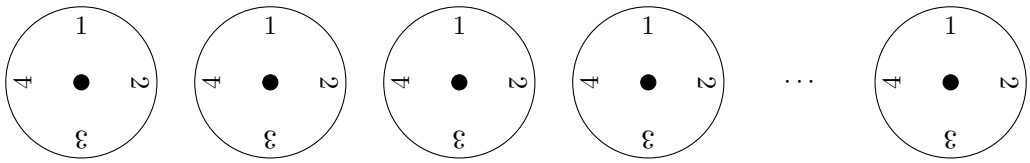
Problem of the Month

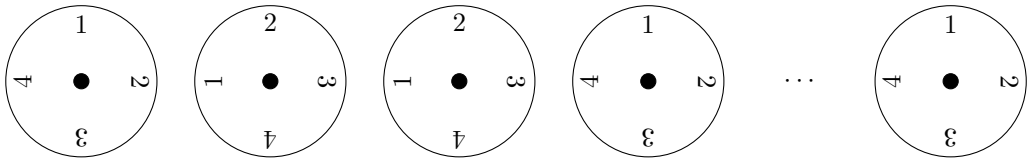
Problem 4: Rotating dials

January 2026

1. Several dials are each labelled with the integers from 1 through 4 in clockwise order. The dials are arranged in a row and initially configured so that each dial shows 1 at the top:



A *move* consists of rotating two adjacent dials in the same direction by the same number of positions. For example, one possible move is to rotate the second and third dials by one position in the counterclockwise direction resulting in the configuration shown:



There are $k \geq 2$ dials and they are initially configured so that each shows 1 at the top. In terms of k, determine how many possible configurations of the dials are attainable by performing a sequence of moves.

2. Suppose now that there is an integer $n \geq 2$ so that each of the $k \geq 2$ dials is labelled in clockwise order by the integers 1 through n beginning with 1 at the top. Find the number of configurations of the dials that are attainable by a sequence of moves. Your answer should be in terms of n and k. Again, a move consists of rotating two adjacent dials in the same direction by the same number of positions.

3. Answer Question 2 with the following additional type of move allowed: rotate the leftmost and rightmost dials in the same direction by the same number of positions.