



## Problem of the Week

### Problem A and Solution

### Shape Sums

#### Problem

Each symbol in the grid below represents a number. The sums of the symbols in the first, second, and bottom rows and the leftmost and rightmost columns are shown.

				20
				17
				?
				15
12	?	?	19	

For example, we know:  +  +  +  = 20

Determine all the missing sums.

#### Solution

There are many ways to solve this problem. Here is one approach.

Since the leftmost column has two triangles and two circles, and the column beside it also has two triangles and two circles, the sum of these columns must be the same. So the sum of the second column must be 12.

From the top row we know that the sum of four triangles is 20. So the value of one triangle must be  $\frac{1}{4}$  of 20 or  $20 \div 4 = 5$ .

Since the value of one triangle is 5, then the sum of two triangles is  $5 + 5 = 10$ . The leftmost column shows the sum of two triangles and two circles is 12, so the sum of two circles is  $12 - 10 = 2$ . Therefore, the value of one circle must be  $\frac{1}{2}$  of 2 or  $2 \div 2 = 1$ .

The bottom row has a sum of 15 and contains two circles, one triangle, and one



square. Since we know the sum of two circles and one triangle is  $2 + 5 = 7$ , then the value of one square is  $15 - 7 = 8$ .

Now we know that the sum of the third column is  $5 + 8 + 8 + 1 = 22$ .

The second row has a sum of 17 and contains one triangle, one circle, one square and one hexagon. Since we know the sum of one triangle, one circle, and one square is  $5 + 1 + 8 = 14$ , then the value of one hexagon is  $17 - 14 = 3$ .

Now we know that the sum of the third row is  $1 + 1 + 8 + 3 = 13$ .

The completed grid is shown.

				20
				17
				13
				15
12	12	22	19	

Alternatively, we can find the sum of the third row without actually finding the value of the hexagon. Notice that the second and third rows have the same shapes in the last three columns, and only the shape in the first column is different. Since we know that the value of one triangle is 5 and the value of one circle is 1, then the sum of the third row must be  $5 - 1 = 4$  less than the sum of the second row. Thus, the sum of the third row is  $17 - 4 = 13$ .



## Teacher's Notes

We could look at this problem algebraically, where we are trying to solve a *linear system of equations*. If we assign variable names to the shapes,

- $t$  = triangle
- $c$  = circle
- $s$  = square
- $h$  = hexagon

then we can describe the information given in the problem as follows:

$$\begin{aligned}4t &= 20 \\t + c + s + h &= 17 \\2c + t + s &= 15 \\2t + 2c &= 12 \\t + 2h + s &= 19\end{aligned}$$

Now, we could use algebraic techniques to find the values of these four variables.

We expect that there is a single solution to these types of problems, and it turns out that there is for this problem. However, for some problems we are not given enough information to determine specific values for each variable. For a problem with four variables, a minimum requirement to guarantee a single solution is that we need *at least* four equations. Also, those equations must be *consistent*. This means that none of the equations can contradict each other. For example, if we added the equation  $t = 7$  to the system in this problem, this would contradict the first equation. A system that is *inconsistent* has no solutions. This is something that students will learn more about in higher level math courses.