



Problem of the Week

Problem A and Solution

Cribbage Scoring

Problem

Cribbage is a card game that is played with a standard deck of 52 cards. There are four suits: clubs ♣, diamonds ♦, hearts ♥, and spades ♠. Each suit has 13 cards identified with either a letter or number as follows: A, 2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q, K. These letters and numbers are called the card's *rank*. Since there are four suits, there are four cards with the same rank in every deck. *Note: In this activity we will ignore the suits and just focus on the ranks.*

To determine the number of points a hand is worth, each player makes combinations using 2, 3, 4, or 5 of their cards. Cards can be used more than once in different combinations. These are the combinations that count for points:

- Each combination that totals 15 is worth 2 points. When making these combinations, an A is worth 1, a J, Q, or K is worth 10, and all number cards are worth their rank. For example:

$$6 + 9 = 15 \text{ is worth 2 points}$$

$$5 + Q = 15 \text{ is worth 2 points}$$

$$2 + 4 + 9 = 15 \text{ is worth 2 points}$$

- Each pair of cards that have the same rank is worth 2 points.
- Each combination of three or more cards that form a sequence of consecutive ranked cards is worth the number of cards in the sequence. For example:

$$3, 4, 5 \text{ is worth 3 points}$$

$$8, 9, 10, J \text{ is worth 4 points}$$

Which of the following hands is worth the most points?

Hand A: 5♥, 10♦, J♠, Q♣, and K♠

Hand B: A♠, 6♣, 7♦, 7♥, and 8♣

Hand C: 5♣, 5♥, 5♠, 8♠, and Q♦

Justify your answer.



**Solution**

We record the combinations in each hand that earn points in the following tables.

Hand A:

| Type of Combination | Combinations | Total Points |
|---------------------------------------|--|---------------------|
| Cards that total 15 | $5\heartsuit + 10\diamondsuit$, $5\heartsuit + J\spadesuit$, $5\heartsuit + Q\clubsuit$, $5\heartsuit + K\spadesuit$ | $2 + 2 + 2 + 2 = 8$ |
| Pairs of cards with the same rank | none | 0 |
| Sequences of consecutive ranked cards | $10\diamondsuit, J\spadesuit, Q\clubsuit, K\spadesuit$ | 4 |

The total points in this hand is $8 + 4 = 12$.

Hand B:

| Type of Combination | Combinations | Total Points |
|---------------------------------------|---|---------------------|
| Cards that total 15 | $7\diamondsuit + 8\clubsuit$, $7\heartsuit + 8\clubsuit$, $A\spadesuit + 6\clubsuit + 8\clubsuit$, $A\spadesuit + 7\diamondsuit + 7\heartsuit$ | $2 + 2 + 2 + 2 = 8$ |
| Pairs of cards with the same rank | $7\diamondsuit$ and $7\heartsuit$ | 2 |
| Sequences of consecutive ranked cards | $6\clubsuit, 7\diamondsuit, 8\clubsuit$ $6\clubsuit, 7\heartsuit, 8\clubsuit$ | $3 + 3 = 6$ |

The total points in this hand is $8 + 2 + 6 = 16$.

Hand C:

| Type of Combination | Combinations | Total Points |
|---------------------------------------|---|---------------------|
| Cards that total 15 | $5\clubsuit + Q\diamondsuit$, $5\heartsuit + Q\diamondsuit$, $5\spadesuit + Q\diamondsuit$, $5\clubsuit + 5\heartsuit + 5\spadesuit$ | $2 + 2 + 2 + 2 = 8$ |
| Pairs of cards with the same rank | $5\clubsuit$ and $5\heartsuit$, $5\clubsuit$ and $5\spadesuit$, $5\heartsuit$ and $5\spadesuit$ | $2 + 2 + 2 = 6$ |
| Sequences of consecutive ranked cards | none | 0 |

The total points in this hand is $8 + 6 = 14$.

Therefore, **Hand B** is worth the most points.