Problem of the Month
Problem 6: Cobbling cards
March 2026

The card game Cobble (not to be confused with the popular card game Dobble, also known as Spot It) is played with a special deck consisting of finitely many cards, each containing finitely many symbols. A Cobble deck satisfies the following properties:

 (i) Each symbol appears at most once on each card.

 (ii) Each pair of cards has exactly one symbol in common.

 (iii) For any pair of symbols, there is exactly one card on which both symbols appear.

 (iv) In the deck, there is some set of four symbols with the property that no three of them appear on the same card.

We denote each card by square braces, with the symbols listed inside. For example, $[A, B, D]$ denotes the card that contains the symbols A, B, and D. The collection of three cards

$$[A, B], [A, C], [B, C]$$

satisfies Properties (i), (ii), and (iii), but not Property (iv). Therefore, it is *not* a Cobble deck.

1. Construct a Cobble deck with seven cards using the seven symbols $\{A, B, C, D, E, F, G\}$.

2. Prove that there does not exist a Cobble deck with fewer than seven cards.

3. Show that in a Cobble deck, any two cards must each contain the same number of symbols. Conclude that each cord in a Cobble deck contains the same number of symbols.

4. Show that in a Cobble deck, for any two distinct symbols, they must appear on the same number of cards, and that number is the same as the number from Question 3. Conclude that each symbol in a Cobble deck appears on the same number of cards.

5. Prove that you cannot have a Cobble deck with 2026 cards.

Challenge. Construct a Cobble deck with 157 cards, or prove that you cannot do so.