Problem of the Month

Problem 2: Kaleidoscopes

November 2026

Hint

(a) It may help to graph the line ℓ and draw the point P on the same axes. See if you can guess where $\Gamma_\ell(P)$ lands.

(b) First try to do the question in the case where one of the two points is on the line ℓ. Try to find some congruent triangles. It may help to label the points on ℓ that bisect the line segment joining Q and $\Gamma_\ell(Q)$ and P and $\Gamma_\ell(P)$.

(c) Use Part (b) and an argument involving congruent triangles.

(d) Draw out two lines that make an angle of 30° with each other. Pick a few points and see where they are sent under the composition $\Gamma_{\ell_2} \circ \Gamma_{\ell_1}$. Once you have a guess as to what's going on, see if you can gather evidence for the correctness of your guess by trying a few more points.

(e) (i) There are finitely many points in the orbit of P. Play around with some specific points and lines when $n = 3, 4, 5$, and 6. If ℓ is a line in the plane, what is the transformation $\Gamma_\ell \circ \Gamma_\ell$? With the lines ℓ_1 and ℓ_2 as given in the problem statement, what can you say about $\Gamma_{\ell_2} \circ \Gamma_{\ell_1}$ composed with itself n times? Part (d) is helpful here.

(ii) Write down all the points in the orbit of P and use Parts (b) and (c).