 or 13. The sum of $6+7=13$, but we have already used 7 in the ones column. There is only one pair of the remaining digits we could use: $4+9=13$ or $9+4=13$. This means we carry 1 into the hundreds column, so the sum of the two digits in the hundreds column must be 14.

The remaining digits are 6 and 8, and fortunately the sum of these digits is 14.

As in part (a), we can arrange these digits in 8 different ways. The 8 solutions are shown.

$$\begin{array}{r}642\\+897\\\hline\end{array}\quad\begin{array}{r}647\\+892\\\hline\end{array}\quad\begin{array}{r}692\\+847\\\hline\end{array}\quad\begin{array}{r}697\\+842\\\hline\end{array}\quad\begin{array}{r}842\\+697\\\hline\end{array}\quad\begin{array}{r}847\\+692\\\hline\end{array}\quad\begin{array}{r}892\\+647\\\hline\end{array}\quad\begin{array}{r}897\\+642\\\hline\end{array}$$