Problem of the Week
Problem A and Solution
Secret Message

Problem

Alicia and Bao want to send messages to each other without other people being able to read them. They set up a code that uses just `0`s and `1`s in the message. To decode the message it takes two steps:

1. Read the message, 4 digits at a time. Then look up each 4-digit code in the tables below and replace it with the given lowercase letter.

Code	0000	0001	0010	0011	0100	0101	0110	0111
Letter	a	b	c	d	e	f	g	h

Code	1000	1001	1010	1011	1100	1101	1110	1111
Letter	i	j	k	l	m	n	o	p

2. Read the new message, two consecutive letters at a time. Then look up each pair of lowercase letters in the tables below and replace them with the given uppercase letter. The word formed by these uppercase letters is the secret message.

Lowercase Letters	eb	ec	ed	ee	ef	eg	eh	ei	ij	ek	el	em	en
Uppercase Letter	A	B	C	D	E	F	G	H	I	J	K	L	M

Lowercase Letters	eo	ep	fa	fb	fc	fd	fe	ff	fg	fh	fi	fj	fk
Uppercase Letter	N	O	P	Q	R	S	T	U	V	W	X	Y	Z

For example, if Alicia sent the code `0100111101001011`, after the first step the message becomes `epel`, and after the second step Bao knows the message is `OK`.

Decode the following message:

`0101001001000101010100110101000001000101010000110101010100`

Try writing and sending secret messages of your own.

Solution

The first step in decoding the message is to convert each group of 4 digits into a single lowercase letter. This is shown in the following tables.

Code	0101	0010	0100	0101	0101	0011	0101	0000	0100	0101
Letter	f	c	e	f	f	d	f	a	e	f

Code	0100	0011	0101	0100
Letter	e	d	f	e

Next we take each pair of lowercase letters and decode them into a single uppercase letter, as shown below.

Lowercase Letters	fc	ef	fd	fa	ef	ed	fe
Uppercase Letter	R	E	S	P	E	C	T

So the secret message is RESPECT.

Teacher's Note

This process of decoding a sequence of `0`s and `1`s is actually very close to the way computers store and interpret simple text. Each character on an English language keyboard has a numerical value associated with it known as its *ASCII code*. That ASCII code is represented by an 8-bit code, where a *bit* is either a `0` or `1`. Inside a computer, simple text data is stored as a sequence of `0`s and `1`s.

Another way an ASCII code is represented is by its 2-digit hexadecimal code. Each group of four bits is equivalent to a single digit in the hexadecimal number system. There are 16 digits in the hexadecimal number system (note that there are 10 digits in our decimal number system). The only difference between how ASCII codes work and this problem is that the hexadecimal digit values are: `0, 1, 2, 3, 4, 5, 6, 7, 8, 9, a, b, c, d, e, f`. This problem uses lowercase letters to represent the hexadecimal digits to avoid confusion with the bits of the secret code and actual hexadecimal digits.